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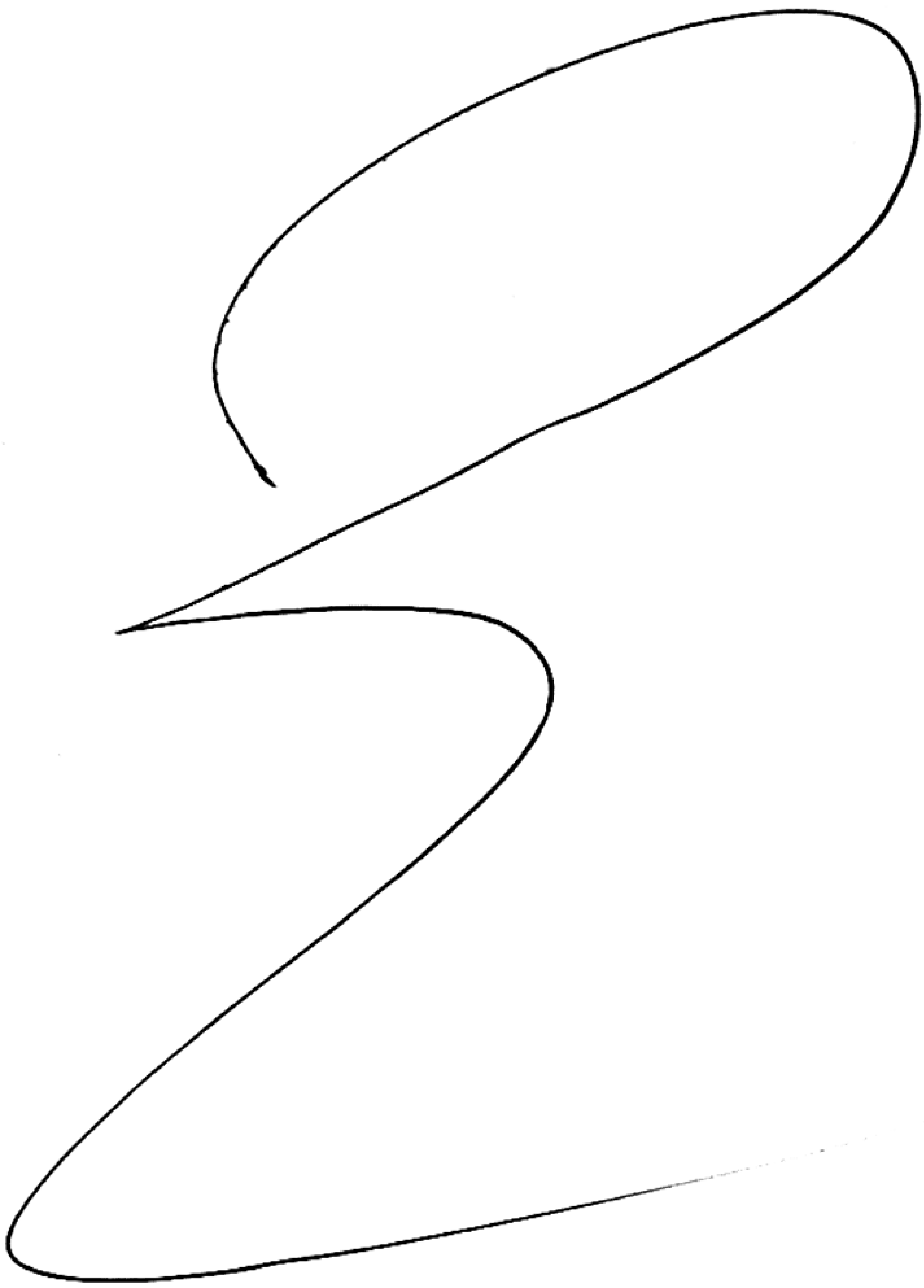

ACOUSTICS

A TEXTBOOK FOR PHYSICS AND ENGINEERING STUDENTS

BY

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PREFACE

For centuries man has been surrounded by sounds in the form of speech, music and even noises, and although the impact on our daily lives is so great, yet there have been few serious attempts to develop and understand the theory of such sounds. It is true to say that in the main the applications of acoustics have preceded theory, and speech, musical instruments and architectural acoustics have been developed chiefly by the experimental method; but it is gratifying to note that the subject is now assuming its rightful place among the physical sciences.

In this book an attempt is made to cover the ground necessary for those preparing for all stages of the General Certificate of Education examination from the ordinary to the scholarship levels; but it is hoped the book will also appeal to a wider field, to include those first-year University students of engineering and others who are interested in the development of the more practical aspects. For this purpose, chapters have been included on topics usually outside examination syllabuses, such as sound recording and reproduction, noise, etc.; and although these topics are not discussed with any degree of completeness, it is hoped that sufficient has been said to stimulate interest in them and to suggest that much still remains to be done in these important fields by acoustic engineers of the future.

It will be obvious to the reader that I am much indebted to several standard text-books on acoustics, notably of course to that great classical work *The Theory of Sound* by Lord Rayleigh. A complete bibliography of works consulted in the preparation of this book will be found elsewhere and I acknowledge with gratitude my indebtedness to these sources. My especial thanks are due to Mr. S. G. Starling for his kindness in allowing me to reproduce diagrams from his *Textbook of Physics*, to Dr. D. R. Griffin of Cornell University, U.S.A., for his interest and practical help in connection with the section on the ultrasonic cries of bats, and to Mr. H. Bagenal for his kind permission to include one of his diagrams on the acoustics of halls.

Messrs. Henry Hughes and Son Ltd. very kindly supplied me with material and illustrations for the sections on echo-sounding

and the ultrasonic flaw detector, and I am indebted to Mr. L. E. A. Bourn, of the John Compton Organ Co. Ltd., for information and illustrations concerning the electronic organ, also to Her Majesty's Stationery Office for permission to use material and illustrations from *Science at War*.

I also acknowledge the permission of various examining bodies to use questions from papers set by them ; the source of each such question is indicated by letters which will easily be recognized.

Finally, I express my sincerest thanks to Mr. A. J. V. Gale for the many suggestions he has made during the preparation of the book, and for the valuable criticism and advice he has given at all stages of the work.

T. M. YARWOOD

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CHAPTER I

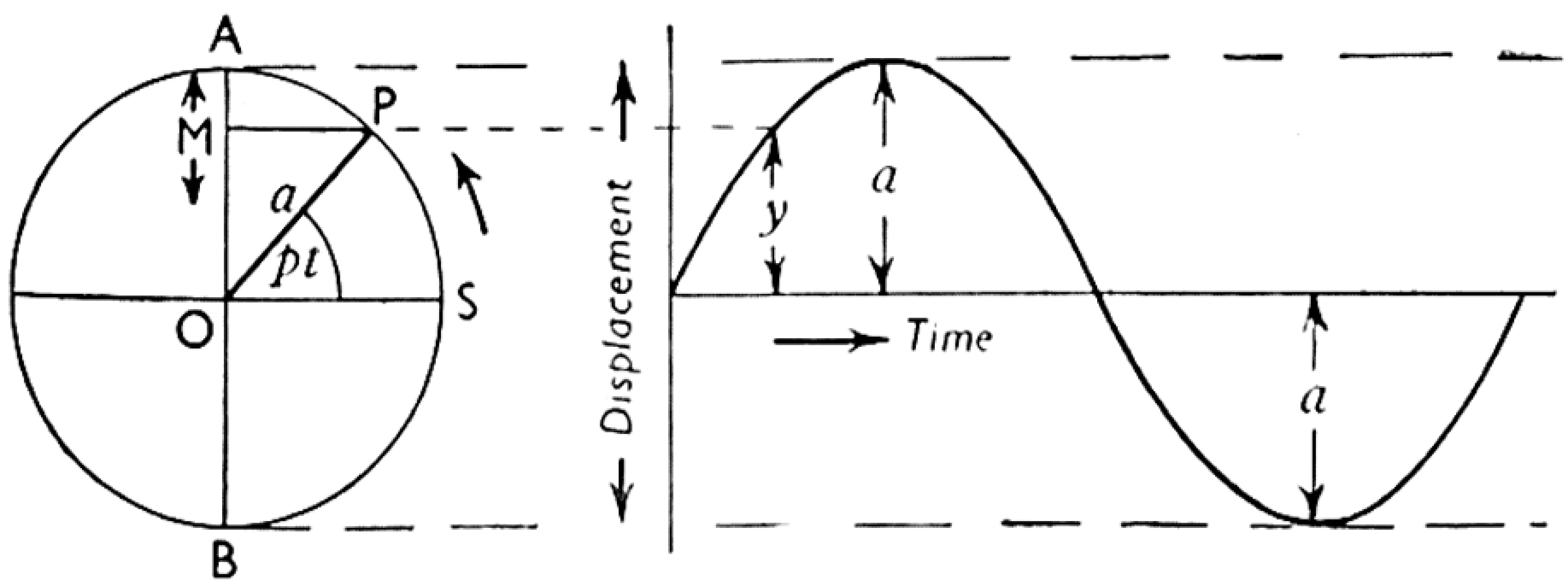
PROPAGATION, TRANSMISSION AND RECEPTION OF SOUND

It is well known that when any object vibrates a sound is emitted, and conversely, when a sound is produced by an object, that object is in a state of vibration. The vibration of the object causes the particles of the air or other medium to vibrate, and by this means the energy is propagated.

The term vibration or oscillation is used so much in sound and other branches of Physics that it is important to understand the nature of such a motion.

SIMPLE HARMONIC MOTION

Consider a point P moving with uniform speed around a circle. The projection of P on a diameter, say AB, of the circle is M, and when P, starting at S, has made a complete rotation, M has travelled along the diameter from O, through A and B and back again. Thus M vibrates backwards and forwards along the diameter AB in a regular fashion.



This particular type of rectilinear vibration which M executes between its extreme positions is called **simple harmonic motion** (S.H.M.) and it may be defined as the orthogonal projection of uniform circular motion.

If the angular velocity of P is p radians per second, the interval between two successive transits of M in the *same* direction through any given position is $2\pi/p$. This is the **period of vibration** (T),

and the reciprocal $p/2\pi$ is the number of vibrations per second or the **frequency** (n).

It is clear from the diagram that the distance OM, which is called the **displacement** of M after a time t, and which may be represented by y, is given by the equation

$$\mathbf{y = a \sin pt,}$$

where a is the radius of the circle of reference and is, of course, the maximum displacement of M. This equation is the simplest equation to represent S.H.M., and if it is treated graphically with t as abscissae and y as ordinates we get a sine curve. For this reason simple harmonic vibrations are sometimes described as *sinusoidal*. From the equation we can deduce the velocity of M, which is

$$v = \frac{dy}{dt} = ap \cos pt.$$

The acceleration of M is therefore

$$\frac{dv}{dt} = \frac{d^2y}{dt^2} = -ap^2 \sin pt,$$

which is equal to $-yp^2$ since $y = a \sin pt$.

As $$\frac{2\pi}{p} = T,$$

we have $$p = \frac{2\pi}{T}.$$

Therefore the equation for S.H.M. can also be written

$$\mathbf{y = a \sin \frac{2\pi t}{T}.}$$

Energy of a particle executing S.H.M. As a particle executing S.H.M. is moving, it will, in general, have a kinetic energy equal to $\frac{1}{2}mv^2$. This energy varies with the velocity, being zero at the end of the motion where the displacement is a maximum and a maximum when the particle is at O. Thus the kinetic energy decreases as the particle approaches maximum displacement and the potential energy must increase. Clearly, the maximum potential energy must equal the maximum kinetic energy, and the total energy of the system may be taken as either the maximum potential energy or the maximum kinetic energy.

Now maximum kinetic energy $= \frac{1}{2}mv^2$

$$= \tfrac{1}{2}mp^2a^2 \quad (\text{since } v = pa)$$
$$= \tfrac{1}{2}\mathrm{m}(4\pi^2)\mathrm{n}^2\mathrm{a}^2.$$

Thus the energy of the vibrating particle is proportional to the square of the frequency (n) and to the square of the amplitude (a).

Also it may be noted that for particles of equal mass having the same energy but different frequencies, the amplitudes must be inversely proportional to the frequencies.

WAVE MOTION

When a sounding object vibrates, for example, a tuning fork, the vibrations of the prongs set up similar vibrations in the air, and a sound **wave** is propagated through the medium.

In a wave motion we have to consider the motion of various particles of the medium, each of which is executing its own periodic motion.

Further, although the form of the wave is similar to the simple harmonic curve, it must be remembered that the harmonic curve represents the successive displacements of a *single* particle, the abscissae representing time, while the wave-form curve represents the simultaneous positions of a *number* of particles, the abscissae being the distance of the mean position of the particles measured from some fixed point. However, since all the particles move in exactly the same way, and in any complete wave-length any single particle is at some time in every phase of the motion, we may look upon the wave curve as also showing the displacement of each particle at different times.

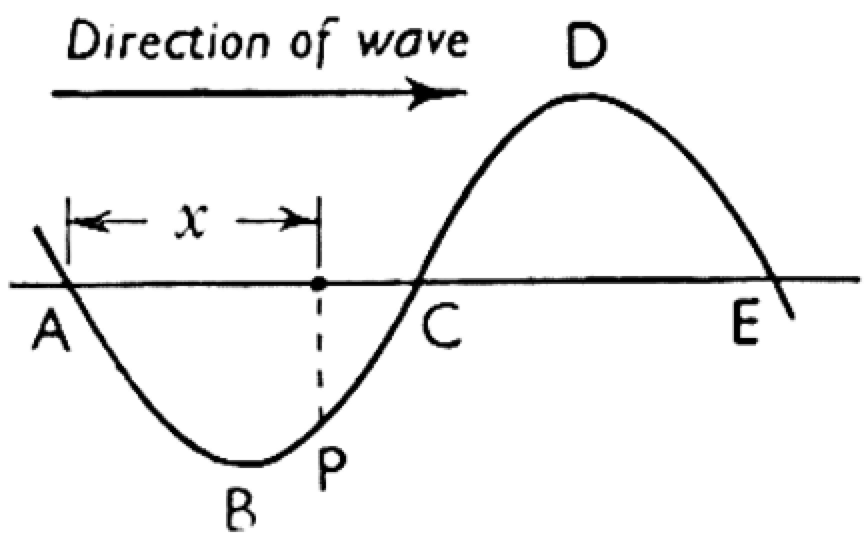
Types of waves. There are two types of wave motion. In one, the particles of the medium move at right angles to the direction of the wave; a typical example of such a motion is a water wave. This type is called a **transverse wave**, and heat, light and electromagnetic energy are transmitted from one place to another by such waves. In the other, the particles of the medium travel forwards and backwards along the direction in which the wave is travelling. This is called a **longitudinal wave**, and it is the type which conveys sound energy through a medium. In the study of sound, however, we shall certainly have to consider transverse waves as well, for these occur in the vibrations of strings, etc. Longitudinal waves are not so easily represented or studied as transverse waves, which will therefore be dealt with first.

TRANSVERSE WAVES

Equation of a simple harmonic wave. Every particle of a medium through which a simple harmonic wave is passing executes a vibration of the type given by the equation

$$y = a \sin pt, \quad \text{or} \quad y = a \sin \frac{2\pi t}{T};$$

but different particles are at different parts of their path, that is, they are out of **phase.** Therefore, the condition of the particles at any particular instant can be represented by a sine curve. Let a complete wave-length be represented by the curve $ABCDE$.



The vibrations of the particles of the medium at A and B are not in the same phase; the particle at B is always a quarter of a period later than that at A. Also the particle at C is half a period later than that at A, and so on; at E the particle is a complete vibration later than A, which means that it is in phase with A. The equation to the motion of the particle at B is therefore

$$y = a \sin \left(\frac{2\pi t}{T} - \frac{\pi}{2}\right),$$

at C it is

$$y = a \sin \left(\frac{2\pi t}{T} - \pi\right)$$

and so on. At E the equation is

$$y = a \sin \frac{2\pi t}{T}.$$

In general, if x is the distance from A, in terms of wave-length, of any particle P in the medium through which the wave is travelling, the equation becomes

$$y = a \sin \left(\frac{2\pi t}{T} - 2\pi \cdot \frac{x}{\lambda}\right),$$

where λ is the full wave-length AE.

$$\therefore \; y = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right). \quad \text{.......................(1)}$$

Now, since $\frac{2\pi}{p}=T$, we have $\frac{2\pi}{T}=p$.

Also, since $\lambda=\frac{V}{n}$ (V is the velocity of the wave), and $\frac{p}{2\pi}=n$, then

$$\frac{2\pi}{\lambda}=\frac{p}{n}\times\frac{n}{V}=\frac{p}{V}.$$

Hence, equation (1) can be written in the form

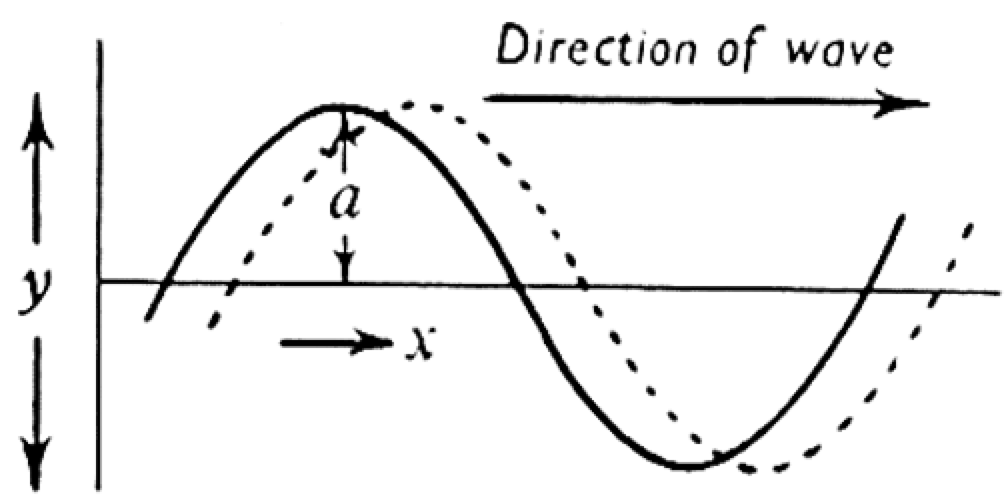
$$y=a\sin 2\pi n\left(t-\frac{x}{V}\right)$$

or

$$y=a\sin p\left(t-\frac{x}{V}\right)\text{ ....(2)}$$

This equation of a wave motion, in either form, is so important and so useful that it is worth while studying it a little further to understand that it really does represent a wave.

If a graph of y against x is plotted, keeping time at a constant value t_C, the result looks the same as that of an ordinary water wave at any instant. This is what we should expect, for the graph *must* be a sine curve.



Full-line graph corresponds to time t_C
Dotted " " " " " t_D

If another graph is drawn on the same diagram corresponding to a different time t_D, it will go up and down in exactly the same way as the previous one ; but the value of x for which the displacement y is zero has changed from x_C to x_D. Hence the wave has moved a distance x_C-x_D in the time t_C-t_D and must therefore have a velocity $\frac{x_C-x_D}{t_C-t_D}=V$.

Further, if we consider the way in which the displacement, y, at any particular value of x varies with time, we find that equation (1) becomes $y=a\sin(pt+k)$, k being a constant, and so the motion must be simple harmonic.

Hence the graph of the equation looks like a wave, it moves like a wave and must be a wave, since a point in the wave moves in a simple harmonic motion.

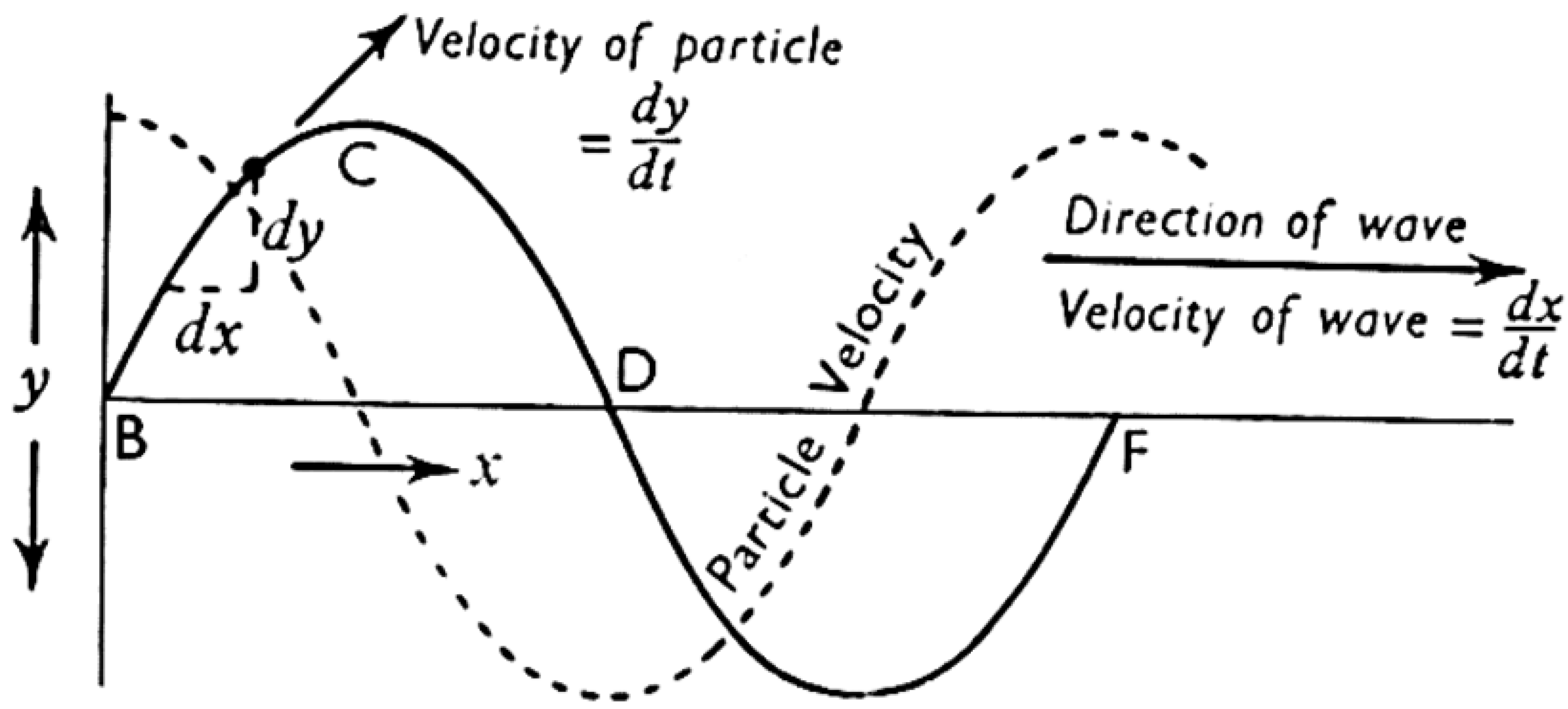
It is the term $\sin pt$ or $\sin 2\pi t/T$ in the equation which indicates

simple harmonic motion, and if the equation has a term of the character

$$\left(t-\frac{x}{V}\right) \quad \text{or} \quad \left(\frac{t}{T}-\frac{x}{\lambda}\right),$$

then this will indicate movement of the wave. This latter statement is of particular importance when we consider stationary waves.

Wave velocity and particle velocity. The **velocity of the particles** of a medium transmitting a wave motion must not be confused with the **velocity of the wave.** In the diagram, the full-line curve is the displacement curve, and it is clear that the velocity of a particle at any instant is dy/dt. But the velocity of the wave at that instant is given by dx/dt.



Hence we have:

$$\frac{\text{particle velocity}}{\text{wave velocity}}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{dy}{dx}$$

which is the slope at any point of the displacement curve.

Since the motion of the particle is given by $y=a\sin pt$, its velocity dy/dt is given by $ap\cos pt$. This is a maximum when $\cos pt=1$. Hence,

$$\text{maximum velocity}=ap=\frac{2\pi a}{T},$$

since

$$p=\frac{2\pi}{T}.$$

Also, $\cos pt=1$ when $pt=0$, π, 2π, etc. Hence, the maximum velocity occurs at B, D, F, etc., on the diagram, and the minimum velocity (zero) at C, E, etc., where

$$pt=\frac{\pi}{2},\ \frac{3\pi}{2},\ \text{etc.},$$

and $\cos pt=0$.

The velocity curve of the particle is shown in the diagram (not to scale) by the dotted line, and it will be seen that it differs in phase from the displacement curve by $\pi/2$. This can also be derived from the wave-equation

$$y = a \sin 2\pi n \left(t - \frac{x}{V}\right).$$

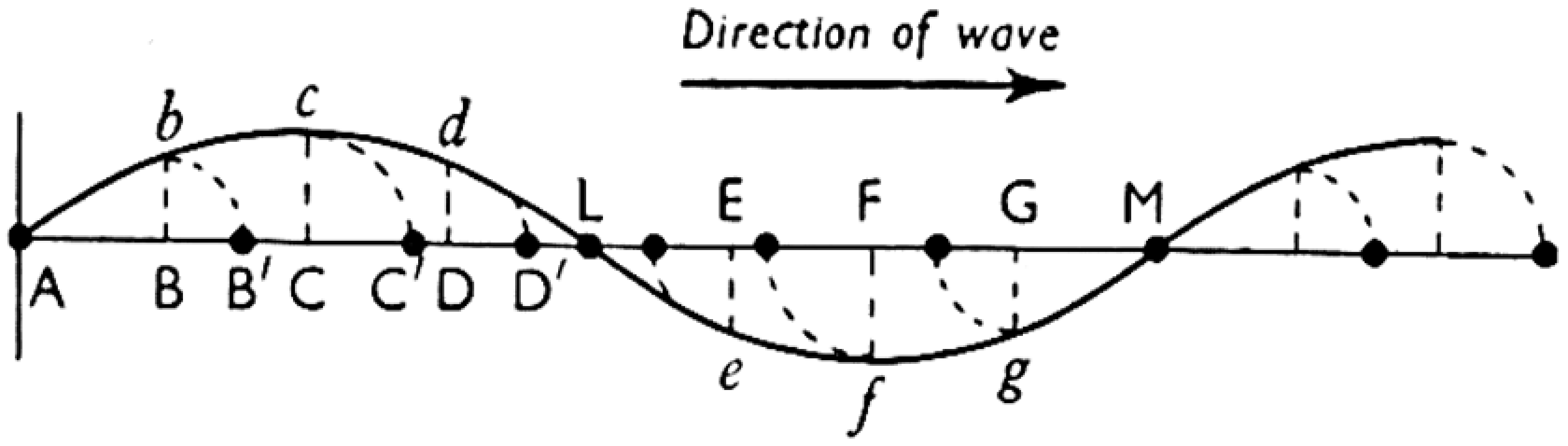
The particle velocity is given by

$$\frac{dy}{dt} = 2\pi n a \cos 2\pi n \left(t - \frac{x}{V}\right)$$

and this represents a cosine curve, which of course differs in phase from a sine curve by $\pi/2$.

LONGITUDINAL WAVES

Sound waves in air are entirely longitudinal ; hence the form of the wave cannot be shown by a sine curve as in the case of a transverse wave, since the particles are moving in the direction of propagation of the wave. A longitudinal wave may, however, be represented diagrammatically to scale by means of a sine curve in the following manner.



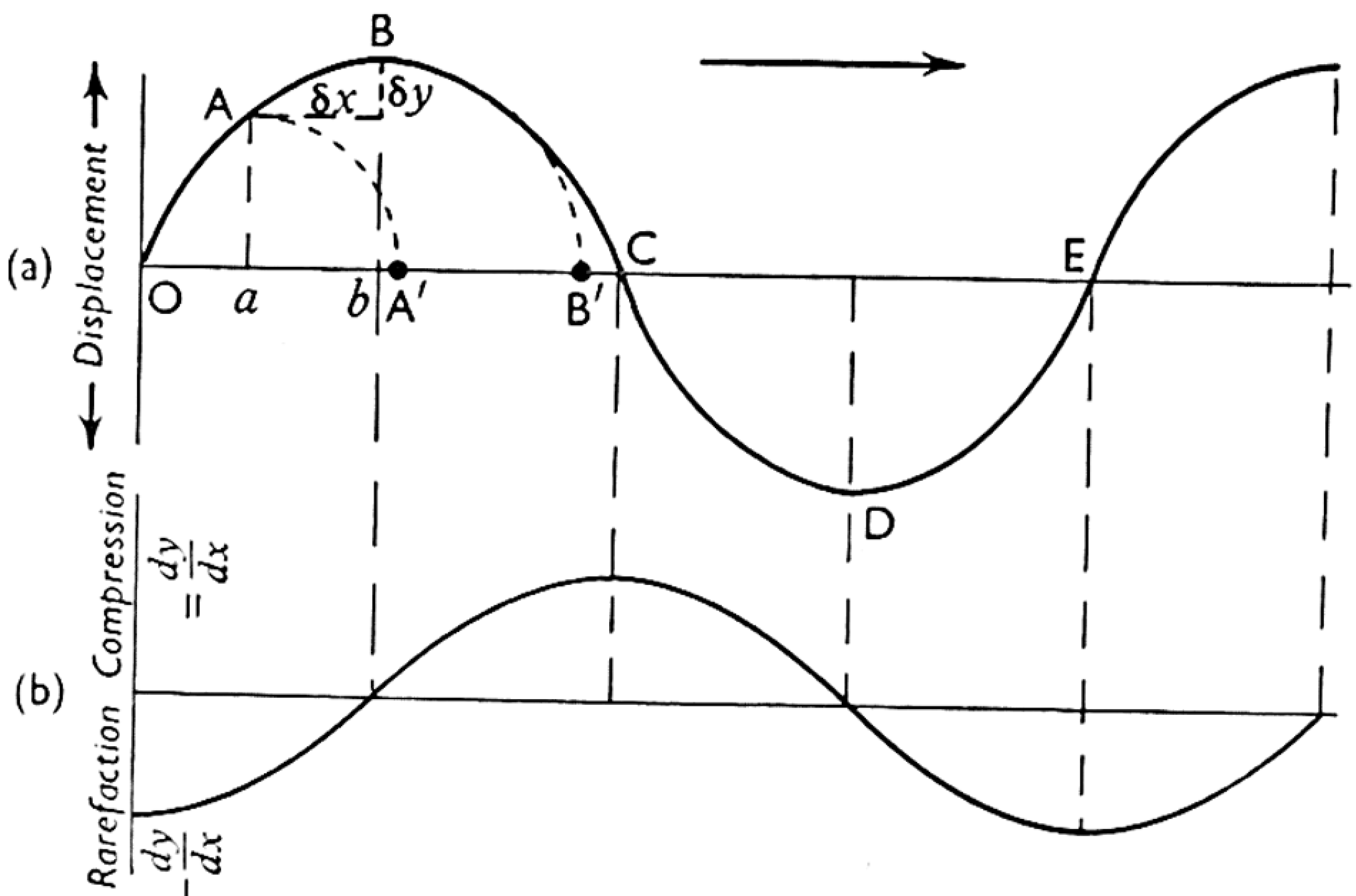
Displacement Curve for Longitudinal Wave

In the diagram, let $ABCDEFG$ be the normal positions of undisturbed particles in a medium. At a given instant during the passage of a wave, their positions may be B', C', etc., that is, they are displaced from their normal positions. If we make

$$Bb = BB', \qquad Cc = CC', \quad \text{etc.},$$

measuring a displacement to the right hand upwards, and to the left hand downwards, we obtain the curve $Abcd$, etc., in which the ordinates represent the displacements of the particles. Such a curve is called a **displacement curve** and is of a sine form, though in the case of an actual air wave the curve is very flat because the displacements of the particles are small. There is no

reason, however, why the displacements should not be represented by ordinates drawn to a larger scale. But whatever the scale the resulting curve is a sine curve ; hence the equation to a transverse wave given on p. 5 may also be used in connection with a longitudinal wave. It will be noticed from the displacement curve that where the curve has a *negative* slope, as at L, the medium is in *compression* ; where the slope is *positive*, as at M, the medium is *rarefied*, and where the curve is horizontal there is neither compression nor rarefaction, the pressure being normal.



Magnitude of Compression and Rarefaction

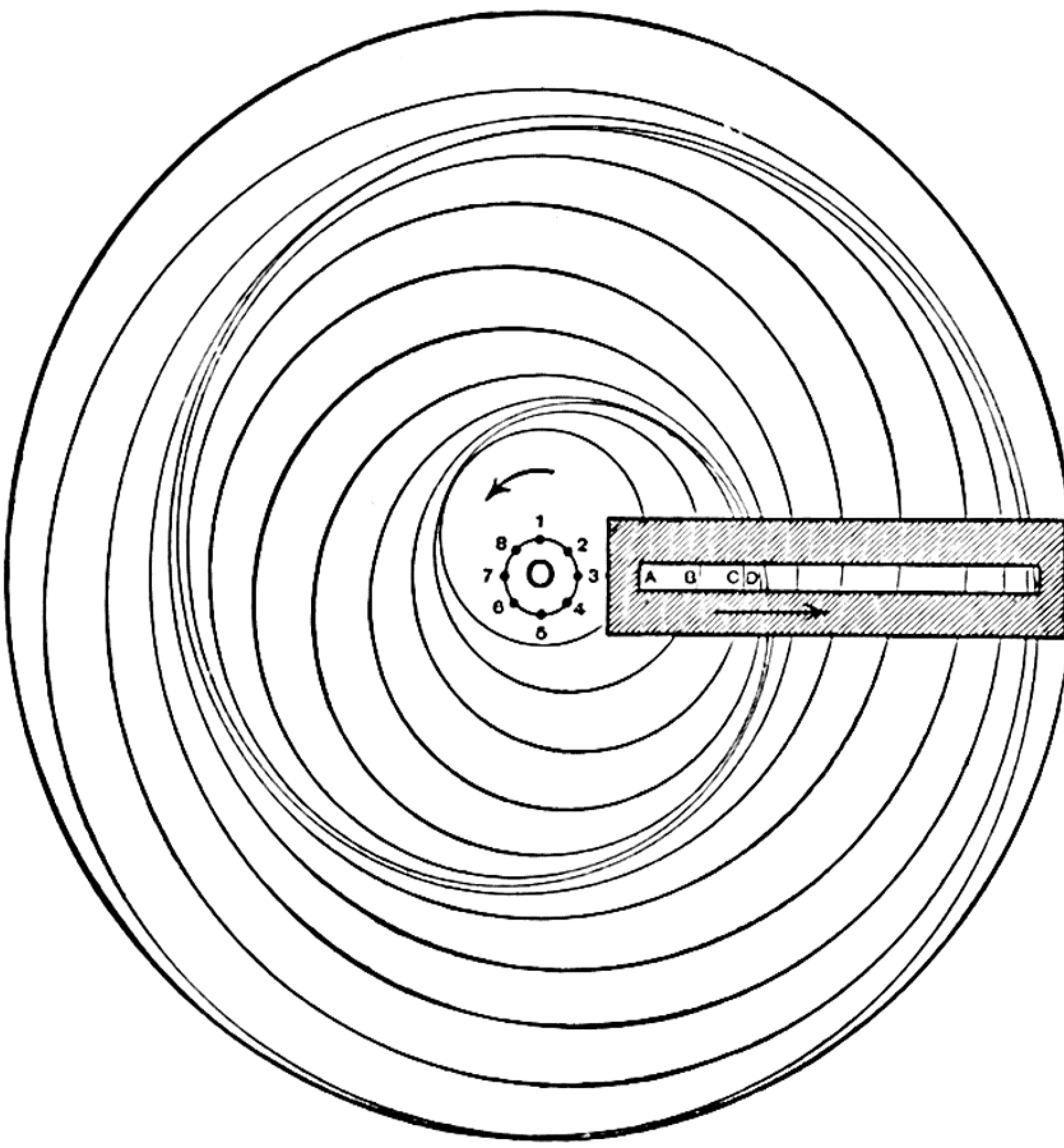
Magnitude of compression and rarefaction. It is possible to draw another curve to represent the amount of compression or rarefaction at each point of a longitudinal wave. Let $OABCDE$ in diagram (a) be the displacement curve for a given wave. Now, if the displacement aA', bB', etc., is represented by y, we have

$$Bb - Aa = \delta y,$$

which is the change in the displacement over the distance ab, or δx. Hence the volumetric strain or the amount of compression or rarefaction is given by $\delta y/\delta x$, and if AB is very small, the *compression is given by dy/dx, which is the slope of the displacement curve.*

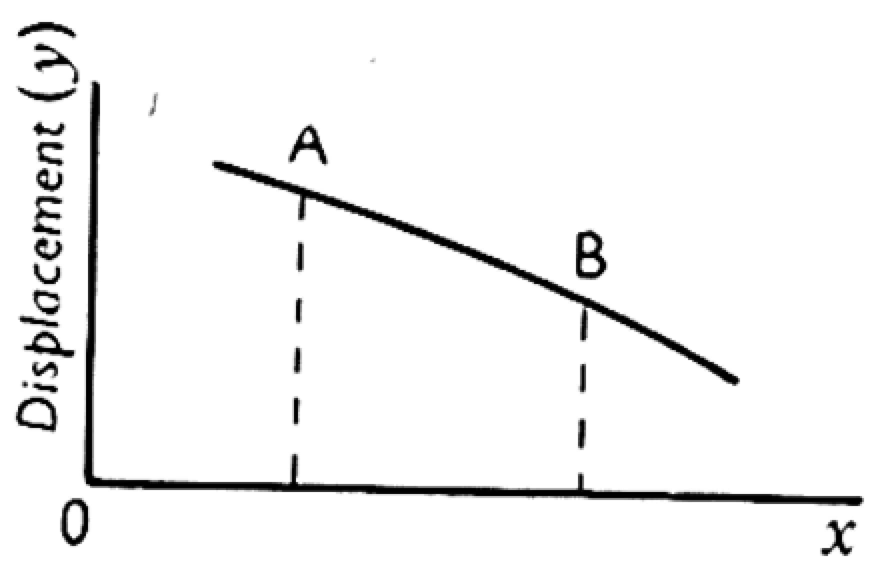
The curve representing the amount of compression and rarefaction is shown in diagram (b).

Crova's disc. The propagation of a longitudinal wave may be illustrated very well by means of **Crova's disc.** A number of circles increasing in radius by small amounts are drawn with centres equally spaced round a small circle as shown in the diagram. A strip of cardboard or metal has a rectangular slot cut in it and is placed with the slot over the circles so that small parts A, B, C, etc., of the circles can be seen. On rotating the disc about the centre O, each small part A, B, C, etc., moves backwards and forwards along the slot over a path equal to the diameter of the circle, 1, 2, 3, etc. Compression and rarefaction waves will then be seen to travel successively along the slot.



Velocity of a sound wave. The velocity of a compression wave can be deduced from a knowledge of the elasticity and density of the medium through which the wave is passing.

Let AB (p. 10) be part of the displacement curve for any compression wave in general, and let k be the bulk modulus of elasticity of the medium through which the wave is passing. Now imagine



two parallel planes drawn in the medium at right angles to Ox, one through A and the other through B. Draw an area s around A and B in these planes, forming a tube of volume $s \, . \, dx$ in the medium, with sides parallel to Ox.

If ρ is the density of the medium, then $\rho s \, . \, dx$ is the mass of the medium in this tube.

The volumetric strain or compression at A is $(dy/dx)_A$, and since

$$k = \frac{\text{stress}}{\text{strain}},$$

the stress (that is, the excess pressure above the normal pressure throughout the medium) at A is $k(dy/dx)_A$. Hence the total force on the end of the tube at A is $sk(dy/dx)_A$. Similarly, the total force at B is $sk(dy/dx)_B$, so that there is a resultant force

$$sk\left\{\left(\frac{dy}{dx}\right)_A - \left(\frac{dy}{dx}\right)_B\right\}$$

on the tube due to the difference in compression at A and B.

Now, $\qquad$ force = mass × acceleration.

Therefore, the acceleration of the medium in the tube is

$$\frac{\text{force}}{\text{mass}} = \frac{sk\left\{\left(\frac{dy}{dx}\right)_A - \left(\frac{dy}{dx}\right)_B\right\}}{\rho s \, . \, dx} = \frac{k\left\{\left(\frac{dy}{dx}\right)_A - \left(\frac{dy}{dx}\right)_B\right\}}{\rho \, . \, dx} \quad \ldots\ldots(1)$$

The particle velocity at A is $V(dy/dx)_A$ (see p. 6), where V is the velocity of the wave, and at B it is $V(dy/dx)_B$. Therefore, the change in velocity of a layer of the medium as the wave moves over the distance dx is

$$V\left\{\left(\frac{dy}{dx}\right)_A - \left(\frac{dy}{dx}\right)_B\right\},$$

and the rate of change in time δt is

$$\frac{V\left\{\left(\frac{dy}{dx}\right)_A - \left(\frac{dy}{dx}\right)_B\right\}}{\delta t}, \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

which is the acceleration.

Hence $$\frac{V\left\{\left(\frac{dy}{dx}\right)_A - \left(\frac{dy}{dx}\right)_B\right\}}{\delta t} = \frac{k\left\{\left(\frac{dy}{dx}\right)_A - \left(\frac{dy}{dx}\right)_B\right\}}{\rho \,.\, dx}$$

or $$V \,.\, \frac{dx}{dt} = \frac{k}{\rho}\,.$$

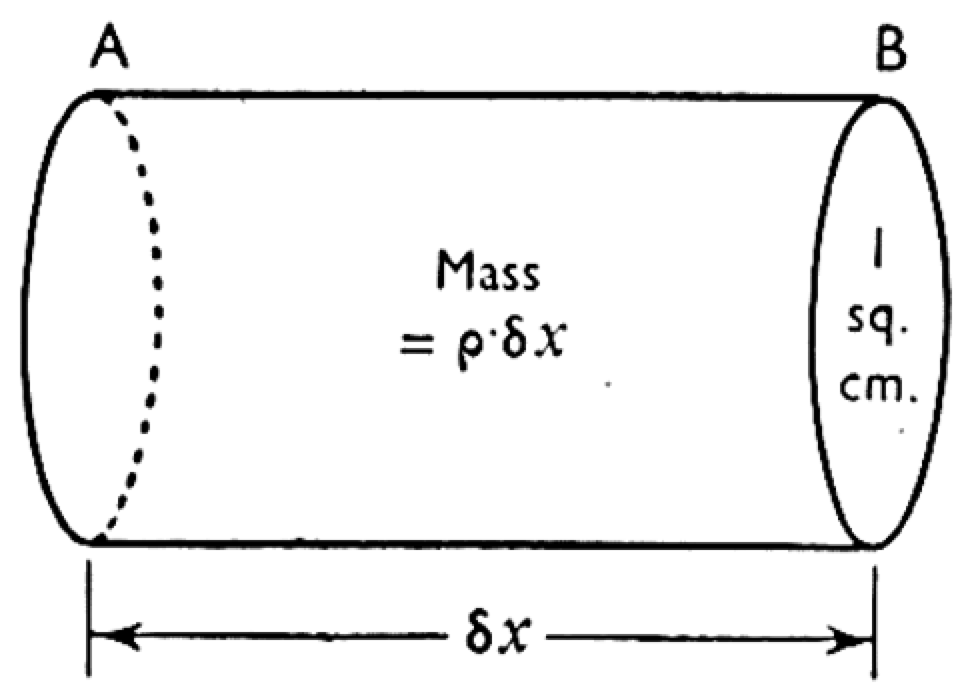
But dx/dt is the velocity of the wave, V.

$$\therefore\ V^2 = \frac{k}{\rho} \quad \text{or} \quad \mathbf{V} = \sqrt{\frac{\mathbf{k}}{\rho}}\,.$$

Alternative proof of $\mathbf{V} = \sqrt{\frac{\mathbf{k}}{\rho}}$. The differential equation which is characteristic of wave-motion is

$$\frac{d^2y}{dt^2} = V^2 \,.\, \frac{d^2y}{dx^2}\,,$$

and assuming this, it is easy to deduce the velocity of a compression wave in a gas.



In the diagram A and B represent two planes in a tube of the medium of unit cross-sectional area, separated from each other by a very small distance δx. Hence, as δx is small, it can be assumed that the rate of variation with distance of a compression at A is the same as at B. Let the displacement at A at a particular instant be dy.

Since the area of the plane is 1 sq. cm., the resulting volume strain at A is dy/dx, and that at B is

$$\frac{dy}{dx} + \frac{d}{dx}\left(\frac{dy}{dx}\right) . \,\delta x$$
$$= \frac{dy}{dx} + \frac{d^2y}{dx^2} . \,\delta x.$$

The difference between these volume strains at A and B multiplied by k, the modulus of elasticity, gives the resultant stress, since $k = \frac{\text{stress}}{\text{strain}}\,.$

Hence the resultant stress $=k\left(\frac{dy}{dx}+\frac{d^2y}{dx^2}\,.\,\delta x-\frac{dy}{dx}\right)$

$$=k\frac{d^2y}{dx^2}\,.\,\delta x\,;$$

and as the area is 1 sq. cm., this is the value of the resultant force between A and B. But the resultant force

$$=\text{mass}\times\text{acceleration}$$
$$=\rho\,.\,\delta x\,.\,\frac{d^2y}{dt^2}\,.$$

$$\therefore\ \rho\,.\,\delta x\,.\,\frac{d^2y}{dt^2}=k\frac{d^2y}{dx^2}\,.\,\delta x$$

or
$$\frac{d^2y}{dt^2}=\frac{k}{\rho}\,.\,\frac{d^2y}{dx^2}=V^2\,.\,\frac{d^2y}{dx}\,;$$

whence
$$V=\sqrt{\frac{k}{\rho}}\,.$$

Use of dimensions. If it can be assumed that V is a function of k and ρ only, an equation connecting these quantities for gases can be obtained by the use of dimensions.

We have: bulk modulus $=\dfrac{\delta p}{\frac{\delta V}{V}}=\dfrac{\text{force per unit area}}{\frac{v}{V}}$.

Now, dimensions of force $=MLT^{-2}$.
Hence, dimensions of elasticity $=MLT^{-2}\times L^{-2}=ML^{-1}T^{-2}$, and dimensions of velocity $=LT^{-1}$.

Suppose velocity $=$ const. $\times$ elasticity$^{\alpha}$ $\times$ density$^{\beta}$.
Using dimensions, we have

$$LT^{-1}=(ML^{-1}T^{-2})^{\alpha}\times(ML^{-3})^{\beta}$$
$$=M^{\alpha+\beta}\,.\,L^{-\alpha-3\beta}\,.\,T^{-2\alpha}.$$

Comparing both sides,
$$\alpha+\beta=0,$$
$$-\alpha-3\beta=1,$$
$$-2\alpha=-1,$$

from which,
$$\alpha=-\beta=\tfrac{1}{2}.$$

$$\therefore\ \text{Velocity}=(\text{const.})\sqrt{\frac{k}{\rho}}\,.$$

In concluding this general discussion on wave motion, it is worth noting that gases, having compressibility only, can only transmit longitudinal waves ; liquids can transmit longitudinal waves and, at their surfaces, on account of surface tension, they can also transmit ripples which are transverse waves. Solids, too, can transmit both longitudinal and transverse waves, since they possess compressibility as well as rigidity and tensile elasticity.

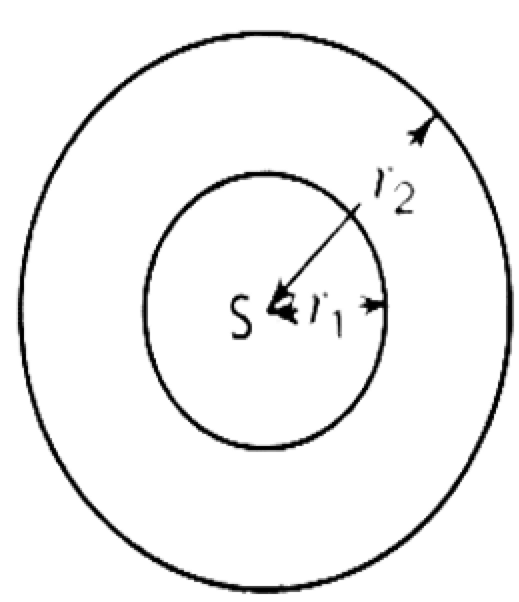
The types of waves most commonly met with in the study of sound are **spherical** waves and **plane** waves. When a sound is emitted by a point source in a homogeneous medium, the energy is transmitted in all directions, the wave-fronts of the waves being spherical. This type is a spherical wave. But if the listener is at a great distance from the source, spherical waves may be regarded as plane waves. The waves passing through a tube of uniform cross-section are also plane waves. By a **wave-front** is meant a surface so drawn that at all its points the wave has the same phase ; that is, the particles of the medium on this surface reach their maximum or minimum displacements at the same time.

Through out density of the medium is Constant.

TRANSMISSION OF SOUND

The propagation of an acoustic wave is accompanied by the transmission of energy through the medium. What happens to the sound energy as it passes through various media and under various conditions will be discussed in subsequent chapters, but we may consider at this stage the energy content of a sound wave at different distances from the source.

Intensity of a sound wave : the decibel scale. When a sound wave emanates from a point source, the energy spreads out uniformly from the source in all directions through a uniform medium, unless, of course, it is interfered with by such processes as reflection and refraction.



Let S be a point source of sound and let I be the **amount of energy per unit area of the medium in one second, the area being normal to the direction of propagation.** The quantity I is called the **intensity** of the energy in the wave.

As the energy spreads out in all directions, the total amount of energy passing through a sphere of radius r_1 is

$4\pi r_1^2 \times I_1$; similarly, if the radius of the sphere is r_2, the total amount of energy is $4\pi r_2^2 \times I_2$.

But the total amount of energy may be assumed to be constant.

$$\therefore\ 4\pi r_1^2 \times I_1 = 4\pi r_2^2 \times I_2$$

whence
$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}.$$

Hence, the **intensity of sound due to a given source varies inversely as the square of the distance from the source.**

The intensity of sound must not be regarded as a mere mathematical quantity of theoretical interest only. It is extremely important in practical acoustics, as, for example, in the consideration of the output of a loudspeaker and in finding the acoustic efficiency of sound generators. Neither must intensity be confused with loudness; the latter is a sensation which certainly depends on intensity, though not entirely, but intensity is a definite and measurable quantity of sound energy. In a plane progressive wave, the intensity I is given by $I = p^2/\rho V$, where p is the root mean square value of the sound pressure, ρ is the density of the medium, and V is the velocity of sound in the medium.

Generally, however, the intensity at any one point is due, not simply to one source of sound, but perhaps to several. In such a case, since it is the sound pressure which produces the sensation of sound, the value of p in the above equation is the root mean square value of the sound pressure at the point concerned. The change in sensation is more nearly proportional to the *fractional* increment of intensity than to the *absolute* increment. Hence, it is convenient in expressing sound intensities to use a scale in which the steps correspond to equal fractional increments, and the one adopted is the **decibel** scale which is based on common logarithms. On this scale, if I_0 and I_1 are two different intensities being compared, the two are said to differ by x decibels where

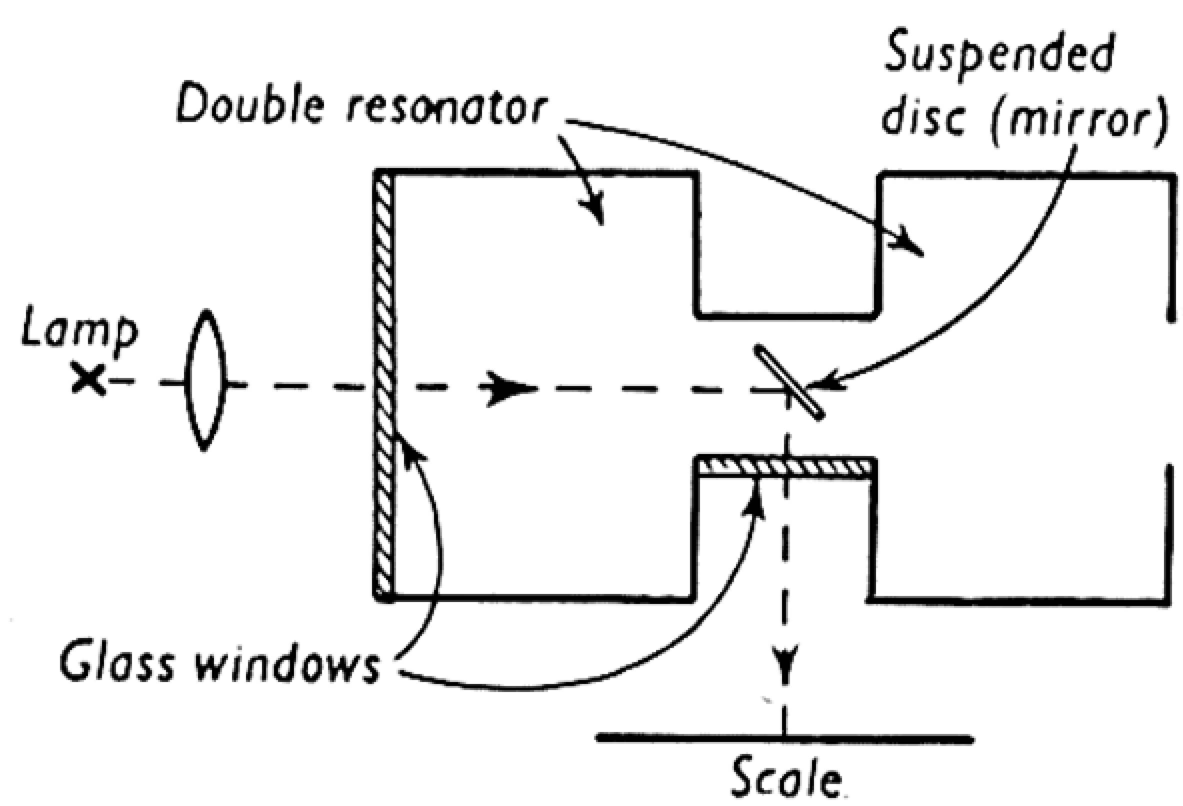
$$x = 10 \log_{10}\left(\frac{I_1}{I_0}\right).$$

It will be noticed that the number of decibels expresses only a *ratio* of intensities; therefore in order to define an *absolute* value of intensity, there must be a datum from which the ratio is to be measured. The value now generally used for this "threshold of audibility" is 10^{-16} watts per sq. cm., corresponding to the

"threshold" at 1,000 cycles per second. Under these conditions, a two-fold increase in intensity corresponds to 3 decibels, a ten-fold increase to 10 decibels and so on, and the whole range of audible intensities from about 10^{-16} watts per sq. cm. to 10^{-3} watts per sq. cm. is covered in 130 decibel steps.

Instruments are made and calibrated so as to read directly the sound intensity levels in decibels above a prescribed zero.

Measurement of intensity. The classical method for the measurement of sound intensity is that employing the **Rayleigh disc.** The basis of this method is that a light disc suspended in a tube through which sound waves are passing tends to set itself so that its plane is at right angles to the direction of motion of the



particles of the medium. It is found that if the disc is suspended and is in equilibrium at a definite angle with the axis of the tube when the sound is not passing, the angle through which it turns on the passage of the waves is proportional to the intensity of the sound. König in 1891 showed that if θ is the initial angle of the disc to the axis of the tube and u^2 is the average value of the square of the particle velocity, the moment tending to decrease θ when the sound passes is given by

$$M = \tfrac{4}{3}\rho_0 a^3 u^2 \sin 2\theta,$$

where a is the radius of the disc and ρ_0 the density of the medium.

It is obvious that M is a maximum when $\theta = 45°$, thus indicating the optimum setting. M can be measured by means of a torsion suspension, and the intensity found by using the value of u^2 so obtained.

Other methods of obtaining the value of intensity are based on pressure measurements. One such method consists in using a

narrow tube with a pin-hole orifice. If this is inserted into a vibratory air column with the other end of the tube connected to one arm of a sensitive manometer, a measurable static pressure difference is recorded when the pin-hole is at a node. Incidentally, this provides a direct experimental method of exploring the pressure distribution in a sound-energy field.

Absorption of sound energy. It was stated on p. 14 that when sound is transmitted through a medium, the power transmission should fall off in accordance with the inverse square law. In practice, however, this is never quite true, for in addition to the normal spreading there is always absorption of energy by the medium. For the causes of the absorption we may look to the viscosity of the medium, heat conduction and, in large-scale transmission, to non-homogeneities in the structure of the medium produced by wind effects, temperature changes and changes in density. The actual damping due to absorption for waves in air has been found to be very small for moderate frequencies, but rises rapidly with increasing frequency. The effect of viscosity is of course of great importance when considering the transmission of sound through the air in tubes, and here, in addition, the loss of energy due to absorption by the walls of the tube must also be considered.

In the transmission of sound waves through water, the damping caused by absorption is not primarily due to viscosity; it is due more probably to the decrease in intensity produced by scattering, which in its turn is caused by the non-homogeneities in the structure of the water. In this case, the absorption causes a more rapid decrease in intensity than would occur with the inverse square law; we shall refer to this point again in Chapter XII.

Scattering of sound. In the previous section, reference is made to the scattering of sound, and a few facts concerning this phenomenon will be helpful. When sound waves meet a rigid obstacle of dimensions small compared with the wave-length of the sound, they are scattered in all directions; hence they do not obey the ordinary laws of reflection.

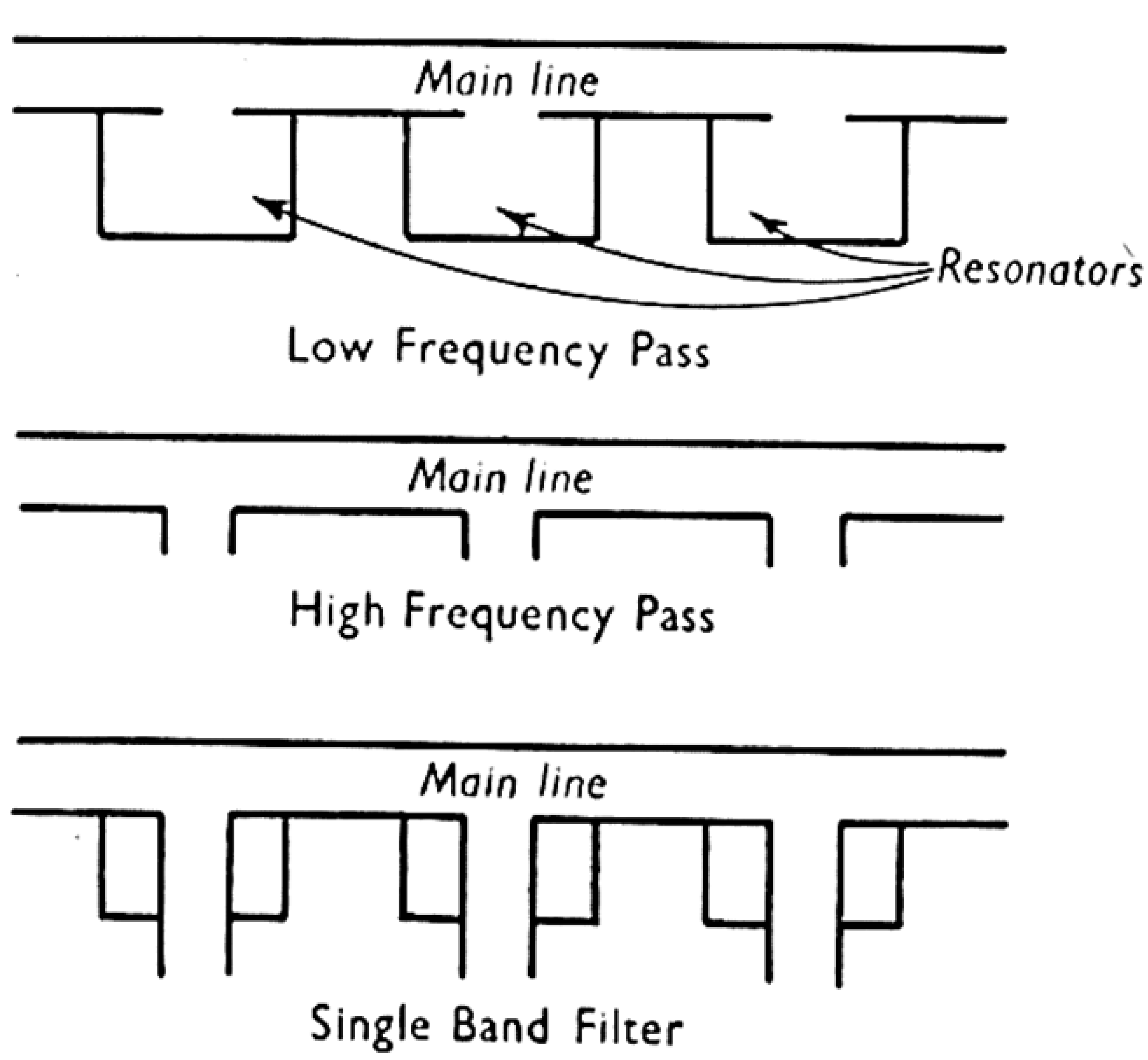
It is found that the *amplitude* (a) of the scattered waves at any point distant from the obstacle is directly proportional to the *volume* (v) of the obstacle and inversely proportional to the square of the *wave-length* (λ), that is, $a = k\frac{v}{\lambda^2}$. It follows from this and the definition of intensity (pp. 3 and 13) that the *intensity* of the

scattered sound varies inversely as the fourth power of the wave-length; this is analogous to the optical law of the scattering of light by very small particles.

It will be seen from the above that the scattering of sound is "selective": the shorter the wave-length the greater the scatterings. Support for this conclusion comes from the fact that during tests, the sound from an aeroplane at the greatest hearing distance is found to be limited to the lowest frequencies in the emitted complex sound.

An illustration of the intensity variation was pointed out by Lord Rayleigh in connection with the so-called "harmonic echoes" (see p. 63). If a complex musical note is sounded near, say, a group of trees, the intensity of the 1st overtone, that is, the octave of the fundamental, in the scattered sound is found to be much increased. In fact, so much, that the scattered sound may thus appear to be raised an octave in pitch.

Filtration of sound. When energy of any type is propagated in the form of a complex wave motion, it is sometimes desirable to eliminate certain constituents of the original wave and allow only energy of definite wave-lengths and frequencies to pass. The process of elimination is known as **filtration**, and it can be applied to sound energy as it is to other forms of wave motion. The problem is to render the channel through which sound is passing selective to certain frequencies or bands of frequencies. This can be done in various ways. For example, the Quincke tube (see



Acoustic filters

p. 80) eliminates the transmission of sounds of definite frequencies, namely, those for which the difference in length of the two parallel branches is an odd multiple of $\lambda/2$ and those for which the sum of the two branch lengths is equal to any integral multiple of λ. It may be noted that this tube is highly selective for wave-length in contrast to a filter, which removes a relatively large band of frequencies.

Acoustic filters can be divided into three classes: (1) low-frequency pass filters, (2) high-frequency pass filters, and (3) filters of the single-band type. A series of Helmholtz resonators (see p. 131) attached to the main channel through which the sound is passing will constitute a low-frequency pass filter. For the high-frequency pass type the branches might consist of simple orifices in the main line; while to obtain the single-band type of filter a combination of the other two types can be used. Similar filters may be employed with liquid media instead of air or other gases. Even a metal rod through which longitudinal waves are transmitted may show filtering properties if the rod is loaded with masses at regular intervals; the rod serves as the acoustic line and the attached masses as branches.

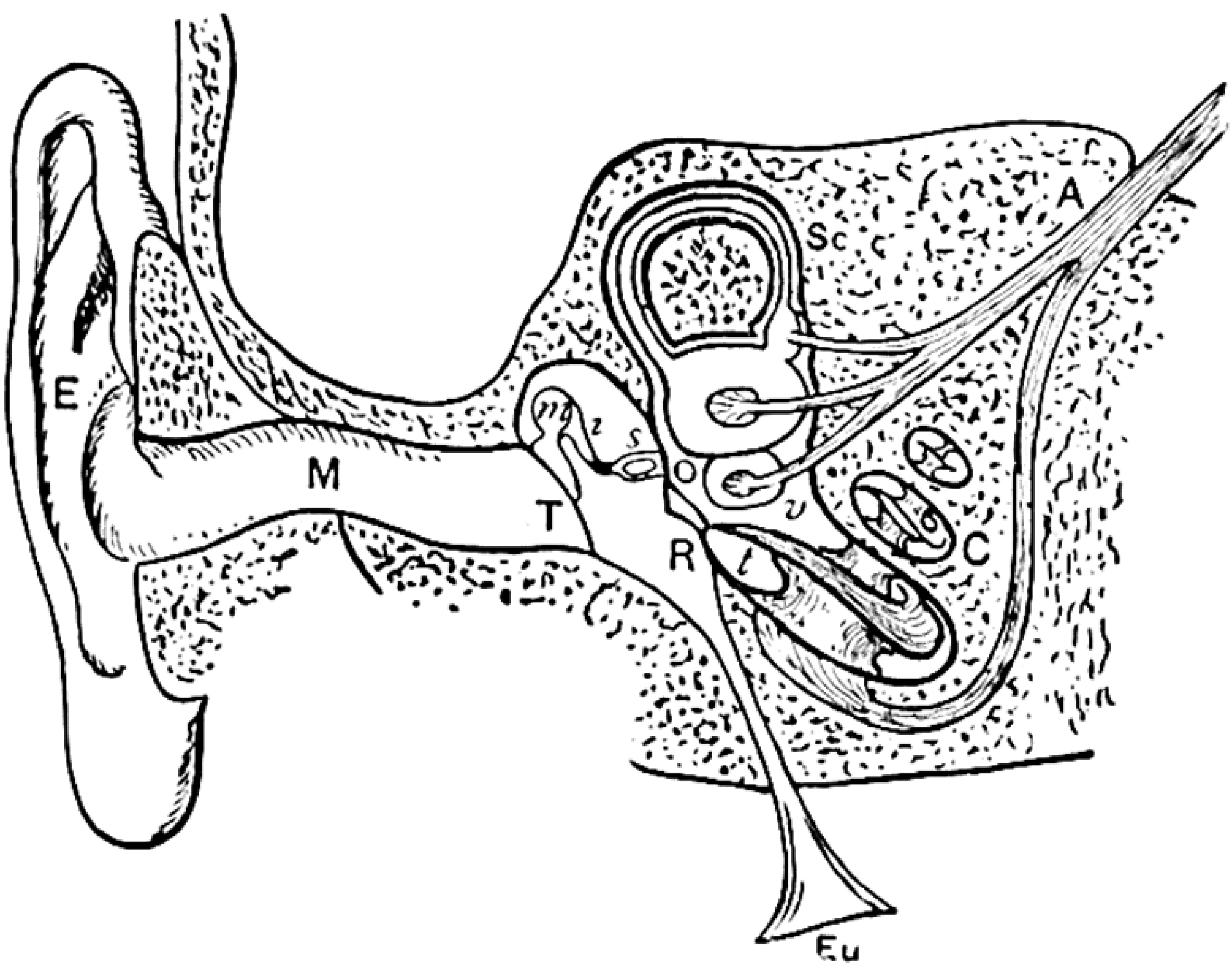
Acoustic filters are used for purifying sounds, for example, removing harmonics from a complex wave form to obtain a pure tone, and in connection with various speech transmission devices.

RECEPTION OF SOUND

In the preceding sections we have briefly discussed the propagation and the transmission of sound: reference must now be made to its reception.

Before any sound can be recorded there must be a receiver, and the choice of a receiver must depend on the medium of transmission; for example, a receiver suitable for air is generally quite unsuitable when the medium is liquid or solid. Further, the selection of a receiver will depend on the frequency and wave-form of the vibration. Sometimes it is important that maximum response to weak signals should be obtained in the receiver, and distortion of the wave-form may be of secondary importance. On the other hand, it may be desirable to obtain a faithful reproduction of the sound wave without distortion, and energy considerations are relatively unimportant. Hence, receivers may be divided into two classes, **resonant** and **non-resonant** receivers. Generally, the former type is used where maximum sensitivity and efficiency are required, and the latter type for faithful reproduction.

The ear. By far the most important receiver of sound is the ear. This organ consists of three parts, the **external** ear by which the sound energy is collected, the **middle** ear or **tympanum** or **drum**, through which the energy is transmitted to the third part, the **internal** ear or **labyrinth.** The external part of the ear is called the *concha*, and it may be noted in passing that in many animals this external appendage is capable of being turned into different positions to assist in determining the direction in which the sound wave is coming.



Structure of the Human Ear

The passage represented by *M* in the diagram connecting the outer ear to the middle ear is closed by the *tympanic membrane*, *T*, often called the *drum* of the ear. The drum proper, however, is the cavity bounded by the membrane on one side and on the other by bony walls except at two places *O* and *R* across which membranes are stretched. *O* is called the fenestra ovalis and *R* the fenestra rotunda. The drum is also in communication with the upper part of the throat by means of the Eustachian tube, *Eu*, which serves to keep the air pressure equal upon the two sides of the tympanic membrane. In the middle ear there is a chain of three small bones (*m*, *i*, and *s* in the diagram) linked with one another, which are connected at one end with the tympanum and at the other end with the fenestra ovalis. Helmholtz has shown that this little chain of bones forms a system of levers, by means of which the movements of the tympanum are diminished in extent, but increased in force in the ratio of 2 to 3.

The internal ear is the real seat of audition, and it comprises the parts called the labyrinth, the semicircular canals, and the cochlea. The labyrinth consists essentially of three semicircular canals of which one, *Sc*, is shown in section in the diagram. All the cavities in the internal ear are lined with delicate membranes and filled with fluid called lymph. The **cochlea** is a spiral canal, shown in section at *C*, and in the cochlea there is an organ called Corti's organ, consisting of innumerable nerve fibres which are an extension of the auditory nerve *A*.

When a compression wave arrives at the tympanic membrane it sets it in vibration, which vibration is transmitted onwards by the three small bones. Thus the fenestra ovalis is set in vibration, and the waves travel through the lymph round the very complex path in the internal ear, so stimulating the nerve fibres and producing the sensation which the brain records as sound. The complete process of audition is not yet fully understood. Helmholtz put forward the ingenious hypothesis that each fibre in Corti's organ was selective to a definite frequency ; hence a composite sound falling upon the ear could be analysed or disentangled by this organ into its constituents. Although this theory has not been altogether upheld, it is certain that the ear does possess this wonderful power of analysis.

Further, it has been established that tones may exist in the ear itself which do not exist in the stimulating tone. For example, Fletcher introduced simultaneously into the ear two pure tones, one a strong one of frequency f and another of variable frequency and adjustable intensity. It is evident that if the first tone generates harmonics in the ear, then the second one will produce beats when its frequency is adjusted to be near $2f$, $3f$, etc. As a result of his investigations, Fletcher found that these internally generated harmonics, which were called **aural harmonics**, undoubtedly exist.

Also, Wegel and Lane, using tones of frequency 1,200 (f_1) and 700 (f_2) c.p.s., detected **aural combination tones**, made up of the sums and differences of the two tones, as follows.

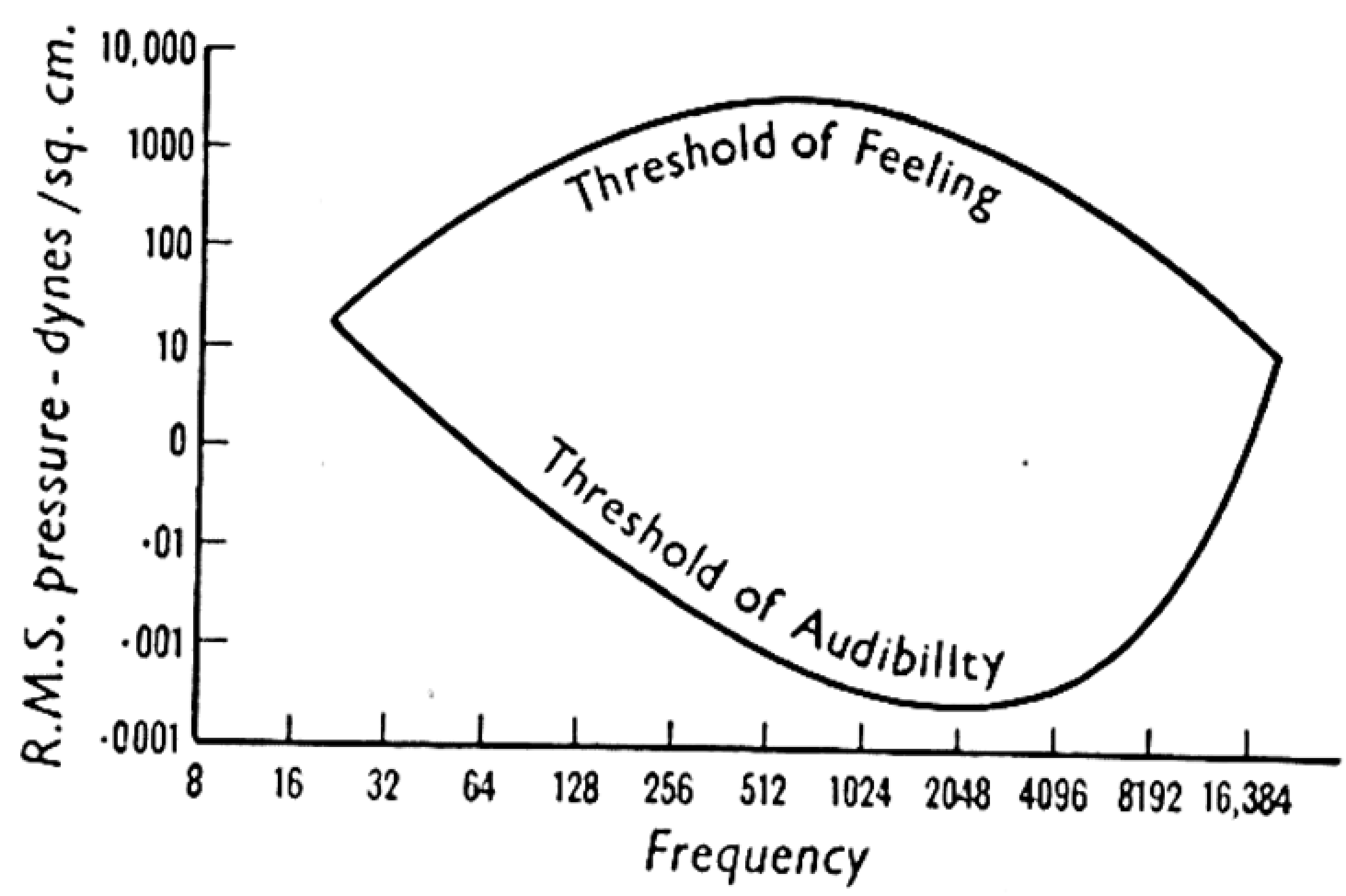
1st order summation and difference tones	1,900, 500.
2nd order difference tones	$200 = (2f_2 - f_1)$
	$1{,}700 = (2f_1 - f_2)$.
2nd order summation tones	$2{,}600 = (f_1 + 2f_2)$
	$3{,}100 = (2f_1 + f_2)$.
Higher order tones	$4{,}300 = (3f_1 + f_2)$
	$3{,}800 = (2f_1 + 2f_2)$.

That these tones *are* produced in the ear itself is proved by the fact that they are not strengthened by a resonator.

Sensitivity of the Ear. A sound may be too faint to be heard or it may be intense enough to cause pain; hence there is a maximum and a minimum pressure limit for audibility. So far as the minimum audible pressure is concerned, the generally accepted pressures are given in the following table for various frequencies.

Frequency	64	128	256	512	1024	2048	4096
Threshold pressure in dynes/cm.²	0·12	0·21	0·0039	0·001	0·00052	0·00041	0·00042

The diagram indicates the range of the average human ear with regard to both intensity and frequency. The upper curve gives the sound-pressures (root mean square) which produce a sensation of *feeling*, and serves as an upper limit to the range of auditory



sensation, while the lower curve indicates the pressures at the threshold of audibility. It would appear then that the ear is most sensitive in the region of frequency 500 to 5,000 c.p.s. Also the curves suggest that persons requiring a pressure of 1 dyne/cm.² can usually follow ordinary conversation, but if the pressure required is round about 10 dynes/cm.², some form of artificial aid to hearing is necessary.

There is also for every frequency a minimum perceptible intensity difference. In this connection, independent investigators have concluded that near a frequency of 2,000 the ear can distinguish, under favourable conditions, from 300 to 400 gradations of loudness between the threshold of audibility and the

threshold of feeling, each step being recognisable by the ear as just perceptibly louder than the one before it.

Audible limits of frequency. The lower and upper limits of frequency for tones audible to the human ear vary according to different observers. The average ear can record sounds of which the frequency lies between about 20 and 20,000. Sounds with a frequency lower than 20 may be heard, but they cannot be distinguished as definite notes; in fact, there is probably no lower limit of audibility for a note, since, as the pitch becomes very low, the note merely becomes resolved into its separate constituents. Sounds with a frequency greater than 20,000 give the listener the impression of a hissing sound, and generally older people cannot hear such a high note as those who are younger; the upper limit of frequency which can be heard decreases with age, being highest for children. Very high and very low pitched sounds of great intensity are felt rather than heard.

The ears of some animals, particularly the dog and the bat, are more sensitive to the high-frequency sounds than the human ear. The frequencies of the notes used in music lie between about 30 and 5,000, and the lowest and highest notes given by a piano have frequencies of about 27 and 3,500 respectively.

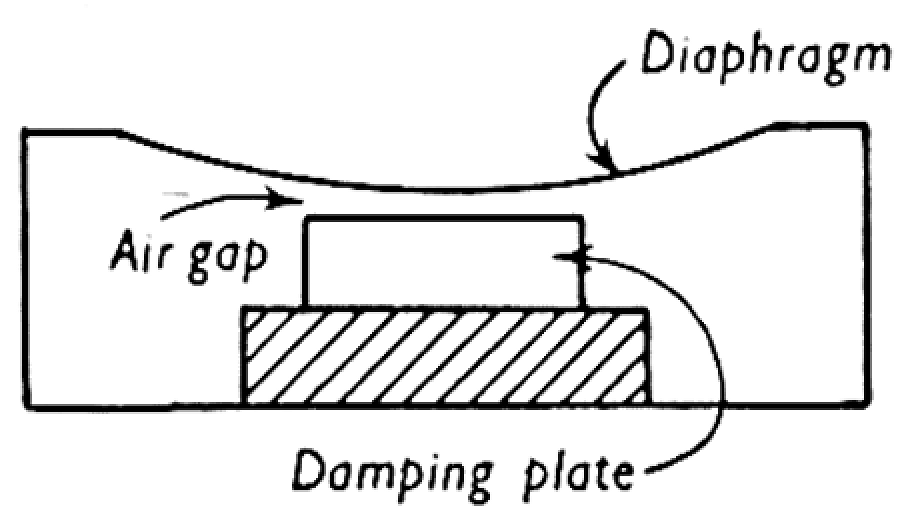
Binaural audition. It is well known that having two ears enables us to localise the direction of a source of sound to a distinct extent. The direction of a sound proceeding from the right or the left is readily determined with fair accuracy, but there is little difference observable between a sound approaching from behind or ahead. For high-pitched sounds of short wave-length, the directional effects can probably be explained by the difference of intensity of the sound reaching the two ears; but when the wave-length of the sound exceeds the perimeter of the head, the intensity difference must be very small and another explanation must be sought. Lord Rayleigh came to the conclusion that the effect depends rather on the phase difference of the sounds as they reach the two ears. The origin of the sound was always attributed to that side on which the phase is in advance. The phase-difference effect is very important in all binaural acoustic finders, but there is little doubt that there is a psychological aspect to the problem as well as a purely physical one. A further reference to binaural audition will be made in Chapter XII.

Hot-wire Microphone. The general type of resonance instrument for receiving sounds consists of a small sensitive object (a diaphragm, disc or hot wire) put in the mouth of a resonator.

One of the most sensitive forms of resonant receiver for sounds

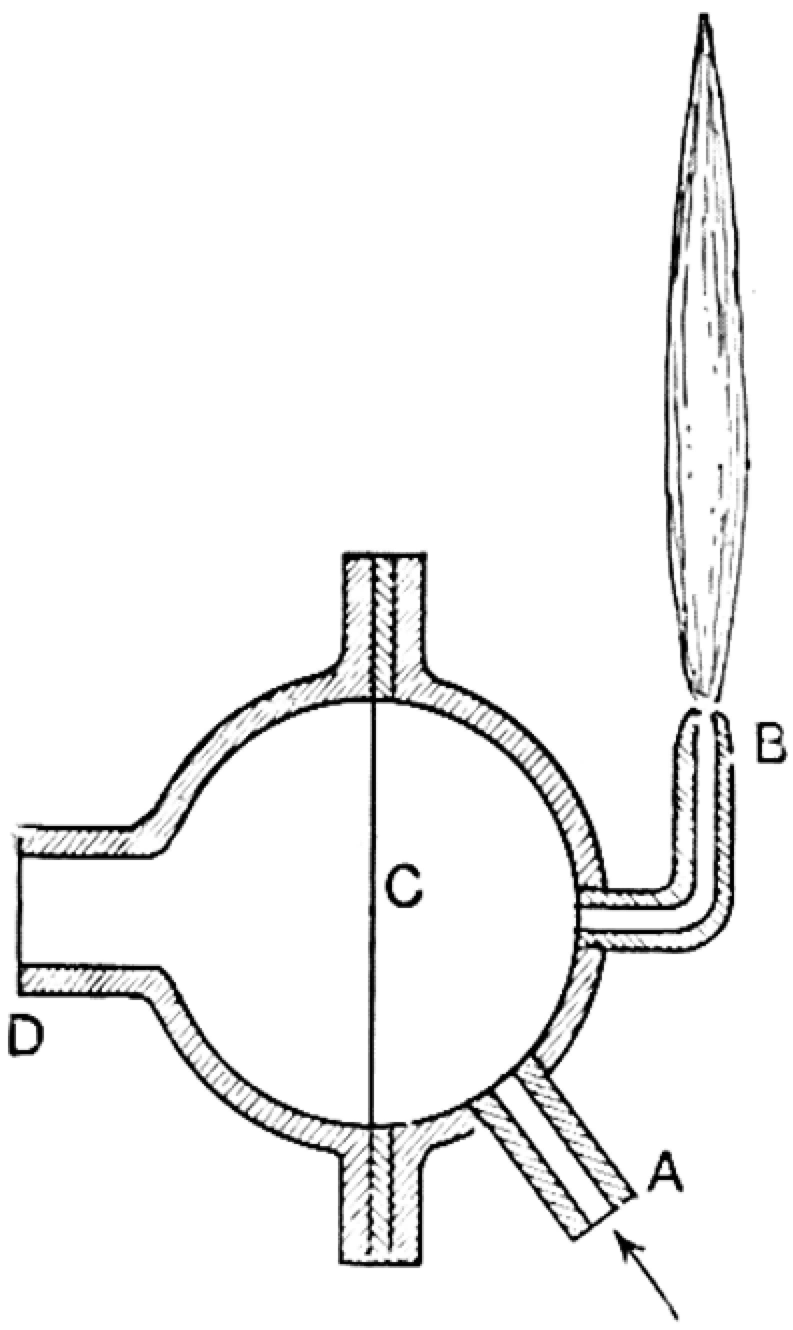
in air is a Helmholtz resonator fitted with a hot-wire microphone. This instrument, devised by Tucker, consists essentially of an electrically heated grid of fine platinum wire (0·0006 cm. diameter) placed in the neck of a Holmholtz resonator. The oscillating air currents in the neck at resonance cool the hot grid, the extent of the cooling being measured, by a Wheatstone bridge, as a change in electrical resistance; this resistance-change is a measure of the intensity of the sound. The instrument is calibrated by observing the cooling produced by steady air streams of known velocity. The sensitivity increases with increase of the heating current, and it is also greater towards the lower frequencies (say 100 per sec.); hence, it has proved itself of great value when used with resonators of low frequency for detecting and locating sounds from distant guns.

Non-resonant receivers. Non-resonant receivers are, as a general rule, much less sensitive than the resonant type at a particular frequency, but they have a good average sensitivity over a wide range. One good example of such a receiver for use in air is **Wente's condenser microphone.** The instrument consists essentially of a tightly stretched thin steel diaphragm (0·001 in. thick) separated from a parallel "damping" plate by an air gap of 0·001 in. approximately. The two plates form an electrical condenser, the capacity of which is varied when vibration takes place, and when a steady voltage is applied to the condenser the vibration results in a fluctuating electromotive force. The output in millivolts per dyne/cm.^{2} is fairly constant over a frequency range of 500–5,000 cycles per sec.; and this freedom from resonance over such a wide range of frequencies renders it most valuable for purposes of sound analysis. This microphone is, however, rather insensitive when compared with, say, the more familiar granular type.



Piezo-electric receivers are perhaps the best examples of the non-resonant type, for their natural frequencies are always very high (10^4–10^6 cycles per sec.; see p. 26). Such receivers are most suitable for use in a medium like water; in air they are rather insensitive.

Sensitive flames. These are very convenient detectors of high-frequency sounds in air and are commonly used to demonstrate the presence of nodes and antinodes in stationary waves of high



frequency, also to indicate the position of the sound focus of a concave reflector. To obtain a sensitive flame, gas is allowed to enter a small chamber by the pipe A, and it leaves by the jet B where it burns, forming a long thin flame. One boundary of the chamber is a thin india-rubber membrane C, and when sound energy from some source enters at D the variations in pressure cause the membrane to vibrate. This, of course, causes a varying pressure to the gas and the jet jumps up and down accordingly.

Sound analysis and recording. It is sometimes desirable to determine the nature of a complex sound, more particularly in regard to the relative amplitudes and the frequencies of the component tones. For example, one problem which concerns the radio engineer is the "fading" which occurs in long-distance wireless signals. In this investigation of the problem it is most convenient if he can get a "visible picture" of the complex sound coming in, as well as hearing it. In other words, he obtains by some means a continuous record of the wave-form of the sound, and he can see how this wave-form changes from instant to instant. Hence, when he employs his remedy, he can see the effect both as regards amplitude and distortion, and he can adjust his treatment until the desired result is obtained. Many attempts have been made to record wave-forms of sound by means of diaphragms and stretched membranes, and the movements of the diaphragms have been recorded in various ways, mechanically, optically and electrically.

Perhaps the most perfect form of sound recording system is that first suggested by Sir J. J. Thomson, namely, a piezo-electric crystal receiver used in conjunction with a cathode ray oscillograph. The latter is a perfect non-resonant recorder of electrical oscillations, having the same sensitivity at all frequencies from zero to the highest "radio" frequency, and the combination is practically distortionless.

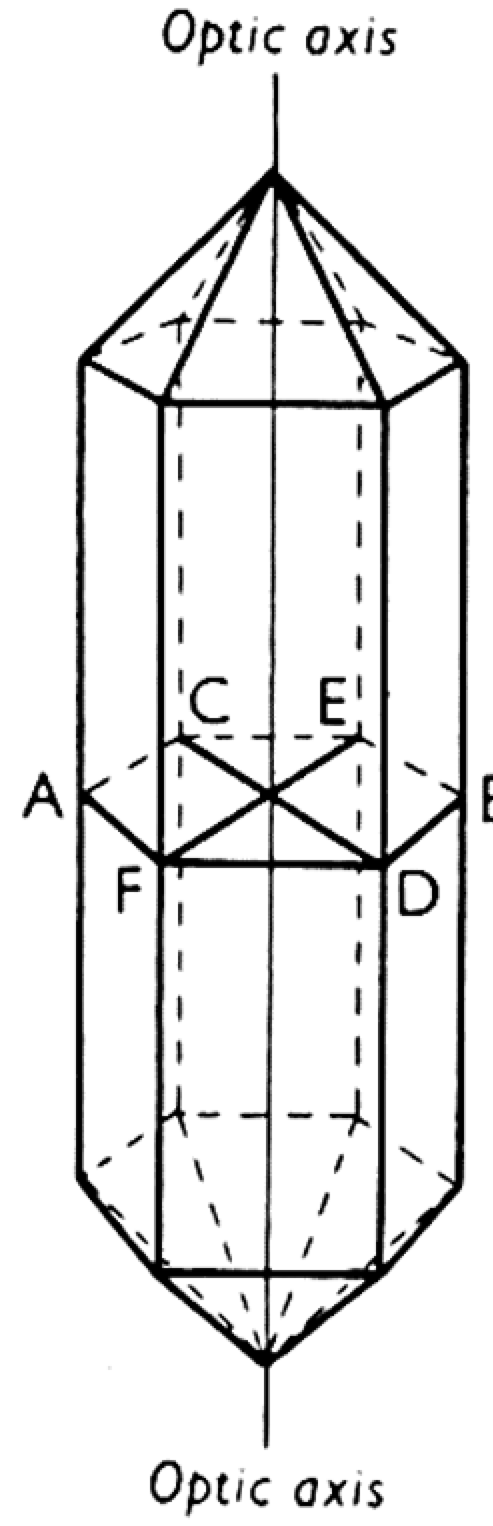
TYPES OF SOUND

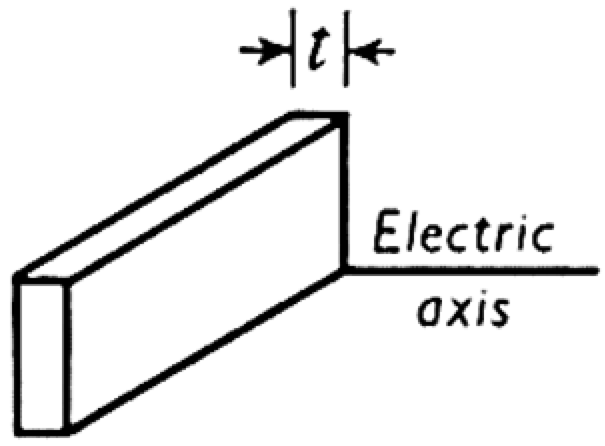
Noises and musical sounds. The sounds to which our ears respond may be divided into two classes, namely, noises and musical sounds. The wave-form of a noise has no set regularity of pattern ; hence it has no definite wave-length or frequency, though, of course, there may be a few regular vibrations amongst the many irregular ones. On the other hand, a musical sound has a regular wave-form even though it may be a complicated one, and a note of definite pitch is the result. Noises are generally unpleasant to the ear and produce a " jarring " effect, whereas musical sounds give a pleasing sensation, no doubt partly due to their rhythmic character.

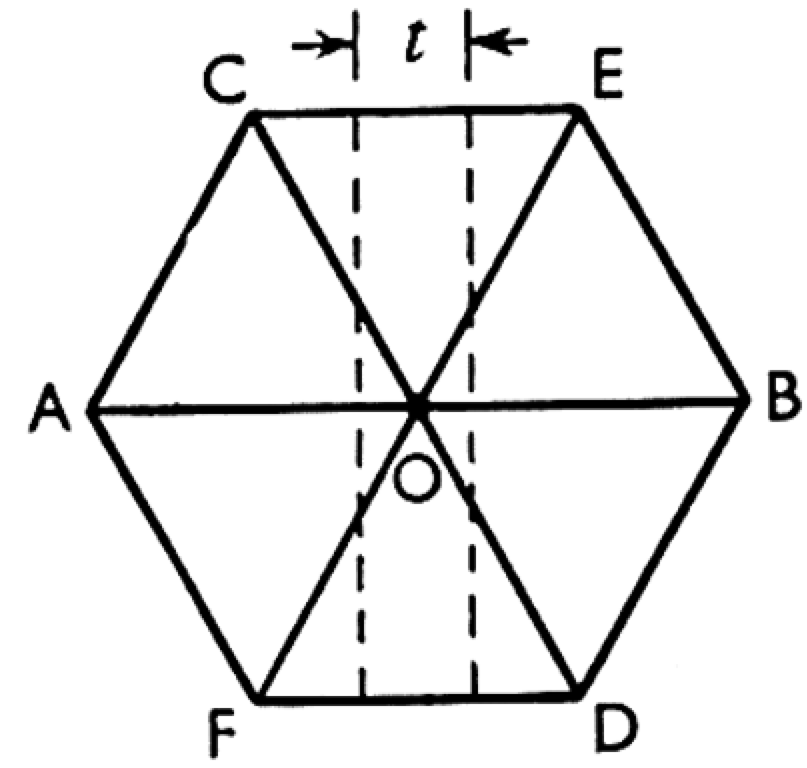
The study of both classes of sounds is important, and although most of the subsequent work in the book will deal with musical sounds, a separate chapter will be given to the study of noise and its suppression.

Sonic and ultrasonic sounds. Those sounds which come within the limits of audibility are referred to as **sonic,** and, as we have seen, the frequencies of such sounds have outside limits for each individual ; they may be referred to as low-frequency sounds. But sounds can be produced of very much higher frequency than the upper limit of audibility, and these " sounds " are referred to as **ultrasonics.** (Previously such " sounds " were referred to as *supersonics*, but the modern custom is to use this term to refer to *velocities* greater than that of sound and to use the term *ultrasonics* to refer to *frequencies* greater than those of audible sound.) Galton's whistle is a simple device for producing high-pitched sounds beyond the upper limit, but the term ultrasonics in its practical applications usually means a much higher frequency than is obtained with Galton's whistle. It is clear that to obtain an ultrasonic " sound ", all that is necessary is to devise some mechanism to vibrate with a very high frequency. Two important methods of doing this will be briefly described here.

Piezo-electric method. Certain crystals, notably quartz and Rochelle salt (sodium potassium tartrate) have the property of changing their linear dimensions when subjected to electrostatic stress, and conversely they develop electrical charges on their faces when mechanically strained. The phenomenon is called **piezo-electricity** and it was discovered by P. Curie in 1880. Rochelle salt shows the phenomenon in a most marked degree ; but because of its more suitable mechanical qualities, quartz has been so far most extensively used in practical applications.







Quartz crystals appear in the form of hexagonal prisms with hexagonal pyramids at each end, and the straight line passing through the summits of the pyramids, or any line parallel to this, is called the optic axis of the crystal.

Consider a section of a quartz crystal cut perpendicular to the optic axis. This will be hexagonal in shape and the lines *AB*, *CD* and *EF* are the so-called electric axes, since along them the piezo-electric effect is most marked. For practical use it is customary to cut out of the crystal a plate or slab with its sides parallel to the optic axis; this is indicated by dotted lines in the diagram. If this plate is inserted between two metal plates and an alternating E.M.F. applied to the plates, the oscillator thus formed will give rise to vibrations of the frequency of the driving E.M.F. The quartz plate has certain natural frequencies, and there will be resonance when the frequency of the applied E.M.F. is equal to one of the natural frequencies of the plate. Resonance is found to be extremely sharp, and it is this property which has made the use of the quartz oscillator so valuable in calibrating wave-meters for radio work, when high-frequency electromagnetic radiation has to be measured.

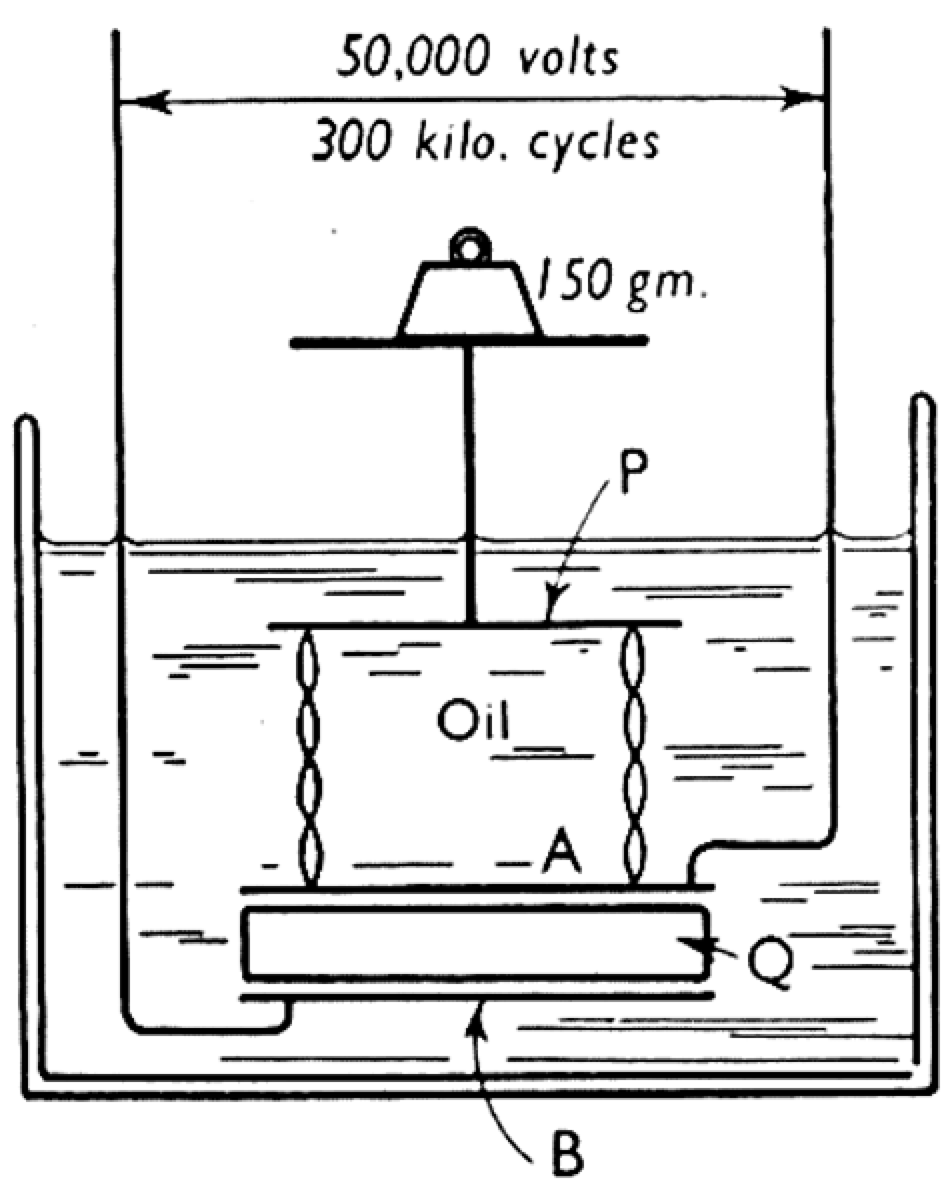
This method of producing ultrasonics was originally developed in 1917 by Langevin, who used an electrical oscillating circuit to provide the applied E.M.F., the tuning being effected by a variable condenser. The frequency used by Langevin was as high as 50,000 cycles per second. In general, the highest fundamental frequency of a quartz crystal cut as indicated above is given by the empirical formula

$$n = \frac{287}{t} \times 10^3 \text{ cycles per second,}$$

where t is the thickness of the plate in cm. A frequency of 50,000 would correspond to a thickness of 5·74 cm.

Magnetostriction method. It is well known that when a rod of a magnetic material is magnetised, it changes in length. This phenomenon is known as **magnetostriction**, and it can be, and is, employed to produce ultrasonics. If such a rod is put in a solenoid and a high-frequency alternating current caused to flow through the solenoid, the rod vibrates longitudinally with the frequency of the A.C., the maximum strain occurring every time the magnetising field is a maximum. If the alternations are sufficiently rapid, an ultrasonic wave will be emitted. Of late years this method has been developed for echo-sounding (see p. 260), and the high-frequency oscillations are obtained from the discharge of a condenser.

Some properties of ultrasonic waves. In 1927 R. W. Wood and A. L. Loomis carried out some interesting experiments in connection with the production of ultrasonics, and their work opened up a large field for future investigation of the physical, chemical and biological effects of this type of radiation. Their apparatus is indicated in the diagram. Q is the quartz plate and A and B are two metal plates near to, or touching, Q. The plates were connected to an A.C. supply of 50,000 volts and 300 kilo-cycles, the plates and the leads being immersed in oil. The frequency of the applied p.d. was adjusted so that it was in resonance with the natural frequency of the quartz oscillator. A glass plate P, 8 cm. in diameter, was then inserted in the liquid, and for certain positions of this plate it experienced a considerable thrust upwards, thus suggesting that the oil above the oscillator had been set into stationary vibrations, the distance betwene P and A being an integral number of half wave-lengths. When one of these positions had been located, the plate P could be



loaded with 150 gm. and remain in equilibrium; the under-surface of P was, of course, a node, a position where the pressure changes are a maximum. When the plate P was removed from the oil, this became heaped up to a height of 7 cm. above the rest of the oil, thus giving an idea of the great pressure produced.

Further, when a mercury thermometer was put into the oil in the above experiment, it registered a temperature of 25° C. But the stem of the thermometer was so hot that it could no longer be held in the hand. The heat was caused by the friction between the rapidly vibrating stem and the fingers.

Ultrasonic waves are destructive to the life of many small organisms; sterilising equipment depending on this effect has been devised. Also, if this type of wave is passed across the boundary formed between water and oil, or even mercury and water, an emulsion is formed.

In recent years a method based upon acoustic vibrations, both low-frequency and ultrasonics, for the dispersal of fog on air-fields, has been tried with some success. In this the idea is to set the fog droplets into to-and-fro motion by the vibrations, so that they coagulate upon encountering one another and thereby reduce the degree of obscuration.

Diffraction of light by ultrasonic waves. In 1932 Debye and Sears noticed diffraction effects when a beam of light was passed through a liquid carrying ultrasonic waves. Light from an illuminated slit was rendered parallel by means of a lens and the beam was allowed to pass through a cell containing a liquid. A lens on the other side produced an image of the slit on a screen. A quartz crystal generating ultrasonic waves was put in the bottom of the cell and arranged so that the longitudinal waves passed through the liquid in a direction at right angles to the beam of light. A diffraction pattern was noticed on the screen similar to that obtained by an ordinary diffraction grating.

The cause of this appears to be due to the variable density of layers of the liquid produced by the acoustic waves. These variations cause corresponding variations in the refractive index of the medium and a scattering process results.

A number of orders can be seen on the screen, and the angular dispersion agrees with that given by the usual equation for a diffraction grating, namely, $\sin\theta = n\dfrac{\lambda_l}{\lambda_a}$, where λ_l is the wave-length of the light, λ_a the wave-length of the acoustic disturbance and n an integer specifying the order.

It will be clear that this experiment affords a method of determining the wave-length of an ultrasonic wave.

Acoustic interferometer. Another method of finding the wave-length and also the velocity of ultrasonic waves, both in liquids and gases, is by using an apparatus, due to Pierce, called the **acoustic interferometer.** In this, a quartz crystal is caused to generate plane waves, the oscillations being maintained by a valve oscillator. The waves are reflected at a movable plate which is parallel to the vibrating surface of the crystal, and stationary waves are set up. The action of the waves is to add a "load" to the crystal and this will be a maximum when the transmitted and reflected waves are exactly out of phase at the surface of the crystal. If therefore a milliammeter is put in the anode circuit of the valve its readings will rise and fall, and if the reflector is moved towards the crystal the current will be maximum for certain positions of the reflector and these positions will be $\lambda/2$ apart.

To find the velocity of the waves, it is now necessary to determine the frequency of the crystal, and this can be done by using a wave-meter in the electrical circuit. The velocity is then calculated from the relationship

$$\mathrm{V} = \mathrm{n}\lambda,$$

where n is the frequency and λ the wave-length of the sound. This is an important and a fundamental relationship which is valid for all forms of wave-motion, longitudinal and transverse.

CHAPTER II

VELOCITY OF SOUND

It was seen on p. 11 that the velocity of sound in any medium is given by $V=\sqrt{k/\rho}$, where k is the appropriate modulus of elasticity and ρ is the density of the medium. If the wave is travelling through a solid rod the modulus of elasticity is Young's modulus, and we have $V=\sqrt{E/\rho}$.

GASES

Newton first established the relationship $V=\sqrt{k/\rho}$ for the velocity of sound, and then deduced the value for k by assuming Boyle's law, which gave $k=p$, the pressure of the gas. This can be shown as follows. Consider a given mass of air or any gas to have a pressure p and volume v and let these change by a very small amount at constant temperature becoming $(p+\delta p)$ and $(v-\delta v)$.

By Boyle's law $pv=\text{constant}$.

Differentiating, we have $p\,.\,\delta v+v\,.\,\delta p=0$

$$\therefore\ p=-v\,.\,\frac{\delta p}{\delta v}\,.$$

But modulus of elasticity

$$k=\frac{\text{stress}}{\text{strain}}=\frac{\text{change in pressure}}{\dfrac{\text{change in volume}}{\text{original volume}}}$$

$$=-\frac{\delta p}{\delta v/v}=-v\,.\,\frac{\delta p}{\delta v}\,.$$

$$\therefore\ p=k.$$

(The $-ve$ sign appears because an increase in pressure causes a decrease in volume.)

Hence, according to Newton, the velocity of sound in a gas is

given by $V=\sqrt{p/\rho}$. But when this equation is used to calculate the velocity of sound in air at 0° C. using the values

$$p=13{\cdot}6\times76\times981 \text{ dynes per sq. cm.}$$

and
$$\rho=0{\cdot}00129 \text{ gm. per c.c.,}$$

it is found that $V=28{,}100$ cm. per sec., which is appreciably lower than the experimental values. Laplace was the first to explain the reason for the discrepancy. He pointed out that the compressions and rarefactions in a sound wave occur so rapidly that the resulting gain and loss of heat produce changes in temperature. In other words, the conditions are *adiabatic* and not *isothermal*, and it is not correct to apply Boyle's law. The adiabatic equation is $pv^\gamma=$ a constant, where γ is the relationship between the specific heats of the gas at constant pressure and constant volume.

In this case $pv^\gamma=\text{constant}$.

Differentiating, we have

$$p\gamma\,.\,v^{\gamma-1}\,\delta v+v^\gamma\,.\,\delta p=0$$

whence
$$\gamma p=-v\,.\,\frac{\delta p}{\delta v}$$

$\therefore$ the adiabatic bulk modulus $=\gamma p$.

Hence
$$\mathrm{V}=\sqrt{\frac{\gamma \mathrm{p}}{\rho}}\,.$$

Using the same data as before, and taking $\gamma=1{\cdot}41$ for air, the velocity of sound in air works out to be 33,160 cm. per sec., which is in closer agreement with the value obtained by experiment.

FACTORS AFFECTING VELOCITY OF SOUND IN AIR

Effect of pressure. From the above equation, it would appear that the velocity of sound depends on pressure. But if temperature remains constant, the pressure of the atmosphere is proportional to the density, by Boyle's law. Therefore, any change in pressure is accompanied by a corresponding change in density in the same ratio, and the value of p/ρ remains unchanged. Hence the **velocity of sound in air (or any gas) is independent of the pressure so long as Boyle's law holds.**

This has been verified by direct experiment, and it has been found that the velocity at a considerable height up a mountain is the same as at sea level.

Effect of temperature. Although any variation in pressure affects the density in the same ratio, it does not follow that the converse is true. Therefore, we must examine what factors are likely to affect the density of the air and consider their effect on the velocity of sound. The most important factor in this connection is temperature. The velocity of sound at 0° C. is given by $V_0=\sqrt{\gamma p/\rho_0}$, and at a temperature $t°$ C. by $V_t=\sqrt{\gamma p/\rho_t}$, where ρ_0 and ρ_t are the densities at 0° and $t°$ respectively.

But
$$\rho_0=\rho_t(1+\alpha t)=\rho_t\left(\frac{T_t}{T_0}\right),$$

where T_t and T_0 are absolute temperatures and α is the coefficient of expansion of air.

$$\therefore\quad V_t=\sqrt{\frac{\gamma p}{\rho_0\Big/\dfrac{T_0}{T_t}}}=\sqrt{\frac{\gamma p}{\rho_0}\cdot\frac{T_t}{T_0}}.$$

Hence
$$\frac{V_t}{V_0}=\sqrt{\frac{T_t}{T_0}},$$

that is, **the velocity of sound in air is proportional to the square root of the absolute temperature.**

Also, we have

$$V_t=V_0\sqrt{\frac{T_t}{T_0}}=V_0\sqrt{1+\alpha t},$$

from which relationship the velocity, measured at any temperature, can be corrected to 0°. This correction amounts to about 61 cm. per sec. per degree for ordinary atmospheric temperatures.

Effect of water vapour. In considering the effect of moisture on the velocity of sound, we must bear in mind that both γ and p/ρ are affected by the pressure of the vapour. It has been shown that in moist air the value of γ is given by $\gamma=1{\cdot}40-0{\cdot}1e/p$, where p is the total atmospheric pressure and e is that portion of it due to the water vapour.

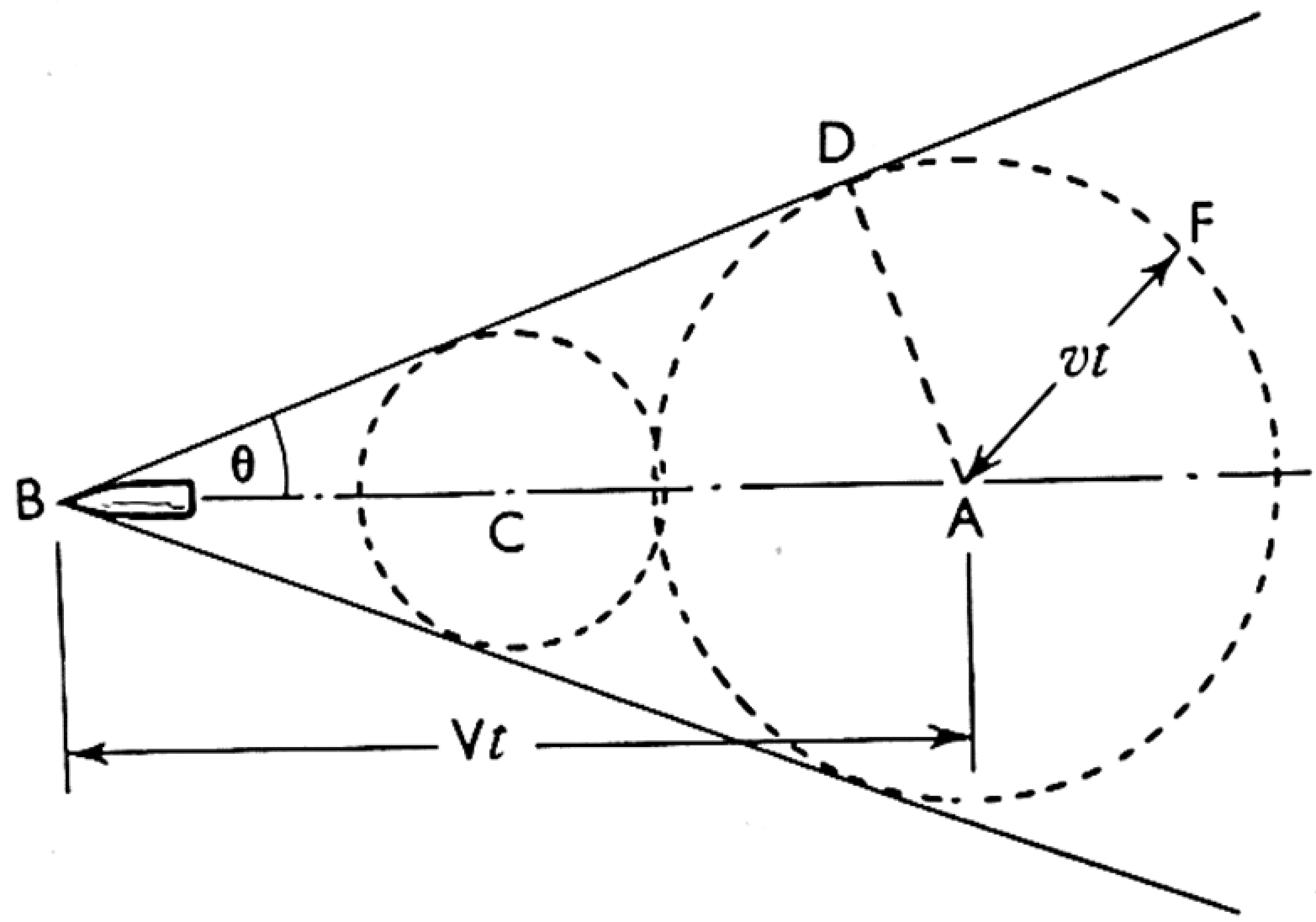
Also, the density of moist air is less than the density of dry air, and when both these variable factors are taken into account, it is found that the velocity in moist air is slightly greater than in dry

air. To quote an example, the velocity of sound in saturated air at 20° C. and a pressure of 76 cm. of mercury is about 0·35 per cent greater than that in perfectly dry air at the same temperature and pressure. The effect of fog on the velocity is not definitely known, but it is probably negligible.

Effect of frequency. For sounds of audible frequency, that is, low frequency, the velocity of sound is independent of the frequency; otherwise, music as we know it would be impossible. But for very high frequencies, say ultrasonics, the study of the passage of a sound wave through a gas will involve further considerations. This problem has been investigated theoretically in the light of the kinetic theory of gases, and it has been shown that for very high frequencies the velocity of propagation in gases should increase slightly with the frequency. A certain amount of experimental work has also been done, and although the earlier work indicated the existence of such an effect, more recent work shows that in air at any rate the effect is not measureable.

Effect of amplitude. The conclusion that the velocity of sound is equal to $\sqrt{dp/d\rho}$ is based on certain assumptions, one of which is that the waves are of small amplitude. For waves of large amplitude, the relationship is not true, and it has been found that velocities far in excess of the normal value have been obtained in **explosive waves.** In such a wave, the curve connecting pressure and density is not a straight line, and the bulk modulus increases as the density is increased by compression and diminishes as the density is reduced by rarefaction. Consequently, the compression wave travels faster, and the rarefaction slower, than a sound wave of small amplitude, and as a result there is a change of wave form. It should also be noted that during the rapid compressions in the early stages there will probably be a rise in temperature which will cause an increase in velocity. A large amplitude wave has therefore an initial velocity much greater than an ordinary sound wave, gradually approaching the latter value as the distance from the source increases. It has been shown that the velocity of sound from an intense electric spark varied from 660 metres per second at a distance of 3·2 mm. from the spark to 380 metres per second at a distance of 18 metres.

Shell waves. In the case of rifles and guns, the bullet or shell, in the early stages of its flight at least, travels faster than sound. On account of this high velocity, the missile gives rise to *V*-shaped compression waves, one at the front and one at the back rather like the wake of a ship.



Suppose a bullet is travelling with a velocity V and it gives rise to a compression wave of velocity v. When the bullet is at A, a compression wave spreads out in all directions, and after a time t it can be represented by the circle F of radius vt. In the meantime the bullet has reached a position B such that the distance AB is Vt. If we consider the conditions after a time $t/2$, the bullet will have reached C and a compression wave of radius $v \cdot t/2$ will spread out from this point, during which time the bullet will have reached B. Therefore, a cone with apex B can be constructed to touch all the spherical waves, as shown in the diagram, and this is the shape of the compression wave. Also,

$$\sin \theta = \frac{AD}{AB} = \frac{v}{V};$$

hence, by observing the compression waves by means of the shadow they produce when illuminated for an instant, it is possible to measure the velocity of a bullet fired from a rifle.

The wave from a gun, such as we have just described, which progresses with a velocity greater than the normal velocity of sound, is known as the " onde-de-choc " and it reaches the observer as a sharp crack. The discharge from a gun, however, is not a single signal, but generally consists of three separate signals. The " onde-de-choc " is followed by the wave due to the explosion of the shell, and also by the gun wave due to the expansion of the gases at the gun. This latter wave, which is called the " onde-de-bouche ", travels with the normal velocity of sound and is the wave used in sound-ranging to ascertain the location of the source.

Velocity of Sound v. Velocity of Aircraft. The value of the velocity of sound is of the utmost importance to the aeronautical designer, particularly as the speed of modern aircraft has now approached that of sound. When the ratio of the two speeds becomes equal to unity the compression waves set up are unable to get away, and the air barrier so formed must offer enormous resistance to the aircraft. Tests have been, and still are being, carried out to obtain information concerning the behaviour of such high-speed aircraft, and until the results of these tests have been given it is idle to speculate on the subject. But it appears certain that the shape and structure of the aircraft-to-come will be very different from that of the machines of today.

Apart from the structural stresses and strains involved in high-speed flying, the question of noise in the aircraft will be of importance. Will the crew of the aircraft be deafened by the noise, or will they hear nothing? Here again it is a matter of speculation until concrete evidence is forthcoming, but probably the answer is that the crew will neither be deafened nor will they be in perfect silence. However, these are all interesting questions to be solved in the future.

Velocity of sound in gases other than air. The equation

$$V = \sqrt{\gamma p/\rho}$$

holds for all gases; hence as all gases have densities differing from that of air, the value of V will be different. Let us assume for a moment that γ is a constant for all gases. We have

$$\frac{V_{\text{air}}}{V_{\text{gas}}} = \sqrt{\frac{\rho_{\text{gas}}}{\rho_{\text{air}}}};$$

that is, the **velocity of sound in a gas varies inversely as the square root of the density of the gas.**

But it is known that γ is *not* a constant for all gases. For air and for those gases which are regarded as diatomic, for example, hydrogen, oxygen, nitrogen, etc., the value of γ is about 1·41. In the case of the rare gases helium, argon, neon, etc., and for mercury vapour, which are all regarded as monatomic, the value is about 1·66, while for triatomic gases like carbon dioxide it is about 1·29. It is interesting to note that the value of γ for water vapour is somewhere about 1·26 (see p. 32). Hence, to calculate the value of the velocity of sound in any gas, the corresponding values of γ and ρ must be known; alternatively, if the velocity of sound in any gas is known, the value of γ for that gas can be

calculated if the density of the gas and the pressure at the time of the experiment are known. This method of finding γ may be used in those cases where the specific heats of the gas cannot conveniently be found directly.

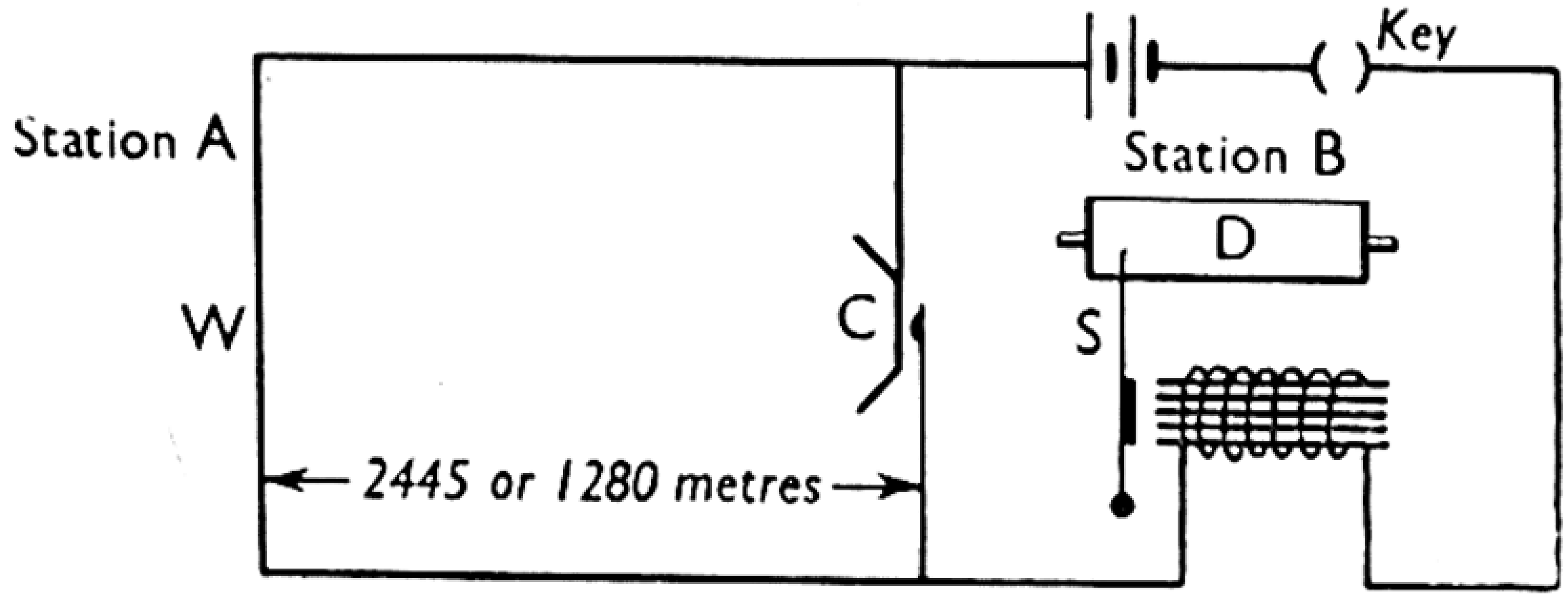
METHODS OF FINDING THE VELOCITY OF SOUND IN AIR AND OTHER GASES

It has already been seen that the velocity of sound in air in any gas can be calculated if sufficient data are known. We will now consider certain experimental methods of obtaining this value.

Historical. The first determination of the velocity of sound which can be considered at all reliable was made about the middle of the eighteenth century by three members of the French Academy of Sciences. Two stations about 30 km. apart were selected and at constant intervals during the night cannons were fired, one at each station. The time elapsing between seeing the flash from the explosion and hearing the report was obtained, and knowing the distance between the stations the velocity of sound could be calculated. The results showed that the velocity of sound in air increased with temperature but was independent of pressure; also that the velocity increased when the sound travelled with the wind and decreased when it travelled against it. In these experiments, however, by taking the mean time of propagation in opposite directions the wind effect was eliminated.

The chief objection to this type of experiment is that the "personal equation" of the observers, which is different for each observer and is difficult to eliminate, introduces errors. Such errors can be very much reduced by employing mechanical means to record the various signals; but here again all such pieces of apparatus have their own "personal" equation. This, however, is much more constant than that of the observer and may be evaluated and then eliminated by experimenting over widely different distances, as was done by Regnault in an important series of experiments in 1864.

Regnault's experiments. In these experiments, the reciprocal firing of guns at stations, at first 2,445 metres and then 1,280 metres apart, was the method employed, and the apparatus shown in the diagram was duplicated, one at each station. W is a wire at one station A connected to the rest of an electrical circuit at the second station B. When the gun is fired at A the wire W is broken and also the circuit at B, thereby causing the style S to make a mark to the left on the rotating drum D, which moves at a



constant and known speed. Thus the instant of the original sound is recorded.

When the sound reaches *B* it is received by the wooden cone *C* which has a membrane at its narrow end, and this is forced outwards to make contact and close the circuit again. Thus the style moves again and makes a mark on the drum to the right. As the speed of the drum is known, the time interval between the two marks can be found and so the time between firing the gun and recording the sound ; hence the velocity of sound can be calculated. To eliminate the effect of wind and other possible varying factors, the experiment was repeated from *B* to *A* and a mean value of the velocity obtained.

Regnault found that the apparatus itself had a " personal equation ", and to eliminate this the experiments were repeated over the smaller distances.

Stone's experiments. In 1871, Stone, who was the Astronomer-Royal at Cape Town, performed a series of experiments in an attempt to eliminate the " personal equation ". Two observers were stationed at distances 641 ft. and 15,499 ft. respectively from a gun which was fired. Each observer reported the arrival of the sound at his station on an electrical chronograph, and the difference between the recordings of the sounds was the time taken for the sound to travel across the distance between the stations, but slightly in error owing to the fact that their " personal equations " were not likely to be the same, especially as the intensity of the sound received by each was different. To eliminate the difference between the personal equations, a smaller gun was fired at such distances from the observers that the intensity of the sound was approximately the same as in the main experiment. The distances were now 162 ft. and 1,483 ft. from the gun, and the recorded difference in the times of reception was 1·265 sec.

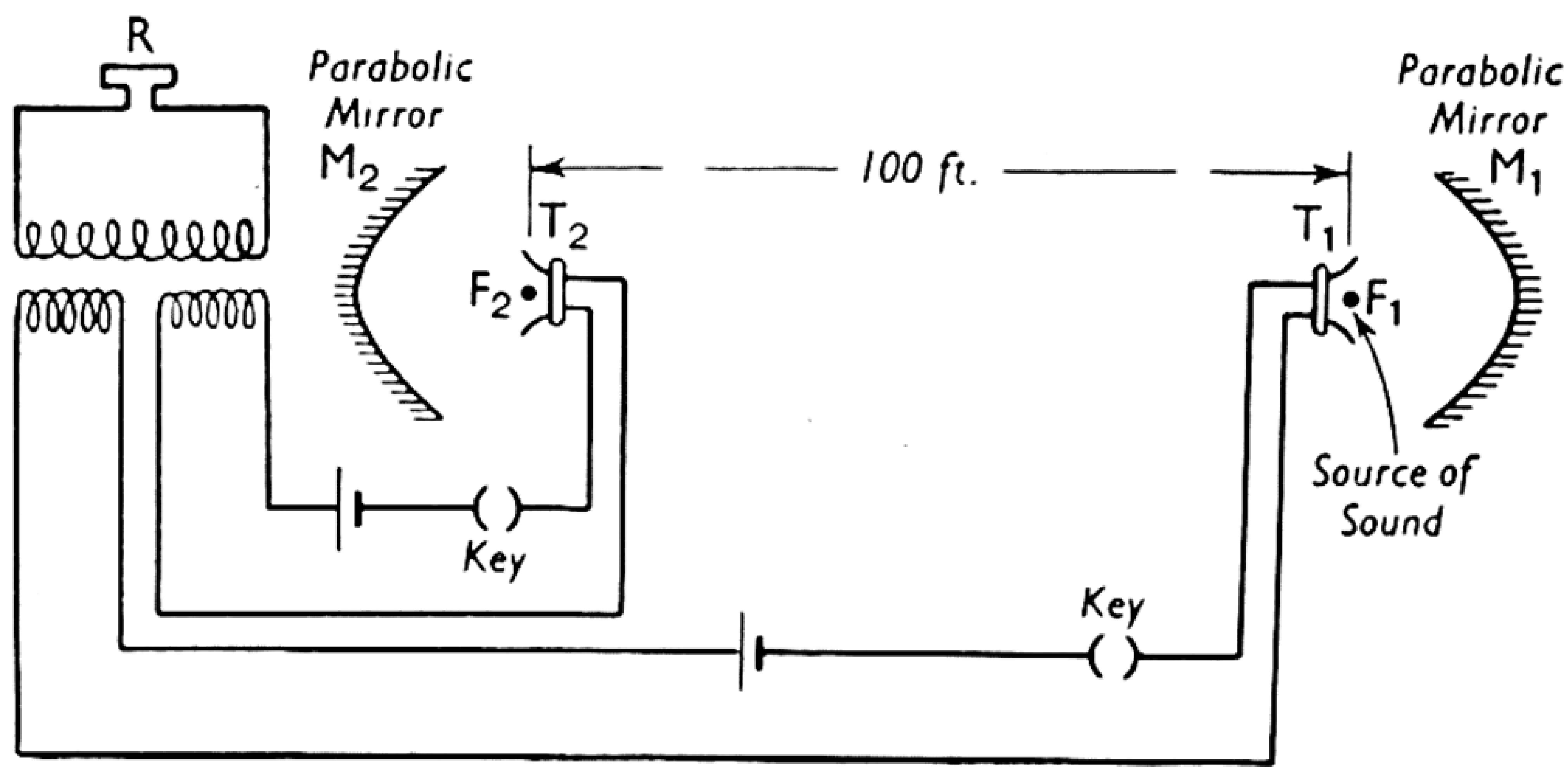
The time for the sound waves to travel across the distance between the two observers was then calculated from the provisional value of the velocity of sound deduced from the first

experiment when no correction was applied. This time was found to be 1·177 sec., a difference of 0·088 sec., and this represents the difference between the two personal equations. When this correction was applied, Stone obtained a value of 332·4 metres per second for the velocity of sound.

Greely, working in the Arctic regions, when conditions were still and the low temperature causes the water content of the air to be small, found that the velocity of sound in air could be represented by the equation $V = (332 + 0{\cdot}6t)$ metres per second, where t is the temperature on the centigrade scale.

The above methods of finding the velocity are essentially long-distance methods, and these certainly have definite disadvantages. In the first place, the original sound must be loud in order to carry the required distances, and we have already seen that the velocity of such sounds is not normal, particularly near the source. Again, it is practically impossible to correct for wind, temperature and humidity changes and variations to any high degree of accuracy. Short-distance methods are to be preferred, and the following method, utilising short distances, was used by Hebb in 1904.

Hebb's method. The basis of the method is to find the wave-length of a source of sound of known frequency and then to apply the relation $V = n\lambda$. The experiments were carried out in 1905 in a room 120 ft. long, and the source of sound was a whistle which was blown so steadily as to maintain a frequency to an accuracy of 1 in 5,000. This source was put at the focus F_1 of a parabolic mirror, of focal length 15 in. and diameter 5 ft., made of plaster of Paris. At the focus also was a telephone transmitter T_1 which



was connected through a battery and a switch to one of two primaries of a special induction coil. At the other end, about 100 ft. away, was a similar arrangement, the transmitter T_2 being connected to the other primary of the induction coil. The secondary of this coil was connected to a telephone receiver R.

When the whistle was sounded, some of the energy passed direct to T_1 and thus actuated the telephone receiver. A parallel beam of sound energy also proceeded from mirror M_1 to the second mirror and so operated transmitter T_2. Therefore the sound heard in the receiver is the resultant effect of the two sounds, which, of course, may differ in phase depending on the distance $F_1M_1M_2F_2$. There must be a difference to correspond to maximum resultant effect, that is, when the two sounds are in the same phase, and also one to correspond with minimum effect. In the experiments, one mirror with its corresponding transmitter was moved towards the other until the maximum sound was heard in R, and then further moved until there was a minimum of sound; the distance between the two positions is $\lambda/2$. A series of observations for maximum and minimum sound was taken and a mean value for λ obtained. By this means the wave-length was determined correctly to within one part in a thousand.

The frequency of the whistle was obtained by tuning it in unison with a fork which was itself compared with a 512-fork by traces on a smoked plate. The 512-fork was compared in the same way with a pendulum and then the pendulum with a clock.

The final determination for the velocity of sound in dry air at 0° C. was $V=331{\cdot}29$ metres per sec. with a probable error of $\pm 0{\cdot}04$ metres per sec.

Hebb repeated his experiments in 1919 and the results gave a velocity of 331·44 metres per second for dry air at 0° C.

Other methods of finding V. Other methods based on widely different principles have been used for finding the velocity of sound in air and other gases. Among these are the echo method, the method of resonance, and Kundt's tube, and all these will be dealt with more fully later.

Perhaps one of the most accurate of these other methods is that based on the same principles as were used in connection with sound ranging in the First World War. The sound of a gun is picked up by two resonant hot-wire microphones at a known distance apart and in the same straight line as the gun. The microphones actuate a string galvanometer, the movements of which are recorded on to a mechanism which can time accurately to the 1,000th part of a second. The velocity is calculated by

dividing the distance between the microphones by the time interval between the two recorded signals. By this method the velocity of sound was 337·16 metres per sec. at 10° C., in dry air and 337·6 metres per sec. at 10° C. in air of average humidity.

Velocity of sound in tubes. We have already mentioned the frictional drag on sound waves when they pass through pipes; hence we should expect that the velocity of sound through the air in pipes should be less than in free air. In this connection, Regnault performed an elaborate set of experiments in 1862–63 with the water mains newly laid in Paris. As a result of these experiments, in which he used pipes of different diameters, Regnault came to the conclusion that the velocity of sound tends to a lower limit when very feeble owing to the great distance traversed, and this limiting speed is greater in wide pipes than in narrow ones. He considered that in the pipe of 1·1 metre in diameter its sides were practically without effect on the velocity, and he gave the value 330·6 metres per sec. as the velocity of sound in air at 0° C. in an infinitely wide pipe.

Blaikley noticed that in Regnault's experiments the decrease in speed found there would, if extended to the diameters used in brass musical instruments (in which Blaikley was interested) lead to smaller values than those actually experienced. He attributed the discrepancy to the roughness of the pipes used by Regnault, and in 1883–84 he carried out a series of experiments in which he used a special form of pipe so that when it was blown only the fundamental note was sounded without any overtones. Smooth brass tubes of five different sizes were experimented with, and Blaikley found that the velocity of sound varied from 324·383 metres per sec. for a pipe of diameter 11·43 mm. to 330·134 for a pipe of diameter 88·19 mm. As a result of his experiments, he inferred that the velocity of sound in free air at 0° C. was 331·676 metres/sec.

VELOCITY OF SOUND IN LIQUIDS

When sound travels through a liquid, the waves are still longitudinal and the elasticity called into play is the bulk modulus, as in gases. The expansion of liquids is so much less than that of gases that it is immaterial whether the changes that take place in the medium are regarded as isothermal or adiabatic, but the modulus of elasticity to be used is no longer easily obtained from the pressure as in gases. If k is the bulk modulus for the liquid, we have $V=\sqrt{k/\rho}$, and this is equal to $\sqrt{1/M\rho}$ where M is

the compressibility. For water at a pressure of 1·25 atmospheres and at a temperature of 15° C., Amagat found the compressibility to be $48{\cdot}9 \times 10^{-12}$ for an increase of pressure of 1 dyne/cm.^{2} The density of water at 15° C. is 0·99912 gm./c.c., and using these figures, the velocity of sound in water works out to be 143,166 cm./sec. This is roughly four times the velocity of sound in air; the greatest contributing factor to this result is, of course, the high value of the bulk modulus for the liquid.

The classical experiments on the velocity of sound in water are those of Colladon and Sturm carried out in 1826 on Lake Geneva. They arranged for a bell to be struck under the water, and the sound was detected some distance away by a large trumpet-shaped receiver, the larger, lower end of which was immersed, while the small upper end was held to the ear of the listener. The striking of the bell was accompanied by the ignition of some gunpowder above the surface of the water, and the time interval between the flash and the sound was measured. The mean temperature of the water was estimated to be 8·1° C., and the value obtained for the velocity was 143,500 cm. per sec., which is in close agreement with the calculated value given above.

The velocity of sound in water, particularly sea water, is of great importance in connection with under-water signalling; hence many experiments have been carried out to find the value of this velocity, using explosion waves, to a considerable degree of accuracy. In 1919 Marti used three hydrophones in a straight line in Cherbourg roadstead, the total base line being 1,800 metres, the actual positions of the hydrophones being accurately located by using a theodolite. The passage of the sound wave due to an explosion under water in a line with the hydrophones was registered by an electric chronograph, with smoked paper and a tuning fork time trace. The result obtained showed that in sea water of density 1·0245 gm./c.c. the velocity of sound is 1503·5 metres/sec. at a temperature of 14·5° C.

Some years later, experiments were carried out by Wood, Browne and Cochrane near Dover, using four hydrophones and a base line of 12 miles. Accurate temperature and salinity observations were made at points along the base line, and a new method was devised to obviate errors caused through the firing of the charge at a point not quite in line with the hydrophones. The times of arrival of the sound at the four hydrophones were recorded on four strings of a six-string Einthoven galvanometer; the ticks of an accurate chronometer were recorded on the fifth

string, and a wireless signal sent from a naval vessel at the instant of firing the charge was recorded on the sixth. Thus the record showed to an accuracy of $\pm 0{\cdot}001$ sec. the various time differences and the total time of travel of the explosion wave to each receiver. The results of the experiments, which were carried out in summer and in winter, showed that with a constant salinity of 35 parts per thousand, the velocity of sound is 1510·4 metres/sec. at 16·95° C. and 1477·3 metres/sec. at 7° C.

Variations in the homogeneity of sea water would appear to be of obvious importance, since the velocity of sound, as will be recalled, is given by $V=\sqrt{1/\rho M}$ (p. 40). The salt content of the water is different from place to place, and the temperature and the density of the water both change with the depth. An increase in the salt content of 20 per thousand brings about a *decrease* in M of only 5 per cent and therefore an *increase* in V of only about 2·5 per cent. This increase in salinity, however, causes an increase in density of about 15 per cent and hence a *decrease* in V about three times the increase due to the effect of M. At moderate depths the effect of change of salinity on velocity is therefore rather small.

It must be noted that the pressure increases with depth, and it has been found that the compressibility decreases with increasing pressure. This decrease is about 1 per cent for 10 atmospheres.

Velocity in liquids in tubes. As in the case of gases, so stationary waves (see p. 81) may be set up in tubes filled with liquids. But on account of the small compressibility of a liquid compared with that of a gas, a correction is necessary due to the yielding of the walls of the containing tube. This yielding produces an apparent lowering of the wave velocity. If V_0 is the theoretical velocity in the liquid, V the actual velocity in a tube of small thickness h, a the internal radius of the tube, k the bulk modulus for the liquid and E Young's modulus for the material of the tube, then

$$V_0=V\sqrt{1+\frac{ka}{hE}}.$$

If the walls are very thick

$$V_0=V\sqrt{\frac{k+n}{n}},$$

where n is the rigidity of the material of the tube.

Kundt and Lehmann have obtained " dust figures " in liquids

as in gases by using fine iron filings, and have measured the velocity in tubes of different diameters and thickness (see p. 45).

If the wave-length of the sound is sufficiently small compared with the diameter of the tube, that is, at high frequency, the correction given above disappears. In 1927 and 1928 Hubbard and Loomis used a quartz oscillator emitting waves at a frequency of 200,000 and at 400,000, and found the velocity of sound in various liquids with an accuracy of 1 in 3,000.

Range of sound in air and in water. It was seen in Chapter I (p. 14) that when a sound is transmitted through a medium, the power transmission falls off approximately in accordance with the inverse square law. This is never quite true on account of other factors, and the wave progresses with exponentially decreasing amplitude, this causing a more rapid decrease in intensity than would occur under the inverse square law. It can be shown that the attentuation factor is equal to

$$\frac{2}{3} \cdot \frac{\eta k^2}{\rho_0 V},$$

where η is the coefficient of viscosity of the medium, $k = 2\pi/\lambda$, ρ_0 is the mean density of the medium and V the velocity of the wave. Thus the attentuation depends on the viscosity and the square of the frequency. The damping for waves in air is very small for moderate frequencies, but it rises rapidly with increasing frequency. The variations in the range of audibility of sound in air must, of course, also depend upon changes of direction of the wave due to reflection, total reflection and refraction, etc. These will be discussed later.

So far as water is concerned, the effect of viscosity on the power transmission of sound waves is practically negligible, at any rate for low-frequency waves. But if the decrease in intensity is *calculated* on the assumption of the inverse square law, it is found that the value obtained is much in excess of the value observed by experiment. Hence, it would appear that the falling off is due to exponential damping chiefly on account of non-homogeneities in the water caused by variations in temperature and density.

Concerning temperature, it has been observed that the range in winter is much greater than in summer, and this is what would be expected from the behaviour of the temperature gradient (see p. 77).

The changes in the density caused by variations in the salt content of sea water have little effect in the open sea, but near the

mouth of a large river the effect is more noticeable; also the effect of change in density with depth and compressibility is very small. It will be obvious, however, that the influence of eddies and currents in the water can affect sound signals considerably. Air bubbles also may be a serious cause of attentuation in sea water.

VELOCITY OF SOUND IN SOLIDS

Since solids are capable of experiencing various kinds of strain, there are possibly many forms of waves propagated. Certain it is that both longitudinal and transverse waves can be set up in a solid on account of the longitudinal elasticity (Young's modulus) and the shear elasticity (modulus of rigidity) respectively. If the solid is a rod and therefore quite free to shrink laterally where stretched and to bulge where compressed lengthwise, the velocity of the longitudinal wave is given by $V = \sqrt{E/\rho}$, where E is Young's modulus. But if the solid is a large mass, in which there is no freedom for bulging sideways at places compressed endwise, for example, the crust of the earth, the velocity is given by

$$V = \sqrt{\frac{k + \frac{4}{3}n}{\rho}},$$

where k is the bulk modulus and n the rigidity.

Calculations based on these expressions show that for a brass rod the velocity of sound is about 345,000 cm./sec., or more than ten times the velocity in air, while if the brass is an extended mass the velocity is about 427,000 cm./sec.

In connection with the transmission of sound through solids, it is interesting to note that the seismic waves which occur in the earth are both longitudinal and transverse. At the earth's surface these two types have velocities of about 7·2 km./sec. and 4 km./sec. respectively; though, down to a certain depth when the velocities remain constant, there is a more or less uniform increase in velocity with depth. The transmission of "waves" caused by small explosions in the earth is the basis of a method of detecting oil strata and other features (see p. 78).

EXPERIMENTAL WORK

Biot's experiment. Biot made observations of the velocity of sound in iron by using 376 cast-iron pipes of a total length of about 950 metres. A bell was struck at one end and the sound travelled through the iron and the air. Thus the observer at the

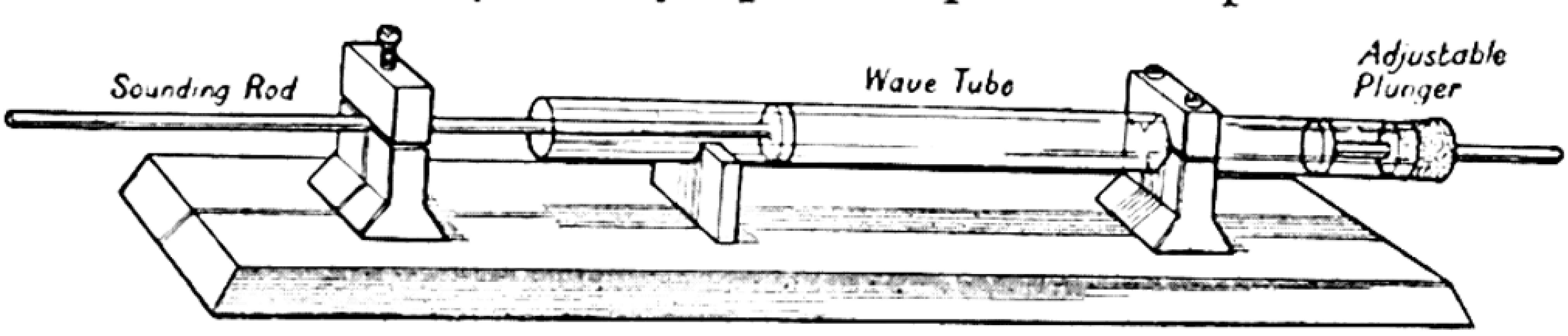
other end would hear two sounds separated by an interval of time (t) which was measured. If L is the length of the tube, V the velocity of sound in iron and v the velocity in air, we have

$$\frac{L}{v} - \frac{L}{V} = t\,;$$

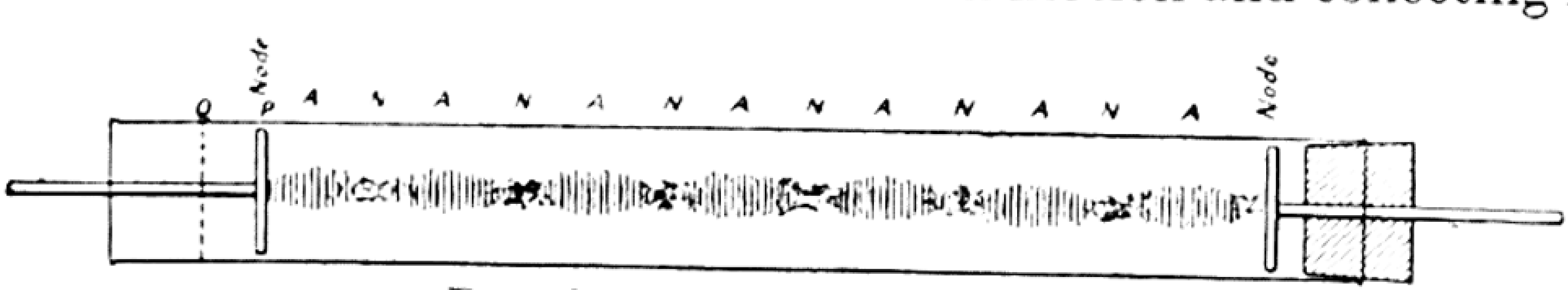
whence $$V = \frac{L}{L - vt}.$$

The value of the velocity for cast-iron was found to be about 350,000 cm./sec.

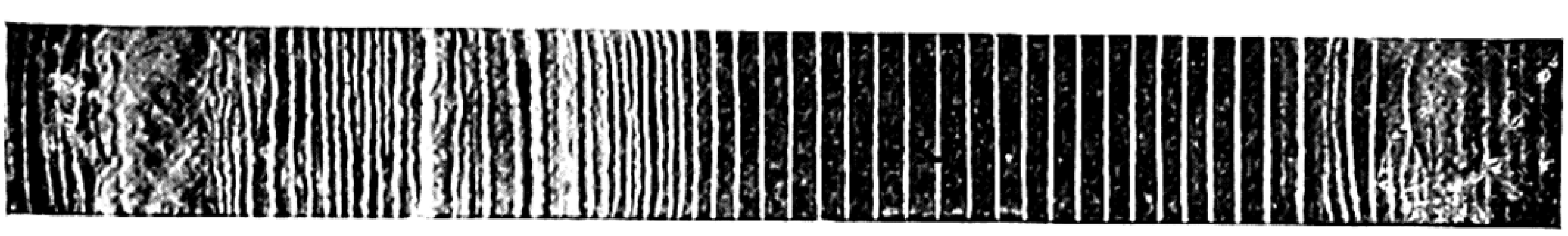
Kundt's tube. This is an apparatus designed to measure the velocity of sound in gases and in solids. In its most usual form, the apparatus consists of a horizontal glass tube about a metre long and about 3 cm. in diameter provided with a movable piston near one end. Near the other end of the tube is another piston attached rigidly to the end of a metal or glass rod which is firmly clamped at its middle point. The tube is thoroughly dried and then a layer of lycopodium powder is spread inside it



along the lower side. When the rod is set into longitudinal vibration by stroking it lengthwise, with a resined cloth if the rod is metal, the air inside is forced to vibrate. By adjusting the position of the movable piston, resonance of the air column is established, and a stationary wave is produced with nodes and antinodes. Hence the powder moves, eventually settling down at or near the nodes where the air is least in motion and collecting in



Dust heaps in a Kundt's tube.



By courtesy of Prof. E. N. da C. Andrade

Photograph of dust striations.

small heaps or ridges at the antinodes. Each piston is approximately at a node. The position of a node at one end and that of a node at the other end are chosen, and the distance between the two is measured and divided by the number of loops to get the mean distance between the nodes. The wave-length of the sound in air is then twice this distance, and the wave-length of the sound emitted by the rod is twice the length of the rod, since the rod is clamped at its middle point. If the velocity of the sound at 0° C. is assumed, the velocity at the temperature of the experiment can be calculated from the relationship

$$V_t = V_0\sqrt{1+\alpha t}.$$

Hence the velocity of sound in the rod can be found, for

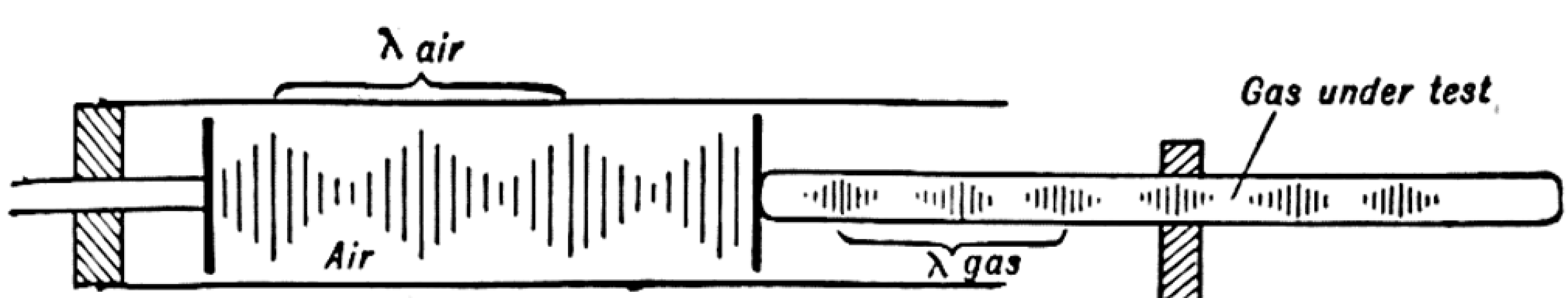
$$V_{\text{rod}} = n \,.\, \lambda_{\text{rod}} \quad \text{and} \quad V_{\text{air}} = n \,.\, \lambda_{\text{air}}.$$

Therefore
$$V_{\text{rod}} = V_{\text{air}} \cdot \frac{\lambda_{\text{rod}}}{\lambda_{\text{air}}}.$$

It should be noted that the presence of the powder introduces a small and variable amount of absorption of energy which results in a lowering of the observed velocity. If, in the experment, the rod is of glass, the stroking might be done with a cloth moistened with alcohol.

Since $V_{\text{rod}} = \sqrt{E/\rho}$, this experiment could be used to find the value of Young's modulus for the rod, and although the strains in the rod do not take place under exactly isothermal conditions, the difference between the isothermal and adiabatic values of E is small, and the value of E obtained should agree closely with that found by the usual statical methods.

The tube can be used to determine the velocity of sound in other gases, and for this purpose it is usually provided with a tap at each end so that the gas enters by one and drives the air out by the other. A later modification of the apparatus for this purpose is that of Behn and Geiger, who used a closed tube of the gas under test instead of a solid rod as the source of sound. The dust pattern inside the gas tube is compared with that in a tube con-



Behn and Geiger tube.

taining air, and as the frequency of the note is the same for both air and gas, we have $V_{gas}/V_{air} = \lambda_{gas}/\lambda_{air}$. When the ordinary pattern of Kundt's tube is used to determine the velocity in a gas, it is not necessary to have a stroked rod to excite the gas. The excitation can be done by using a loudspeaker attached to a valve oscillator at one end, while the other end can be closed. The frequency of the oscillator is varied until resonance is set up.

In 1922 Lang, by using a short steel bar 5 cm. long and exciting the vibrations by striking one end, observed the nodes and antinodes in a Kundt's tube up to a frequency of 50,000 cycles per second. The mean velocity of sound in the air contained in the tube was found to be 339·3 metres per sec. at 22·8° C. and in the steel bar 5,120 metres per sec. approximately. The values of velocity at ultrasonic frequencies are found to be the same as at audible frequencies.

Bubble effect in underwater explosions. Although not connected primarily with the velocity of sound, it is instructive to consider briefly the problem of underwater explosions. It has been observed that charges exploding underneath ships and submarines are often much more destructive than explosions of similar charges in the water at the side or above them, although the shock wave from an underwater explosion is the same whether the charge explodes above or below the ship. In shallow water, this can be understood because there is a strong reflection of the blast from the sea-bed which adds to the blow; but this explanation does not hold in deep water.

The explanation seems to be that when the charge explodes in the water, it forms an extremely hot bubble of gas which expands at a tremendous rate. Consequently, the water is pushed away and acquires a high momentum. It continues to move outwards after the gas has expanded to the degree at which it is in equilibrium with the pressure in the water before the explosion. Thus the gas bubble expands beyond its equilibrium position and presently the water walls of the bubble begin to move inwards and rush towards each other with enormous speed compressing the approximately spherical bubble of gas back again into a small bubble. Then there is a sudden reversal, and the bubble once more begins to expand a second time with explosive violence. A sequence of photographs of an underwater explosion is shown on Plate 1 (facing p. 54).

The bubble is really an oscillating system, the movement of which is relatively slow at the stage of full expansion and very fast at the stage of maximum contraction. Three or more ex-

pansions and contractions may occur, sending out three or more shock-waves, in decreasing order of strength. If the charge explodes above a vessel, the bubble oscillates, but it also rises through the water. Hence the second and third shock waves will occur further and further away, besides being weaker and weaker. If the charge explodes under the vessel, the second and third shocks from the explosion might be much nearer and so more destructive.

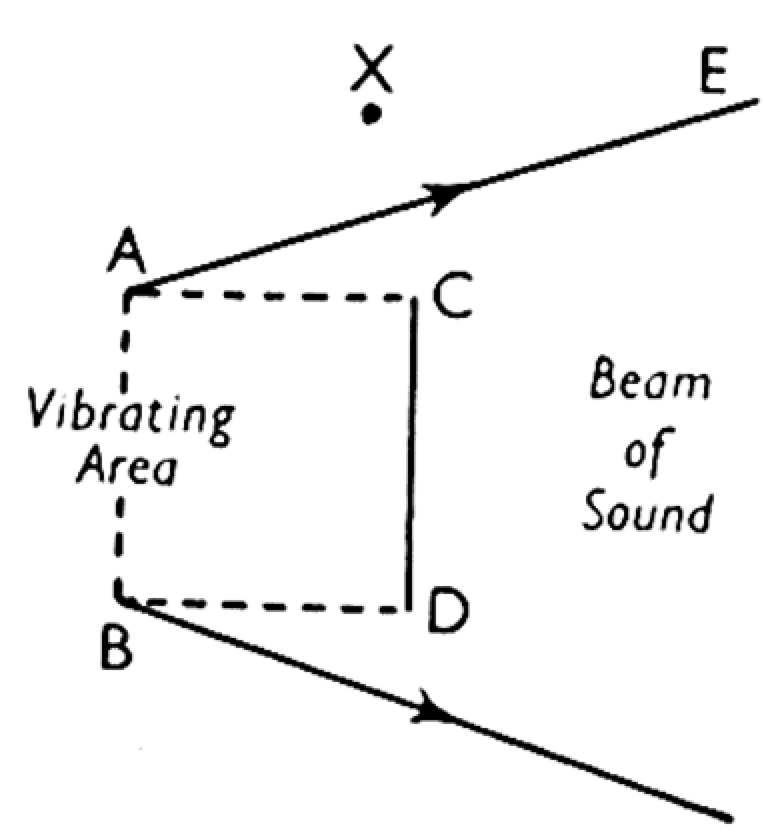
CHAPTER III

PROPERTIES OF SOUND: REFLECTION

Introduction. As sound energy is propagated by wave motion, it is to be expected that it will have certain properties similar to light and other forms of radiant energy, although of course the medium of transmission is very different. It is important, however, when considering the properties of sound, to remember that the wave-lengths of audible sound, at any rate, are large compared with those of light and heat, and consequently, although they may obey the same general laws as light concerning reflection, refraction and so on, yet there will be differences to be noted.

Beam of sound. It is well known that when light is allowed to pass through a hole in a wall, a more or less sharply defined beam is produced. This is also the case when a projector is used; hence the intensity does not fall off as it would do if the light were allowed to spread out.

It is often desirable to prevent loss of intensity when sound is transmitted, and under certain conditions sound energy can also be concentrated in a beam. But whatever type of energy is concerned, it is necessary that the dimensions of the aperture through which the energy passes should be large compared with the wave-length. Consider a vibrating circular area of diameter AB, the dimensions of which are large compared with the wave-length of the sound emitted. For those points along CD, that is, in a beam at right angles to the vibrating source, the Huyghens wavelets will reinforce each other; but at points outside this beam, such as X, there will be interference due to the effects of wavelets from different parts of the source with consequent loss in intensity. Hence most of the sound is confined within a cone-shaped space represented roughly by AE and BF.



If the wave-length of the sound is decreased, that is, the frequency is increased, the beam becomes sharper, while if the frequency is decreased, the beam spreads out more.

If the size of AB is decreased to less than a wave-length, there will be practically no interference and the energy will be spread out. Hence we see that to produce an effective beam the dimensions of any vibrating source, or even a hole in a surface through which energy is sent, must be of the order of a considerable number of wave-lengths.

To amplify the above. Suppose we have a circular piston source of radius r radiating waves of length λ. The sound is a maximum along the axis of the source, where the disturbances from the various points of the vibrating source arrive in the same phase. Away from the axis, the intensity is less, diminishing to a minimum when the difference in the distance from the nearest and farthest points of the source are about $\lambda/2$, so that the effects of each point are neutralised. Still farther away from the axis, another maximum occurs, equal in intensity to about 0·017 of that along the axis, to be followed by other minima and maxima, similar to diffraction rings in optics.

The angle at which the first silence occurs is found to be

$$\sin^{-1}\left(\frac{0{\cdot}610\lambda}{r}\right).$$

Thus, provided that r is many times the wave-length, the central beam of sound will be confined to a small angle.

When r is not greater than $\lambda/4$, the intensity is nearly the same in all directions, and we have a non-directional source of sound. By using a Langevin type of piezo-electric quartz plate, R. W. Boyle has shown that the sound distribution in water is in good agreement with theory. In one example, $r=7{\cdot}65$ cm., $n=135{,}000$, V (water) $=1{\cdot}5\times10^5$ cm./sec.; hence $\lambda=1{\cdot}11$ cm. and the angle of the central beam works out to be 5°.

Double source. The vibrator just considered is a single source, that is, it radiates from one face only, all contributions to the disturbance originating at the source being in the same phase. But a vibrating diaphragm suitably mounted in a ring will radiate energy from both faces, though such waves will be opposite in phase, and this type of vibrator is termed a double source. Such a source, situated in an open space in air or water, is inaudible from any point in its equatorial plane, the maximum sound being received at right angles to this plane.

Also, it is reasonable to expect a double source to be a poor

radiator of sound energy, since two wave trains of opposing nature are set up; this is particularly so at low frequencies. There are numerous examples of a double source. In all stringed instruments, the transverse motion of the vibrating string sets up two out-of-phase disturbances originating on opposite sides of the string. A tuning fork is really a *double* double source, since each prong sets up its own pair of disturbances; the cone of a loudspeaker, vibrating freely in air without any kind of baffle, is another example of a double source

Sources used with mirrors, trumpets, etc. A source used in conjunction with a mirror or a trumpet will exhibit directional properties only if the dimensions of the mirror are not small compared with the wave-length; high-frequency sounds give of course, better effects than low-frequency. The area of the source should also be small, but in this case, if a large output of sound is required, the amplitude must be large, and this militates against efficient transmission both in air and water. The use of mirrors, etc., is probably more suited to directional reception of high-frequency sounds rather than to transmission.

Multiple sources. A number of small "point" sources in line and suitably arranged will have definite directional properties. Suppose we have m equidistant sources all in a straight line and identical in phase, amplitude and frequency. It can be shown that the resultant amplitude r at any distant point P is given by

$$r = \sin\left(\frac{m\pi d}{\lambda} \cdot \cos\alpha\right) \Big/ m \sin\left(\frac{\pi d}{\lambda} \cdot \cos\alpha\right),$$

where d is the distance between the sources, each of amplitude $1/m$, and α is the angle between the line of the sources and P.

If $d = \lambda/2$, then

$$r = \sin\left(\frac{\pi}{2} \cdot m\cos\alpha\right) \Big/ m \sin\left(\frac{\pi}{2} \cdot \cos\alpha\right),$$

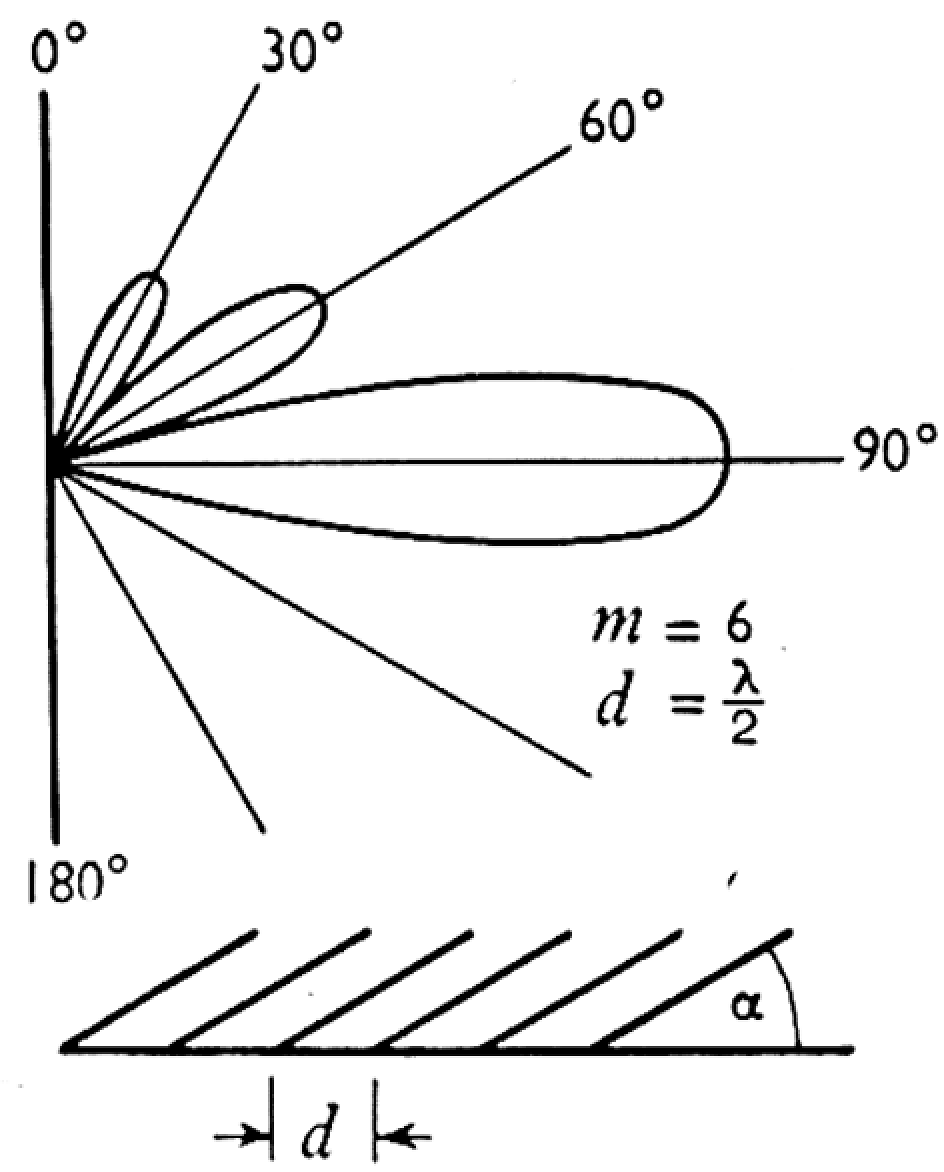
and this is equal to zero when

$$\sin\left(\frac{\pi}{2} \cdot m\cos\alpha\right) = 0,$$

that is, when $\left(\frac{m\cos\alpha}{2}\right)$ is an integer.

For example, if there are six sources spread $\lambda/2$ apart, and if we make

$$\frac{m\cos\alpha}{2} = 1, 2 \text{ and } 3 \text{ in turn,}$$



the values of α become 70° 48′, 48° 42′ and 0°, giving the direction OP of zero amplitude. The primary maximum occurs when $\alpha = 90°$, and there are secondary maxima at $\alpha = 60°$ and 30° approximately. The polar distribution from 0 to π is shown in the diagram, the value of r being plotted radially. If d has other values, some of the secondary maxima may approach the primary in magnitude.

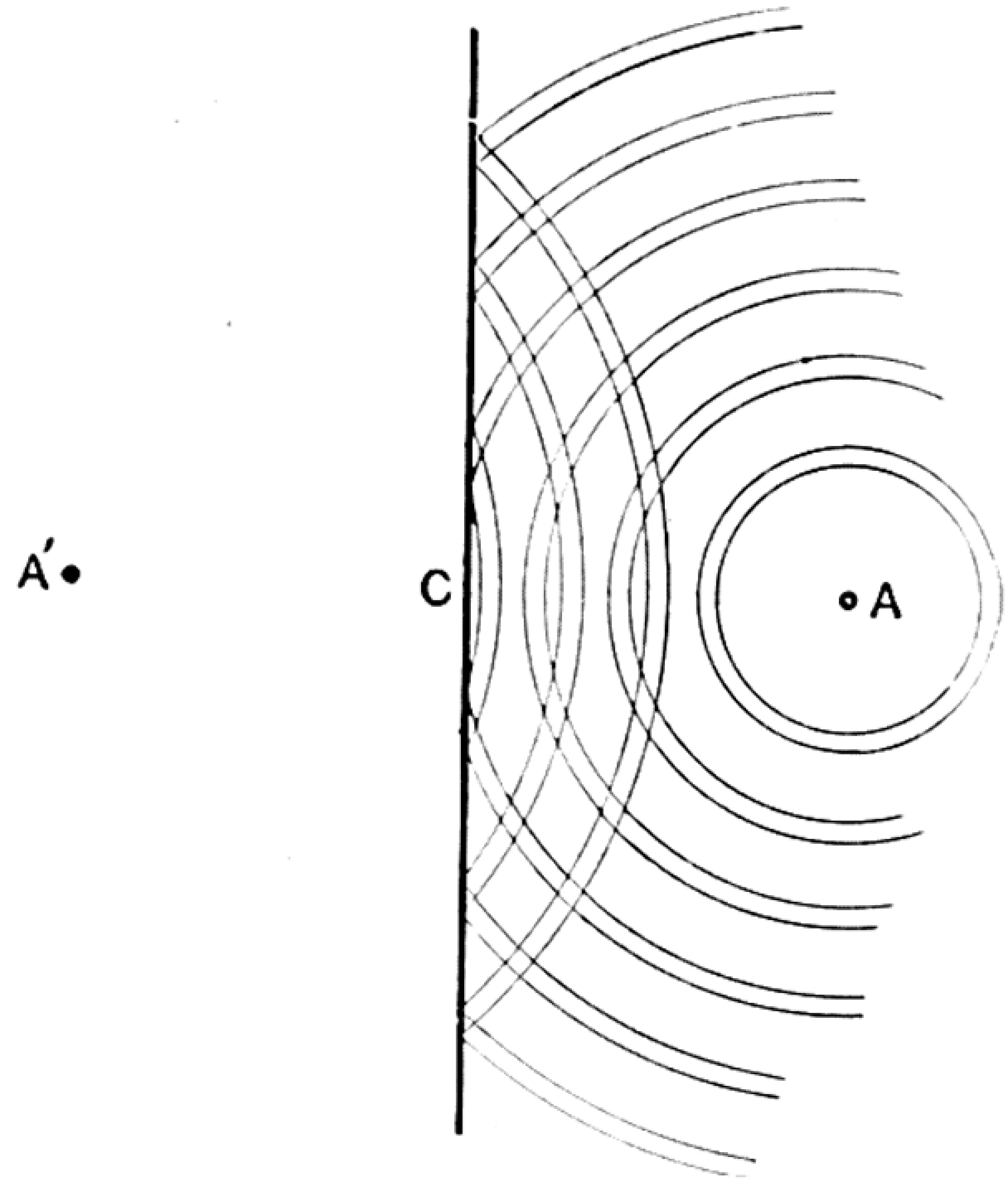
It appears then from the above that a vertical line of sources spaced $\lambda/2$ apart will give a concentration of energy in a horizontal plane at right angles to the line of sources. This principle is used in some sound amplifying systems.

REFLECTION OF SOUND

Reflection of sound may occur under a variety of conditions, as we shall see in the succeeding pages, and it can be stated here that, in general, it occurs whenever there is a discontinuity of the medium. It must also be borne in mind that, with sound waves, we have often to deal with wave-lengths comparable with the dimensions of reflecting objects, and the phenomena must then eventually be regarded primarily as diffraction rather than reflection.

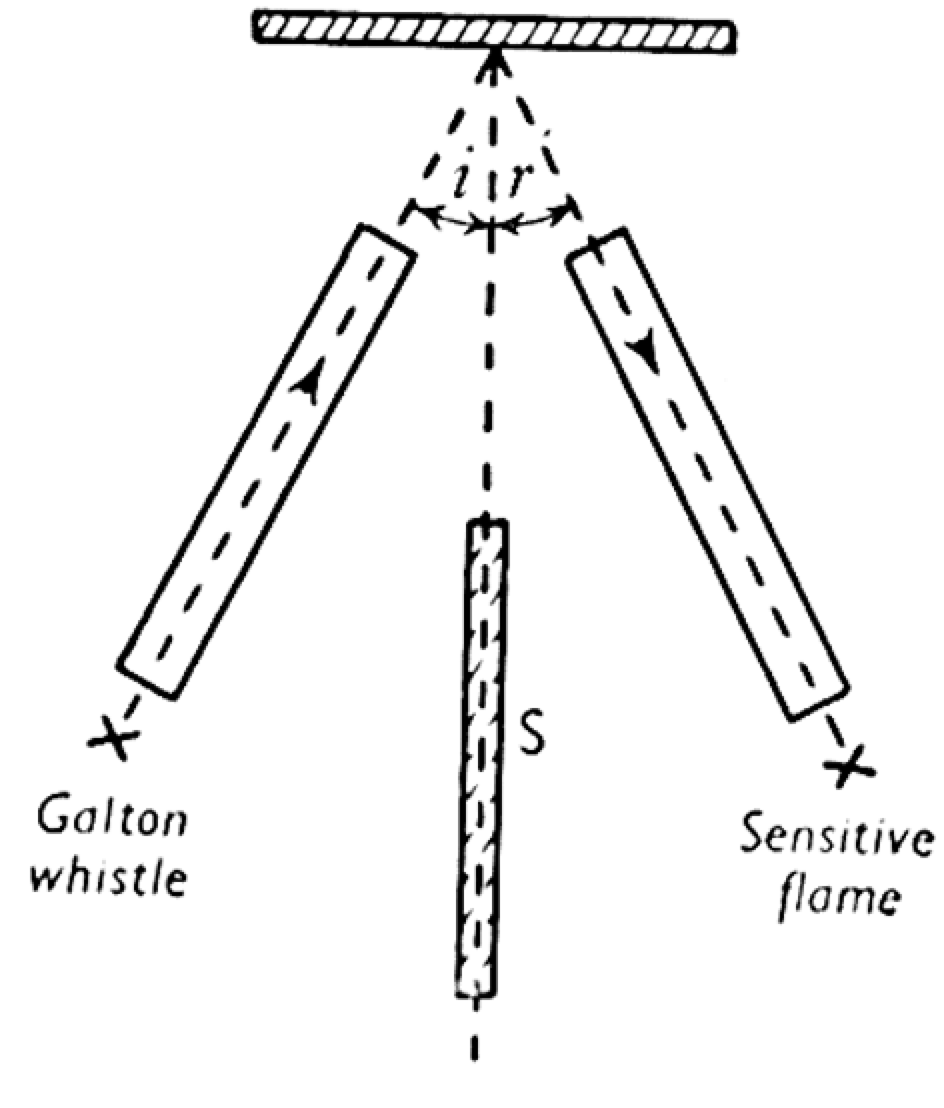
Reflection at a plane surface. When a compression meets a rigid surface, it cannot go beyond the surface, but has its direction reversed. Thus the wave is reflected, and for regular reflection the same law is obeyed as in light, namely, that the angle of reflection is equal to the angle of incidence. As each part of the spherical wave reaches the surface its direction is reversed ; the reflected wave is still spherical with the same curvature, but it is travelling away from the surface as though the centre of the disturbance were at A'. Hence A' is said to be the **acoustic image** of A.

If the size of the reflecting surface is small compared with λ, the reflection will not be in the form of a beam but will be scattered in all directions, and, of course, between the two extremes there will be graduations from a well-defined beam to diffuse scattering. In

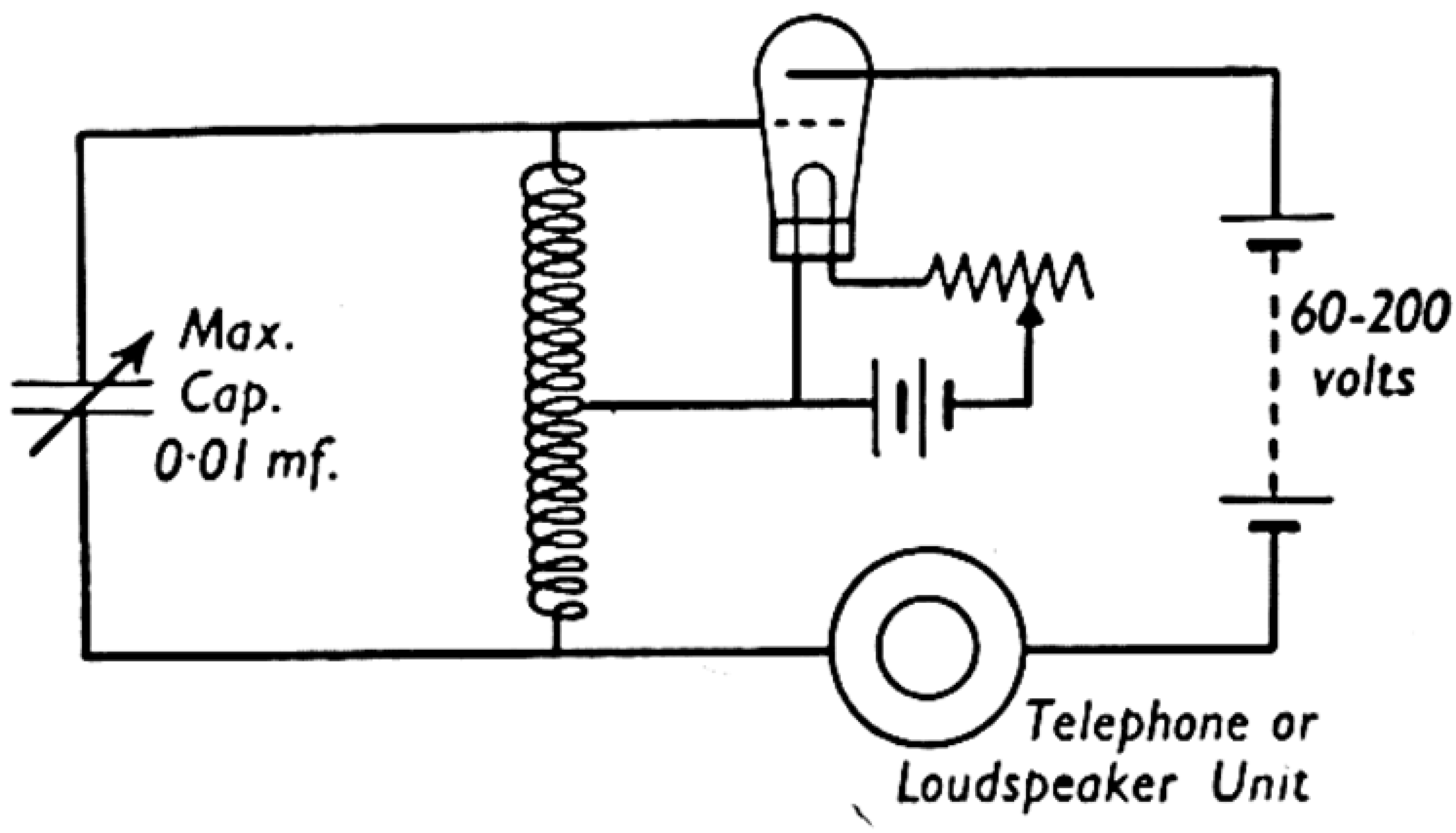


this connection it is interesting to notice that whereas for reflection of light a smooth reflecting surface is necessary on account of the short wave-lengths, a rough surface can be used for sound provided that the variations from planarity are small compared with the wave-lengths of the sound waves.

To demonstrate reflection of sound at a plane surface a source of sound such as a Galton whistle may be used, with a sensitive flame as a detector. The whistle is put at one end of a cardboard or metal tube and the flame at one end of another tube, both tubes being inclined towards the reflecting surface as shown in the diagram. The whistle is sounded and the surface turned until the flame shows its maximum disturbance, thus indicating that the sound has travelled along the lines of the tubes. It will be found that the angles of incidence and reflection are equal. A screen S should be arranged to shield the flame from the direct sound from the whistle.



For experimental work in connection with reflection, refraction, etc., of sound it is best to use a source of high frequency, and besides the Galton whistle, a convenient source is a valve oscillator. The diagram shows a simple oscillating circuit, due to S. R. Humby, which of course can be used with a sensitive flame as detector.



Reflection of plane waves at the boundary of two media. Although this problem involves the refraction of sound, yet it is important to consider what fraction of the original energy is reflected when a sound wave reaches the surface of separation of two different media.

If r and a are the amplitudes of the reflected and incident waves respectively, Rayleigh showed that the relation between the two is given by

$$\frac{r}{a}=\left(\frac{\rho_2}{\rho_1}-\frac{\cot\theta_2}{\cot\theta_1}\right)\bigg/\left(\frac{\rho_2}{\rho_1}+\frac{\cot\theta_2}{\cot\theta_1}\right),$$

where ρ_1 and ρ_2 are the densities of the first and second media, and θ_1 and θ_2 are the angles of incidence and refraction.

If we assume that the law of sines of optical refraction holds in this case, we have

$$\frac{\sin\theta_1}{\sin\theta_2}=\frac{V_1}{V_2},$$

where V_1 and V_2 are the velocities of sound in the two media. Therefore the equation above becomes

$$\frac{r}{a}=\frac{\dfrac{\rho_2}{\rho_1}-\dfrac{V_1\cos\theta_1}{V_2\cos\theta_2}}{\dfrac{\rho_2}{\rho_1}+\dfrac{V_1\cos\theta_1}{V_2\cos\theta_2}}, \qquad \text{.......................(1)}$$

which is the general expression for any angle of incidence.

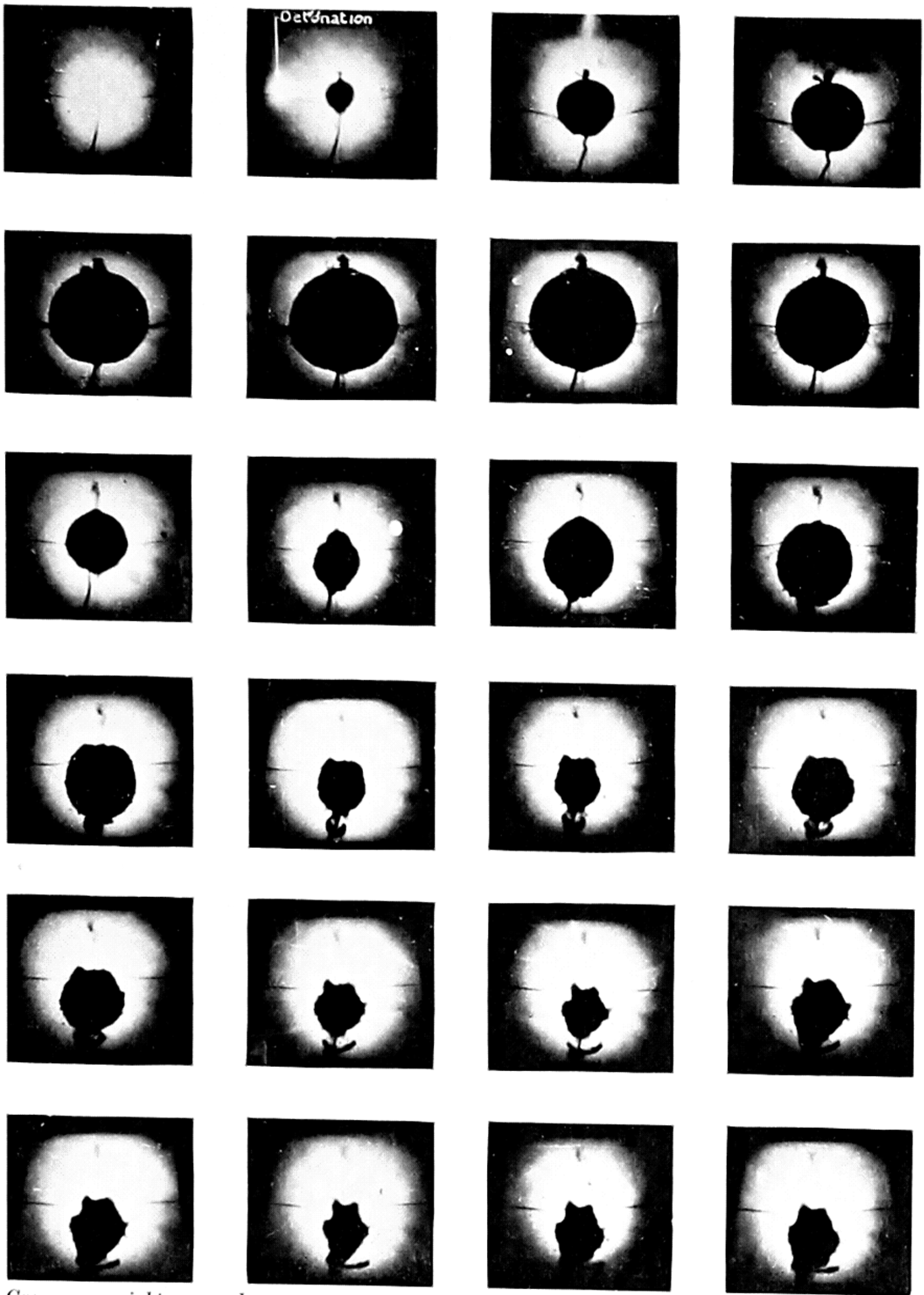
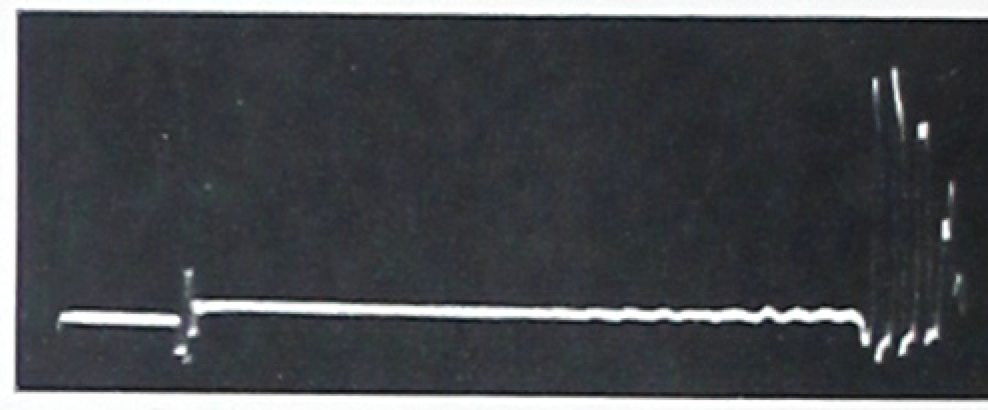


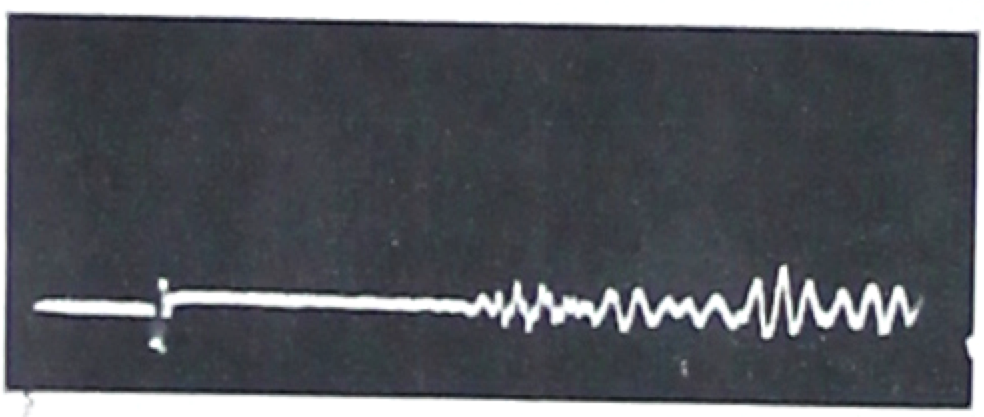
PLATE 1. Sequence of photographs showing the oscillatory increase and decrease in the size of the bubble of gas from an underwater explosion. The time between the first four pictures is 1/750 sec., and between subsequent pictures 1/250 sec.

(*a*) Rough oiled surface.

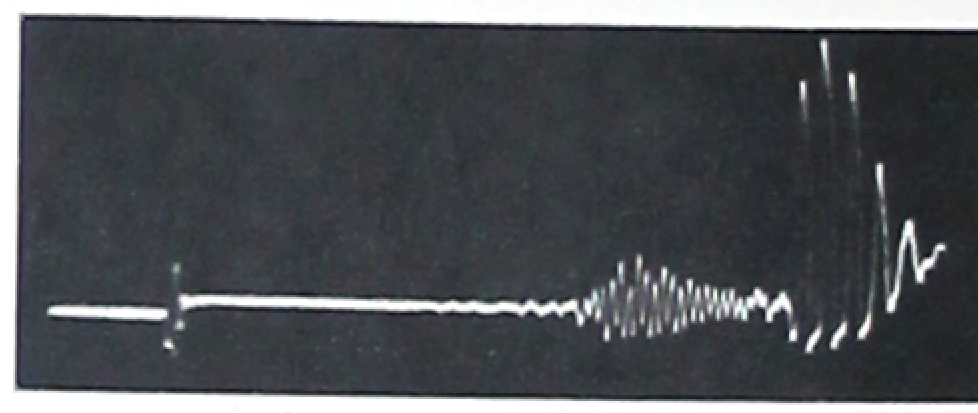


(*b*) Smooth oiled surface.

SOUND SAMPLE.



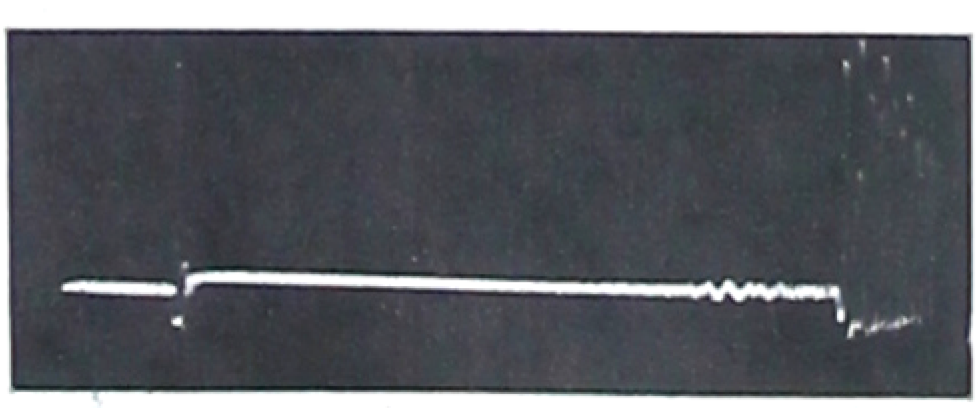
(*c*) Rough oiled surface.

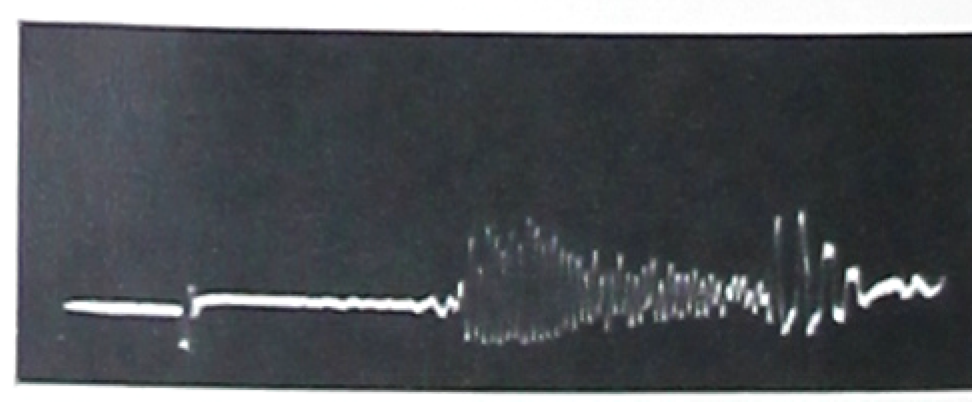


(*d*) Smooth oiled surface.

SAMPLE WITH FLAW.

(*e*) SAMPLE WITH BAD FLAW.





(*f*) SAMPLES COVERED WITH TIN AMALGAM.

By courtesy of Messrs. Henry Hughes & Son, Ltd.

PLATE 2. Oscillograms obtained by testing a square section steel sample under various conditions.

If the incident wave is normal to the surface of separation, then $\theta_1 = \theta_2 = 0$, and the equation becomes

$$\frac{r}{a} = \frac{\rho_2 V_2 - \rho_1 V_1}{\rho_2 V_2 + \rho_1 V_1} \quad \text{.........................(2)}$$

It will be noticed that in this case the relation between r and a depends only on ρ and V. The product ρV is known as the **radiation impedance** or **resistance** for unit area, and since $V = \sqrt{k/\rho}$, where k is the bulk modulus of elasticity

$$\therefore \ \rho V = \sqrt{k\rho}.$$

Further, it will be seen that the expression for the value of r/a does not involve frequency; therefore, the intensity, which is proportional to the square of the amplitude of the reflected beam, is independent of the frequency of the source.

As an example of the above, consider the case of a sound wave in air incident normally on the surface of separation between air and water. We may take the value of ρV for air to be about 40 and for water 14×10^4, and from these figures we get $r = 0{\cdot}9994a$. Thus there is almost complete reflection even at normal incidence. The reflection would be zero and the transmission a maximum when $\rho_1 V_1 = \rho_2 V_2$, that is, when the two impedances are equal.

Whenever the product ρV for the first medium differs from that for the second, there will be both a reflected wave and a transmitted wave.

Equation (2) above is a general equation which holds for all fluids in general. It can, of course, be adapted to apply to gases only by substituting $\sqrt{\gamma_1 p/\rho_1}$ for V_1, etc. The equation then becomes

$$\frac{r}{a} = \frac{\sqrt{\gamma_2 \rho_2} - \sqrt{\gamma_1 \rho_1}}{\sqrt{\gamma_2 \rho_2} + \sqrt{\gamma_1 \rho_1}} \quad \text{or} \quad \frac{\gamma_2 V_1 - \gamma_1 V_2}{\gamma_2 V_1 + \gamma_1 V_2}.$$

If we take $\gamma_1 = \gamma_2$, as is often the case, the equation reduces to

$$\frac{r}{a} = \frac{\sqrt{\rho_2} - \sqrt{\rho_1}}{\sqrt{\rho_2} + \sqrt{\rho_1}} \quad \text{or} \quad \frac{V_1 - V_2}{V_1 + V_2}.$$

Reflection at a plate of finite thickness. This problem is analogous to that of optical reflection from a thick plate of glass, the reflected wave being the resultant of multiple reflections at the two boundary surfaces.

It can be shown that the amplitude-relation for normal incidence is given by :

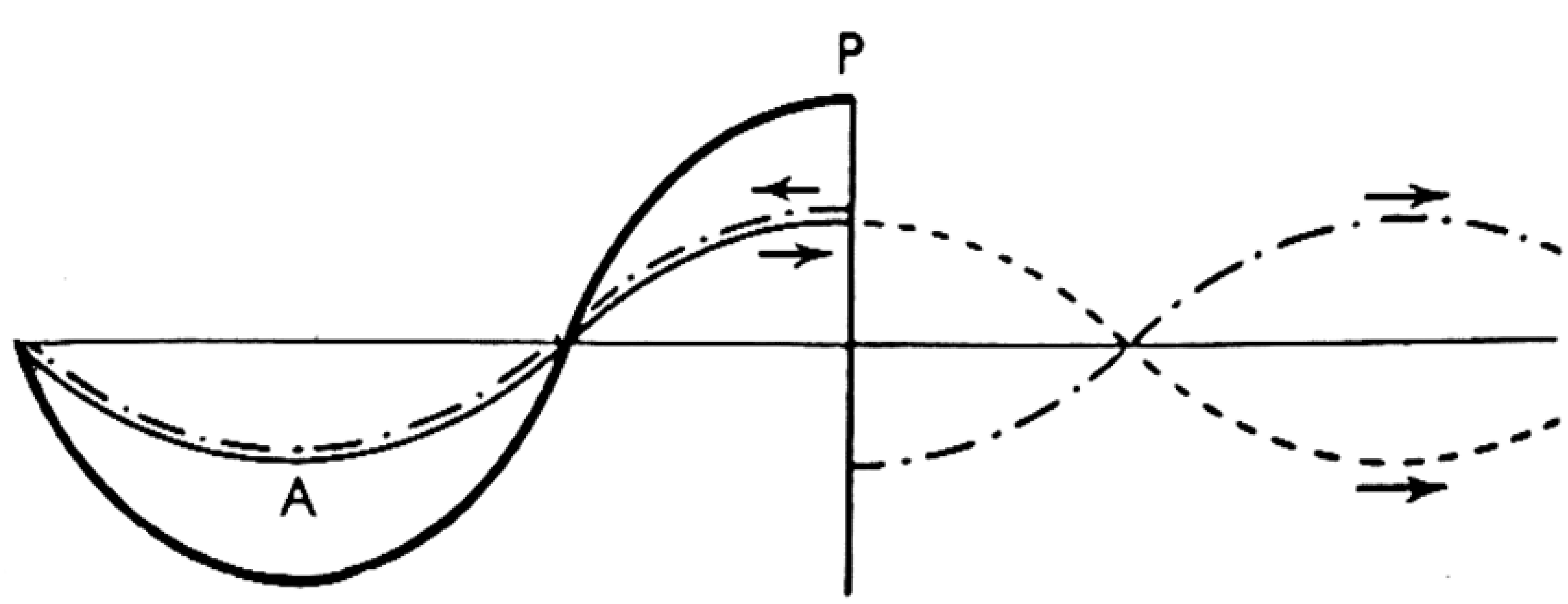
$$\frac{r}{a}=\left(\frac{\rho_1 V_1}{\rho_2 V_2}-\frac{\rho_2 V_2}{\rho_1 V_1}\right)\div\left\{4\cot^2\frac{2\pi l}{\lambda^2}+\left(\frac{\rho_1 V_1}{\rho_2 V_2}+\frac{\rho_2 V_2}{\rho_1 V_1}\right)^2\right\}^{\frac{1}{2}},$$

where l is the thickness of the plate and λ is the wave-length of the sound in it. From this equation, it will be seen that the reflected amplitude fluctuates between zero and a maximum as the thickness of the plate varies. If l is equal to zero or a multiple of $\lambda/2$, the reflected amplitude is zero ; but when l is a multiple of $\lambda/4$ we get maximum reflection. Hence a *quarter-wave plate* will reflect a maximum and will transmit a minimum of incident sound energy.

The relationships for the reflection of plane waves from flat plates have been verified in the case of high-frequency sound-waves passing through water in which the plate was submerged.

From the last two sections, it appears that solid media in air are practically perfect reflectors of sound, but that in water they are relatively good transmitters. Also, an air film in water or in a solid constitutes a perfect reflector, with a reversal of phase at reflection. These facts are, of course, important in the case of sound reception under water.

Reflection in a closed tube. When sound waves travel through a tube, they may be regarded as plane waves. If a tube is closed at one end, the previous discussion shows that a compression wave started at the open end will be reflected at the closed end. Let a compression wave, represented by the curve A be incident upon a rigid wall P. A pressure is exerted on P ; hence P reacts on the air and gives rise to two waves, one to the right of P and the other to the left. It will be obvious that these waves are opposite in phase.



The two waves thus set up by P are represented by the chain-curves in the diagram. It will be seen that the wave to the right cancels out the continuation of the incident wave (dotted curve) to the right. This again is to be expected, since, as the wall is rigid, no energy is transmitted through it. The wave to the left is the reflected wave, and P is a position of maximum variation of pressure. It will be noted in this case that a *compression is reflected as a compression*, and the resultant effect of the two curves is indicated by the thick black line. The diagram, of course, only indicates the conditions in the medium at *one given instant*, but other curves can be drawn to show what is happening at other times. The equation of the incident wave is

$$y = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right) \quad \text{(see p. 4)};$$

and the wave to cancel the continuation of this is

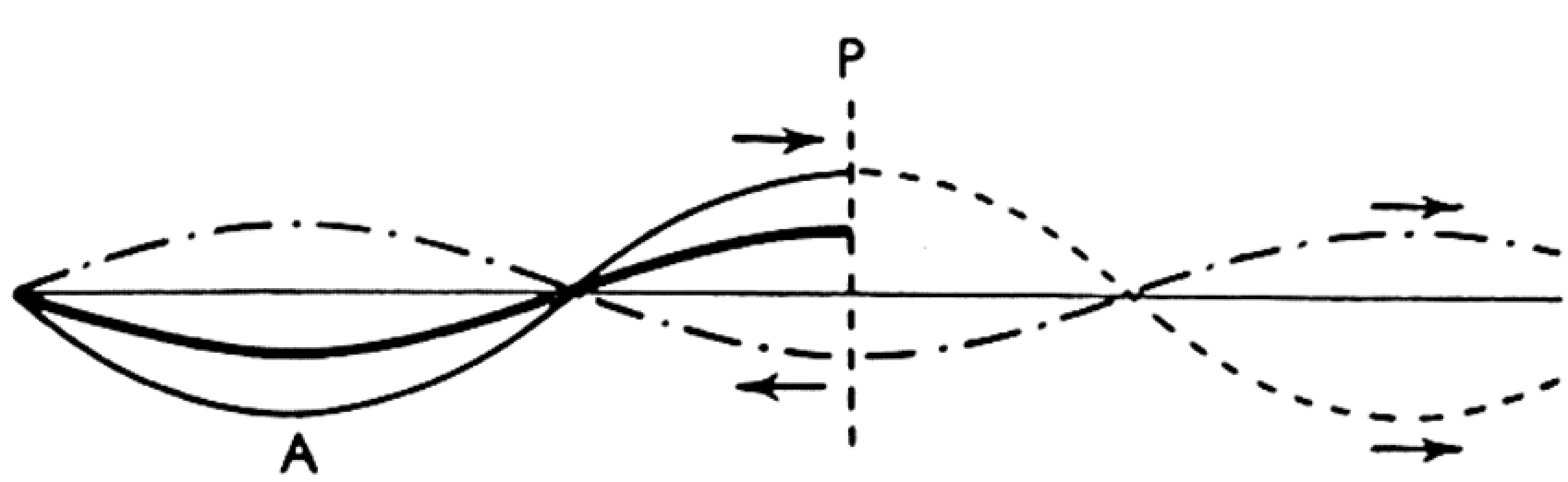
$$y = -a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right).$$

The twin wave to this is

$$y = a \sin 2\pi \left(\frac{t}{T} + \frac{x}{\lambda}\right),$$

and this is the reflected wave.

Reflection in an open tube. Reflection of a compression wave can also take place at the end of an *open* tube, because there is a discontinuity of the medium owing to pressure changes. When a compression wave reaches the end of the tube, expansion takes place in all directions, and the tendency of the open end is to destroy the excess pressure in the compression. Let A be the curve for a compression wave travelling along the tube and reaching the open end at P. The tendency always is to bring pressure to normal, the air moving to the right of P when a compression arrives and to the left when a rarefaction arrives. Thus



a condition is set up at P whereby some of the energy emerges to the right of P and is transmitted, and some is reflected. In the diagram the chain-curve to the right of P *partly* neutralises the continuation of the incident wave (dotted curve) but not completely owing to the transmission, and the twin wave to this is the reflected wave moving to the left. These waves are in phase with each other at P, for a movement of air outwards from P causes a rarefaction to start in both directions ; and conversely, a movement inwards starts compressions. Thus in this case a *compression is reflected as a rarefaction*, and vice versa.

The equation of the incident wave is

$$y = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right),$$

and the wave to cancel partly the continuation of this outwards is

$$y = -a' \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right).$$

Note the change in amplitude.

The reflected wave is

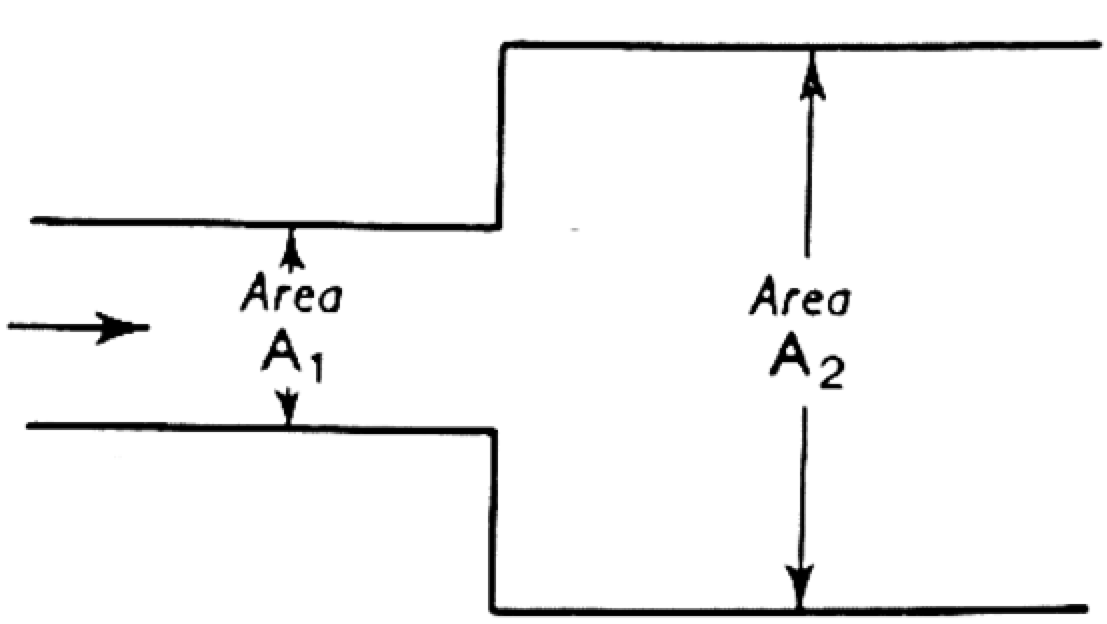
$$y = -a' \sin 2\pi \left(\frac{t}{T} + \frac{x}{\lambda}\right).$$

These equations and those in the previous section will be referred to again in dealing with steady vibrations or the so-called stationary waves.

Reflection at a change of area in a tube. When a sound wave travelling through a tube comes to an abrupt change in the cross-sectional area, part of the energy will be transmitted and part reflected. Here again there is a discontinuity of the medium on account of the change in the area. It can be shown by reasoning beyond the scope of this book that the ratio of the reflected intensity to the incident intensity is given by

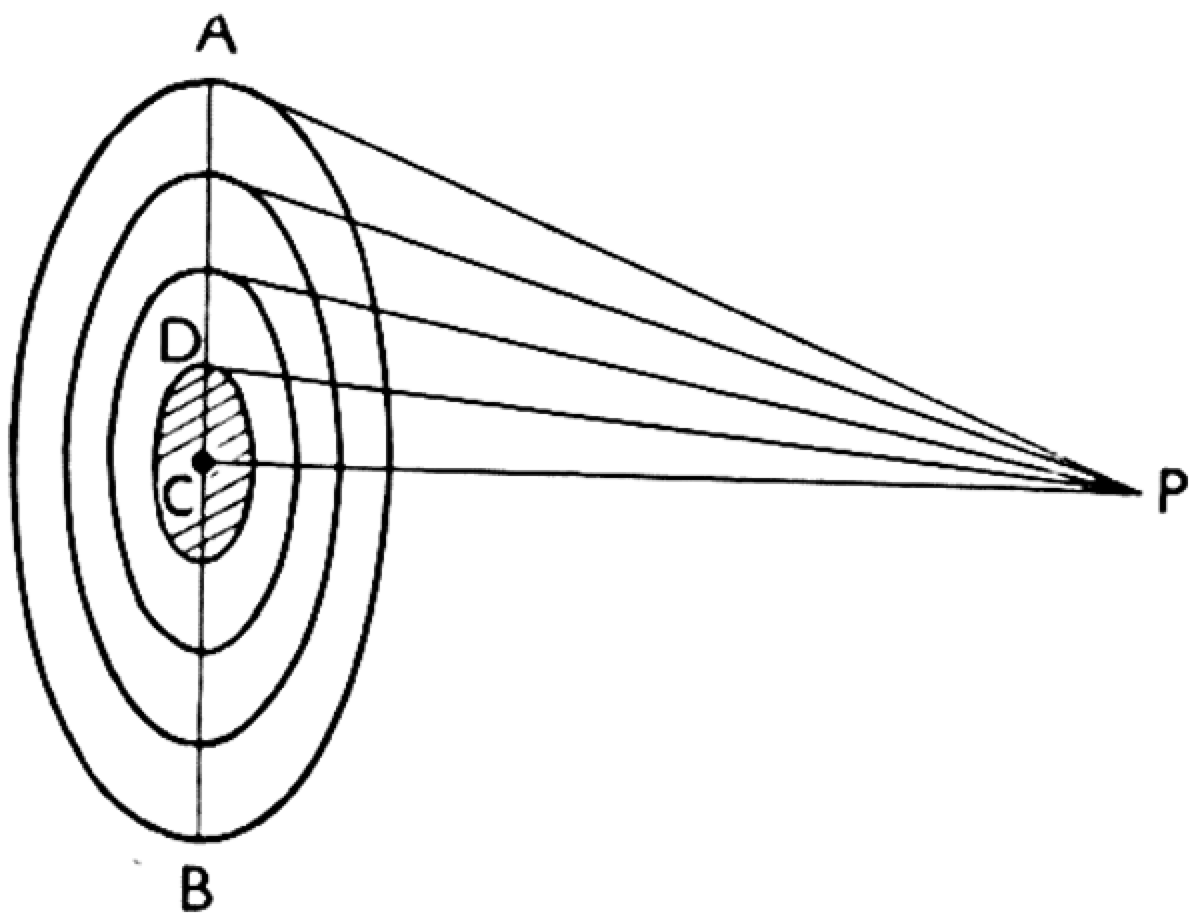
$$\left(\frac{A_2 - A_1}{A_2 + A_1}\right)^2,$$

where A_1 and A_2 are the two areas of cross-section, and as this value is independent of the sign of $A_2 - A_1$, the result is the same for either direction of the wave.



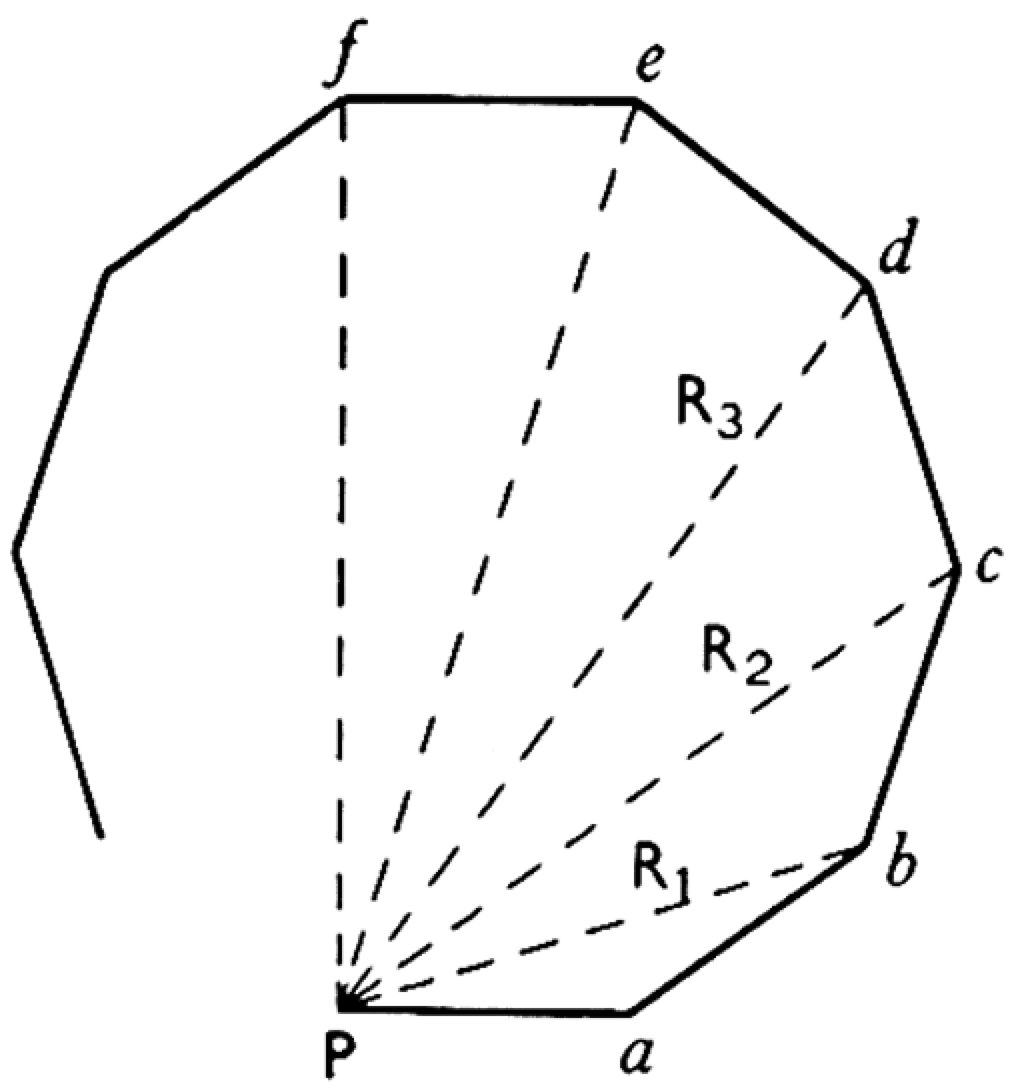
When $A_2 > A_1$, that is, the sound is passing from smaller to larger cross-section, the incident and reflected waves are opposite in phase, thus corresponding to the passage of sound from a dense to a rare medium. If $A_2 < A_1$, the two waves are in the same phase, corresponding with transition from a rare to a dense medium. The change in phase is always either zero (when $S_2 < S_1$) or π (when $S_2 < S_1$). If $S_2 = S_1$ there is of course no reflection at all. The transmitted wave is always in phase with the incident wave.

Selective reflection at a plane surface. It is interesting to examine more fully the effect at a point of a plane wave reflected from a plane surface. To do this, consider the effect at a point P on the axis of a circular plate ABC which is vibrating so that all points of it are in the same phase. The resultant effect at P will



be due to the superposition of the effects from each point of the plate. Consider the central part of the plate, the circle shaded in the diagram, to have a radius such that the path difference between PD and PC is small compared with the wave-length of the sound emitted. Now draw a series of concentric circles with centre C of such radii as will ensure that each annulus has the same area as the central circle; this will be the case when the radii are in the ratio of the square roots of the natural numbers.

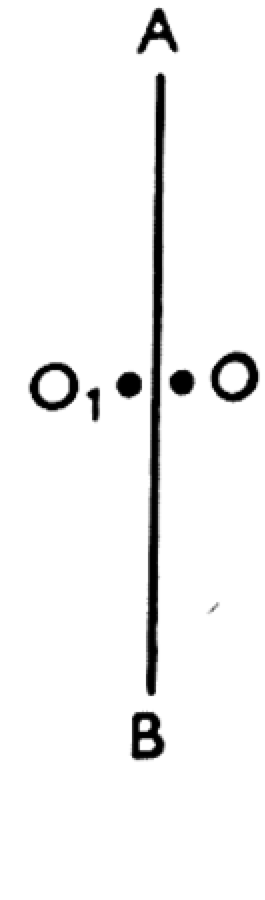
It is clear that the effect produced at P by each annulus of the vibrating plate will differ in phase and in amplitude, though we may assume for the moment that the amplitudes are equal. Therefore to obtain the resultant effect at P, we can draw a vector diagram (see p. 60). The effective amplitude at P due to



the central circle is represented by Pa, and the effect due to the first annulus is represented by ab. Thus the resultant effect due to both is given by Pb. Similarly, we can consider the effect due to all the annuli, and we shall eventually reach an annulus such that the path difference between its distance from P and the distance PC is exactly $\lambda/2$. This is represented by ef, and the resultant effect at P, represented by Pf, will be a maximum. After this stage is reached, the resultant decreases as further annuli are considered, and becomes a minimum when the path difference is λ. It is clear that, as the area of the plate is increased, there will be a succession of maxima and minima, which depend on the frequency of the sound.

Hence, we conclude that for a given frequency there is an optimum size of reflector to give maximum effect; and conversely, a reflector of definite size will produce a maximum effect for one definite frequency.

In connection with a vibrating plate, it is instructive to consider what happens when such a diaphragm is put in close proximity to a plane reflecting surface. If the source is at O, very near the plane surface AB, the direct sound travelling to the right of O will be reinforced by that reflected from the wall, equivalent, as we have seen, to the sound from the acoustic image O_1. These two sounds are in the same phase, and as O and O_1 are separated by a distance small compared with λ, we have the equivalent of two equal sources radiating energy to the right of O. Hence, the total energy radiated is twice that radiated without the reflected surface. Such a reflector is called a **baffle plate**, and is found in some forms of loudspeakers, etc.; the principle is also used in connection with the canopy arranged over the head of a speaker in the open air.



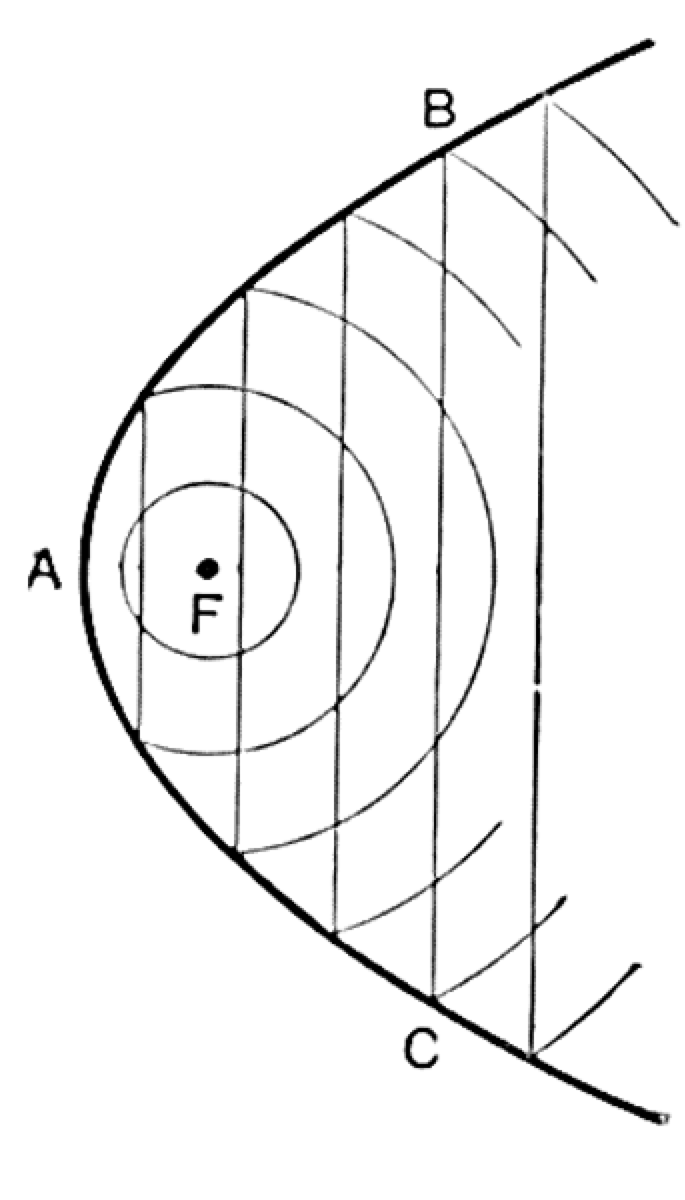
It should be noted that if the source is in a dense medium and the reflector is the boundary between this and a much rarer medium, the opposite effect is

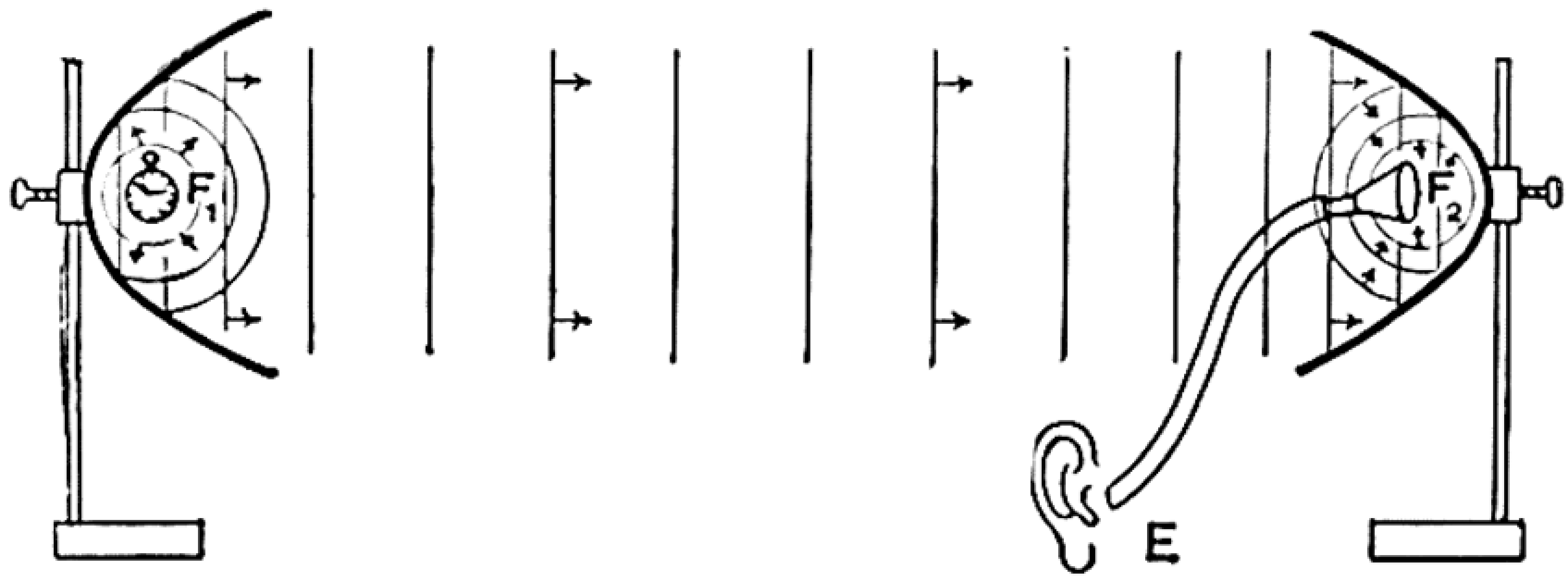
produced and much less energy is radiated. This is because the sound from O and that from its acoustic image are now opposite in phase.

Sound shadows. In order to obtain sound shadows comparable to those produced in optics, it is essential that the surface to cast the shadows should be large compared with the wave-length of the sound. The ticks of a watch contain some components of high frequency, and so are effectively screened by a relatively small surface, but a man's voice requires a larger screen. In these two cases the wave-lengths are of the order of a few inches and several feet respectively; hence the linear dimensions of the reflecting screens must be in the same proportion to obtain the same degree of screening. It is interesting to notice that behind the screen the sound appears to have changed in quality, the high-frequency sounds being screened more perfectly than those of lower frequency; in this way the screen can be regarded as a sound filter.

Reflection at a curved surface. When a compression wave meets a curved surface, reflection occurs as in the optical case, and the reflected wave has not the same curvature as the incident wave, the exact form depending on the curvature of the surface which may be spherical or parabolic or any other shape. If a parabolic reflector is used, the reflected wave is plane when the source is put at the principal focus. Conversely, when a plane wave strikes such a "mirror", it is reflected so that the sound energy is directed towards the focus, similar to the parallel case in optics.

The above effect of a parabolic reflector requires, however, a certain qualification, concerning both optics and sound. More careful study has shown that the reflected wave in optics does not come to a focus at a mathematical point, but at a region which has the dimensions of the order of a single wave-length. Sound behaves in a similar way, and on account of the relatively long length of the sound waves, it will be realised that sound focusing is very rough compared with the case of light; high-frequency sounds are, of course, much more sharply focused than low ones.





Reflection of sound by a parabolic mirror can be demonstrated in the following way. A watch, the ticks of which contain some sounds of high frequency, is put at the focus F_1 of a parabolic reflector. The plane waves emitted are allowed to fall on the second parabolic reflector, the axis of which is coincident with that of the first. The waves are then reflected and become spherical, and the energy is brought approximately to the focus F_2. If a funnel is held in this neighbourhood, and an india-rubber tube from it is brought to the ear E, the ticking of the watch may be heard.

Whispering gallery effects. The well-known Whispering Gallery at St. Paul's Cathedral, London, owes its peculiar acoustical properties to the reflection of sound by the walls, though perhaps the exact mode of action is still not fully understood. Lord Rayleigh pointed out that the sound tends to creep round the inside of a curved wall without ever getting very far from it ; also that the abnormal loudness with which a whisper is heard is not confined to the position diametrically opposite to that occupied by the whisperer, and therefore, it would appear, does not depend materially on the symmetry of the dome. Whispering is generally heard more distinctly than ordinary conversation, and in this connection it is significant that whispers contain a higher proportion of high-pitched sounds than ordinary speech.

Rayleigh further suggested that the propagation of earthquake disturbances is probably affected by the curvature of the earth acting like a whispering gallery. It is also possible that sounds travelling long distances in the sea are dependent on a similar action and on repeated reflection at the surface and the bottom.

ECHOES

A common result of the reflection of sound is the production of echoes. If a person makes a short sharp sound in front of a suitable reflecting surface, the reflected sound may be heard as a definite and separate sound by the listener after a certain time

interval, this constituting the echo of the original sound. The human ear is unable to distinguish separate sounds unless the time interval between the two is about $\frac{1}{10}$ second. Hence, to produce an echo which can be detected by the ear, the reflecting surface must be a minimum distance from the listener, this depending on the velocity of sound. For example, if the velocity of sound is taken as 1,100 ft. per sec., the minimum distance works out to be 55 ft.; of course, the greater the distance, the bigger the time difference between the two sounds which reach the listener. If the time difference is less than $\frac{1}{10}$ sec., the reflected sound merges with the original sound and gives rise to **reverberation.** Both real echoes and reverberation are of great importance in connection with the acoustics of buildings and will be referred to again in Chapter XIII.

It will be obvious that the phenomenon of echoes affords a quick and moderately accurate method of finding the velocity of sound in air if the distance between the source and reflecting surface is known, also the time interval between the detection of the original sound and its echo.

Echoes may be classified under various headings. We have already mentioned the single echo and the overlapping echo, namely reverberation. Another type is the **multiple echo** which is due to a number of successive reflections, and of course, very often, prolonged reverberation is caused by overlapping multiple echoes.

Sometimes an echo is caused by the scattering of the sound by many small objects, this type being called a **diffuse echo.** Two other types will be mentioned in greater detail.

Harmonic echo. If a sound has a complex wave-form, the overtones, which are of higher frequencies than the fundamental, will be scattered or diffusely reflected more than the fundamental. Since the amplitude of the scattered sound varies inversely as λ^2, and the intensity varies inversely as λ^4, an observer near the source will hear the returning echo to be raised in pitch by perhaps one or two octaves, depending on the nature of the original sound.

Musical echo. When a sharp sound is made near one end of a row of palings, the resulting echo takes the form of a musical note. The successive palings each reflect the sound or perhaps the higher overtones, and the listener receives a succession of reflections, which, if sufficiently rapid, will give a note of a definite pitch, depending on the spacing of the palings and the position of the observer. For example, if δx is the path difference between the successive reflectors and the listener, and δt is the time interval

between the sounds heard, then $\delta t = 2\delta x/V$. Therefore, the frequency of the note will approximate to $V/2\delta x$. If δx is taken as 4 in. and $V = 1{,}100$ ft. per sec., the frequency will be about 1,650 cycles per sec., which is quite a high-pitched note.

The row of palings may be regarded as an echelon structure, and it can serve as the equivalent of a grating by which complex high-frequency sounds can be analysed.

PRACTICAL APPLICATIONS OF THE ECHO PRINCIPLE

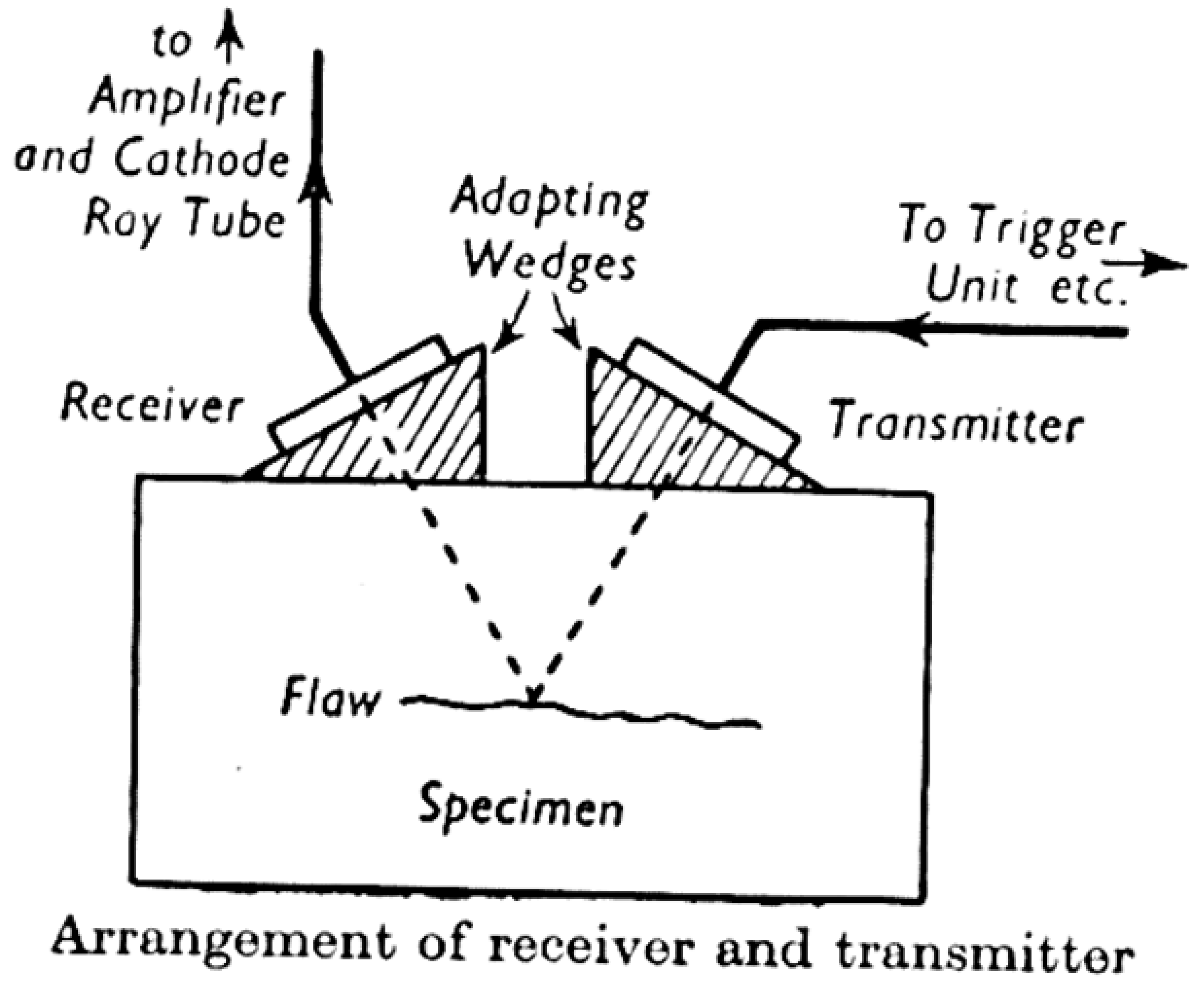
Apart from the utilisation of echoes for the determination of the velocity of sound, the principle has been increasingly applied in recent years to practical problems in various spheres of activity. In 1912, after the sinking of the liner *Titanic* by collision with an iceberg, a British engineer named Richardson suggested that icebergs might be detected by the echo of a pulse of sound waves emitted from the approaching ship. It was recognised that high-frequency waves, which can be readily " beamed " like a searchlight, are necessary in order to secure precise definition of the direction of the berg, and in 1914 the only known practical method of producing such ultrasonics was from oscillators made of mica and caused to vibrate by electrical stresses. Great strides have, of course, been made in this direction since then, and the method has been applied with success in the detection of icebergs. It must be remembered that since a portion of the berg is above the water-line, a more modern method of detection is by the use of Radar, which, however, cannot be employed for the detection of objects totally submerged. Therefore, for locating such objects, ranging from under-water mines to whales and shoals of fish and submarines, the echo method is invaluable. This problem will be more fully dealt with in Chapter XII. There are also other examples in which the principle of the echo is prominent, and two will be considered here.

Ultrasonic flaw detector. Most known materials will transmit ultrasonic waves a millimetre or less in length over a useful range in distance, and this opens up the prospect of detecting flaws in the interior of bodies opaque to light. One method which of course has previously been used is by X-rays, on account of their high penetrative power, and the gamma rays from the pin head of a " radium bomb " can penetrate steel 8 inches thick, forming a picture on an X-ray film to show any flaw in the metal. When comparing X-rays with the ultrasonic waves, it must be remembered that the former are not refracted or reflected appreci-

ably from the boundary of a solid or liquid medium, whereas the latter are generally reflected and refracted heavily. This very behaviour of ultrasonic waves really presents a difficulty in the detection of flaws, since even at normal incidence the energy due to transmission from the transmitter to the receiver may be small in comparison with the energy which has suffered multiple reflections between the parallel surfaces of the sheet of metal. As a result, a flaw which would otherwise be capable of detection is completely obscured by the multiple reflections.

In the method devised and developed by Messrs. Henry Hughes and Son, Ltd., Ilford, London, the difficulty is overcome by sending out a train of waves so short that it occupies a space less than the distance between the boundaries of the medium being investigated. Also the transmitted and reflected wave trains are shown on a cathode ray tube in such a way that they do not interfere, and it is possible to detect a very small echo even though it may be followed by reflections containing much more energy. This effect is secured by employing an electronic triggering system which first sets the cathode-ray time base in operation, so starting the spot on its way across the screen, and then a very short time later causes a short transmission to be sent out. The echo and the subsequent reflections cause a transverse deflection of the spot, and the result is an oscillogram of the sort shown in Plate 2 facing p. 55. The transmission time mark on the left is due to electro-magnetic induction effects accompanying transmission and picked up by the receiver amplifier.

The transmitter and receiver are composed of similar crystals of quartz, each 2 cm. in diameter, approximately 0·1 cm. thick, with a natural frequency of 2·5 mega-cycles. The transmitter is



Arrangement of receiver and transmitter

mounted on a steel wedge, and this directs the beam of ultrasonic energy into those regions of the specimen which are to be tested for flaws. The receiver is also tilted by a similar wedge to make it sensitive to the reflected energy. It should be noted that the maximum beam intensity is on a line perpendicular to the line of the quartz and the maximum receiver sensitivity is along the same line. Intensity falls off each side of this maximum at a very slow rate for a few degrees, and then much more rapidly, so that it is zero at perhaps 8° off the axis on either side. At greater angles from the axis there are a number of other maxima and minima (see p. 52), but it is the main beam which is the important one in searching for slight flaws. It is clear then that the maximum echo will be received from a given flaw if it is located on the intersection of the axes of the transmitter and receiving systems. Therefore, if it is desired to detect an echo from a different distance, it will be necessary to alter the separation of the wedges, or alternatively, to use wedges which incline the beam axis at a different angle.

The equipment briefly described above was designed to detect flaws within a region of approximately $\frac{1}{2}$ in. from the surface to 12 ft. from the surface, and between these limits useful results have been achieved. The nature of the surface has a profound effect on the clarity of the indication obtained (Plate 2, facing p. 55). Using a sound sample of steel and applying the transmitter and receiver to the *rough* side, merely oiled, gives the indication contained in oscillogram (*a*). This should be compared with (*b*), which is the oscillogram for the same sample but using a *smooth* oiled surface ; in these two cases there is bottom echo only. Oscillograms (*c*) and (*d*) are for a sample with a flaw, and here again the smooth oiled surface (*d*) indicates the flaw more clearly than the rough (*c*). Oscillogram (*e*) for a badly flawed sample is interesting since there is practically no bottom echo.

It has been found that the use of an amalgam instead of oil very much minimises the effect of the rough surface. Oscillograms (*f*) and (*g*) were obtained using a tin amalgam on the same surface as that employed for (*a*) and (*c*).

Flaws can be detected in samples with curved surfaces by the use of special adaptors.

Ultrasonic cries of bats. It is well known how cleverly bats can avoid obstacles while flying in complete darkness, and much investigation has been undertaken to find the cause of this skill of manoeuvre. The pioneer of these investigations is Prof. H. Hartridge, formerly of St. Bartholomew's Hospital Medical

College, London, who, in 1920, advanced the hypothesis that during flight bats emit a short wave-length note, and that this sound is reflected to the bat from objects in the vicinity. This hypothesis is now generally accepted as correct, and much more investigation into the nature and origin of such ultrasonic sounds has been made by Prof. Hartridge, and also by Dr. R. Galambos and Dr. D. Griffin in the United States, by Dijkgraaf and others. The whole problem is of interest chiefly to students of wild life and of the science of flight, but it is mentioned in this book on account of the acoustic principles involved.

It seems certain that a bat produces different kinds of sounds, either through the nose or the mouth. This point does not yet appear to be agreed upon, but according to Dr. Griffin the origin of the sound emitted by the bats he experimented on is the mouth. In 1950 it was confirmed experimentally by F. P. Mochres that certain kinds of bats do employ the nasal apparatus rather than the mouth for sound emission. Prof. Hartridge states that there are four of these sounds ; (1) a buzz which is not audible unless the listener is quite close to the animal, (2) a signalling tone of about 7,000 c.p.s. which is probably used for sending messages to neighbouring bats ; (3) the ultrasonic tone which probably has a frequency of about 50,000 c.p.s. and which is used for the task of " echo-location " as Dr. Griffin has termed it, and (4) a click which is usually audible anywhere in a small room. Dr. Griffin pointed out in 1942 that the ultrasonic pulse and the click always occur simultaneously, the buzz being merely the rapid repetition of the click at rates as high as 60 per second. In 1945 the same investigator further tentatively suggested that the audible click results from the abrupt starting or stopping of the ultrasonic pulse. He obtained accurate pictures of the actual sound waves present in the bat's pulse by photographing the record of the bat's cry obtained on a cathode-ray oscillograph (see Plates 3 and 4, facing pp. 70 and 71). Although Dr. Griffin suggested that further analysis of the records would be necessary to come to a definite conclusion, the picture seems to indicate that no low-frequency waves are recorded, and also that the duration of the pulse is only about 0·001 second. Dr. Griffin has pursued his investigations, and some of his findings were published in 1950 and 1951, these relating chiefly to the common little brown bat. A brief summary of the conclusions reached concerning the acoustical properties of the ultrasonic pulses is as follows.

Duration of pulse. Most of the oscillograph records show a duration of less than two milliseconds. The exact duration

indicated by any particular photograph depends upon what Dr. Griffin calls the intensity resolution—the ratio of peak amplitude to the lowest amplitude which can be resolved from the baseline or background noise.

Intensity. The sound pressures developed by bats' ultrasonic cries are surprisingly high, at least when measured within 5 to 10 cm. of the animal's mouth. The average sound pressure measured is about 60 dynes per sq. cm., whereas the pressure of the noise of a pneumatic drill is about 2 to 3 dynes per sq. cm., though it must be emphasised in the latter case that the distance involved may be a few feet. With such a high intensity, it is understandable how bats can hear the echoes of the pulses even after they have been returned from very small objects at some distance.

Harmonics. Many of the photographs of bat ultrasonic pulses show harmonics, which are usually shown as bright spots or lines along the rising or falling parts of the individual sound waves, and it appears that the higher frequency is not an exact multiple of the fundamental.

Frequency. A very striking fact found from analysis of typical ultrasonic pulses is that the frequency is not constant ; it undergoes a sort of frequency modulation throughout the duration of the pulse. The highest frequencies are found at the beginning of a pulse, and a typical case is where the initial frequency is 74 kc. and the end 43 kc., a drop of almost an octave. The highest frequency recorded to date is 120 kc.

Low-frequency components. Recent photographs show low-amplitude waves of about 10 kc. which seem to have roughly the amplitude required to account for the audible click. The low-frequency components appear in front of the ultrasonic pulse, and a problem yet to be decided is whether the components are an unavoidable accompaniment of the ultrasonic pulse.

Acoustic location of insect prey by bats. The ultrasonic sounds emitted by bats have recently (1951) been analysed while the animals were flying under *natural* conditions, and it has been found that when flying through small caves and when leaving roosts in buildings, bats emit pulses virtually identical to those previously described and measured in the laboratory. This suggests that bats appear to search actively for their insect prey by the same process of " echo-location " previously shown to be the basis of obstacle avoidance.

The student who desires further information in this interesting subject should consult the original papers of the investigators named above, or study the articles that have appeared in *Nature.*

CHAPTER IV

MORE PROPERTIES OF SOUND

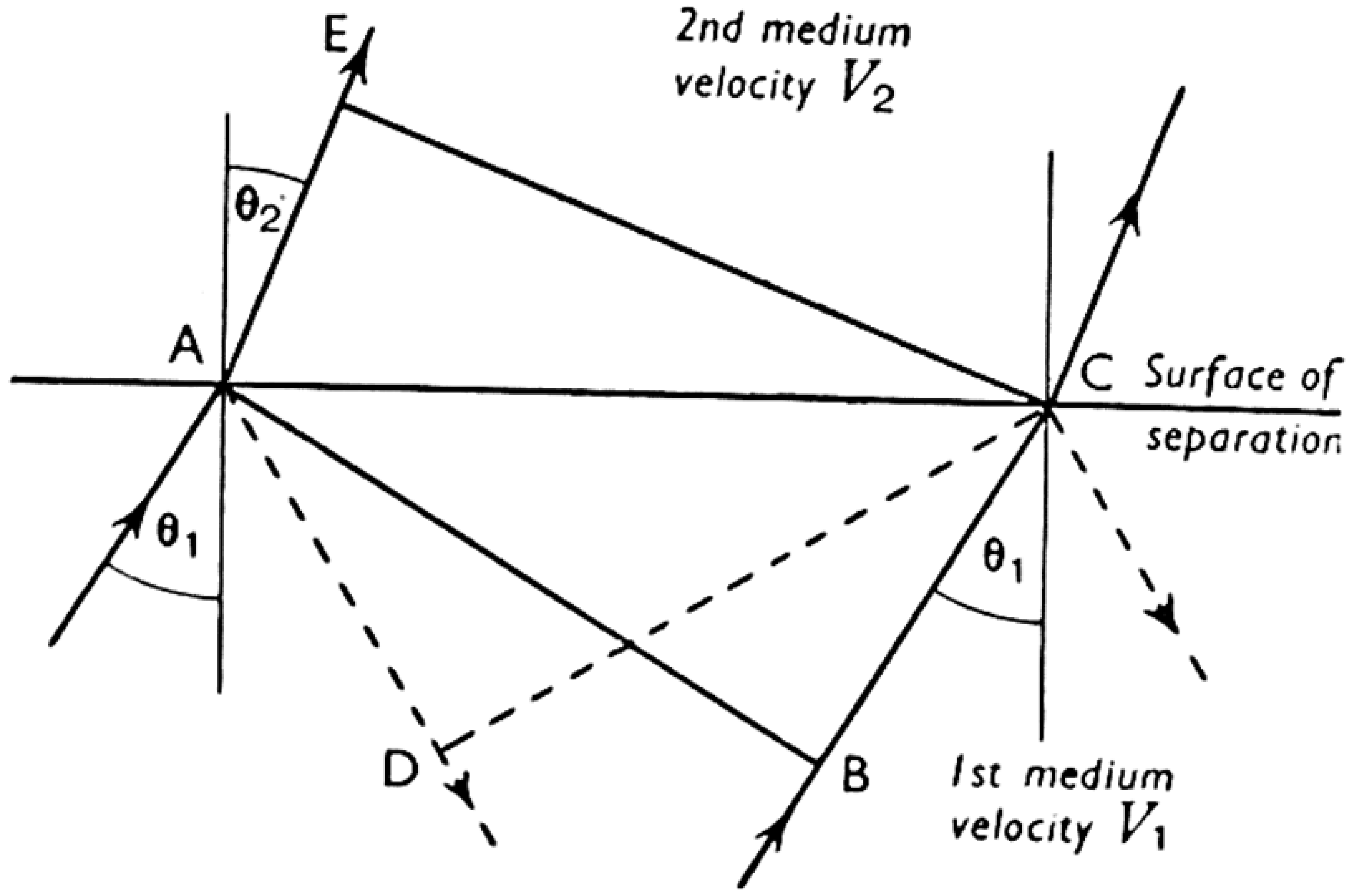
REFRACTION

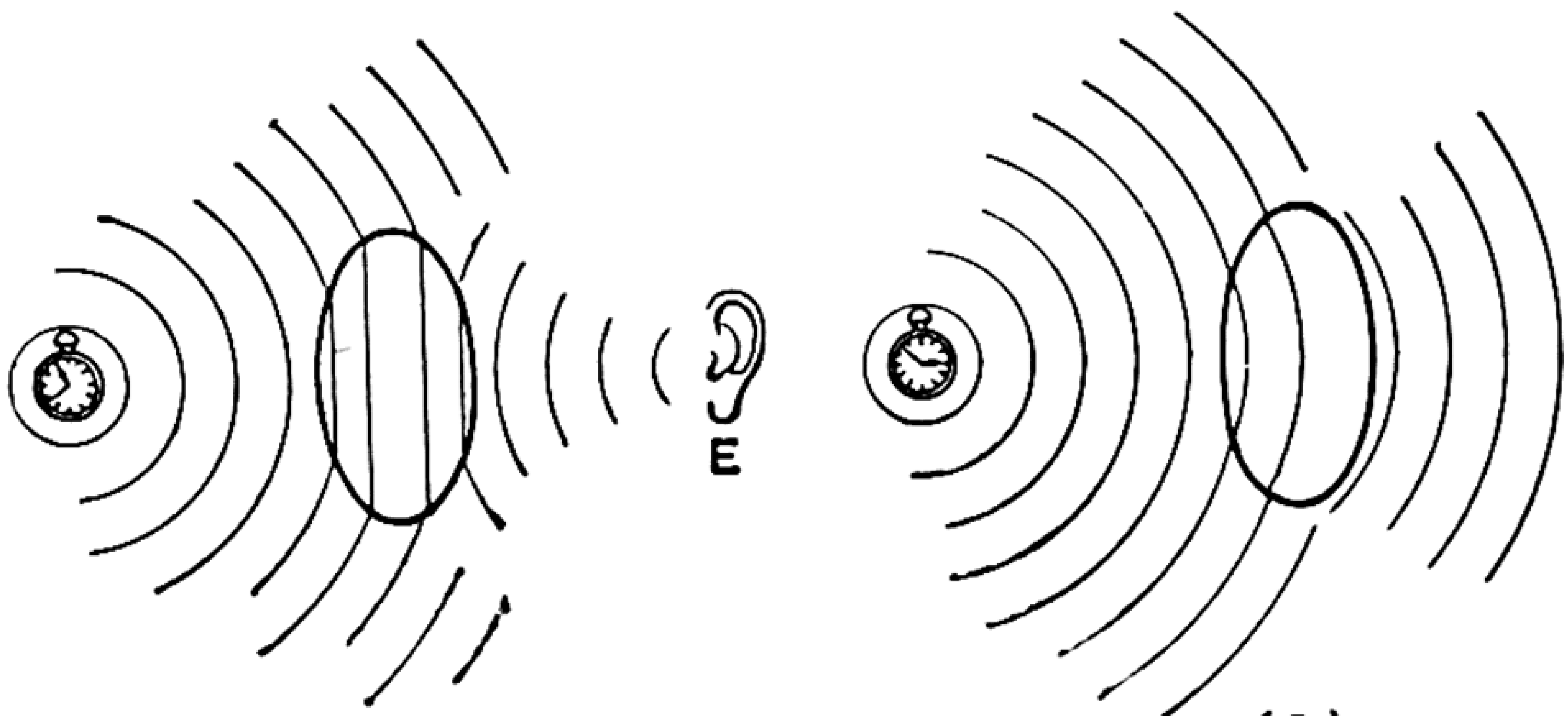
WHEN plane waves of sound cross the bounding surface between two different media, the direction of propagation is changed in accordance with the sine law as in optics, and we have the relationship

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{V_1}{V_2},$$

where θ_1 and θ_2 are the angles of incidence and refraction respectively and V_1 and V_2 are the corresponding velocities of sound in the two media. Hence rays are bent towards or away from the normal according as the velocity in the first medium is greater or less than that in the second.

In the diagram, the case when V_1 is greater than V_2 is illustrated and the refracted rays are bent towards the normal. AB is the wave-front of the incident wave and CE that of the refracted wave. The reflection which occurs at the boundary is represented by the dotted portion, and CD is the wave-front of the reflected wave.





(a) (b)

Sondhauss in 1852 demonstrated refraction of sound through prisms containing various gases, and determined the value of μ (the refractive index) relative to air. He also demonstrated the focusing action of a convex lens of carbon dioxide enclosed in a thin envelope of collodion. The refraction of sound in gases can be conveniently demonstrated by constructing a lens-shaped vessel such as an india-rubber balloon with a gas which is denser than air, say carbon dioxide (*a*). The source of sound can be a ticking watch, and if this is arranged on one side of the lens, it will be found that the sound is brought to a focus at E. Thus this lens is the counterpart to a converging lens in optics. If the gas is less dense than air, say hydrogen or coal gas, the resulting lens is of the divergent kind and no focusing is obtained (*b*).

Total reflection. As in the case of optics, there is an angle of incidence for which the reflection which occurs at the boundary of two media is total and no energy is transmitted. This critical angle is given by the relationship

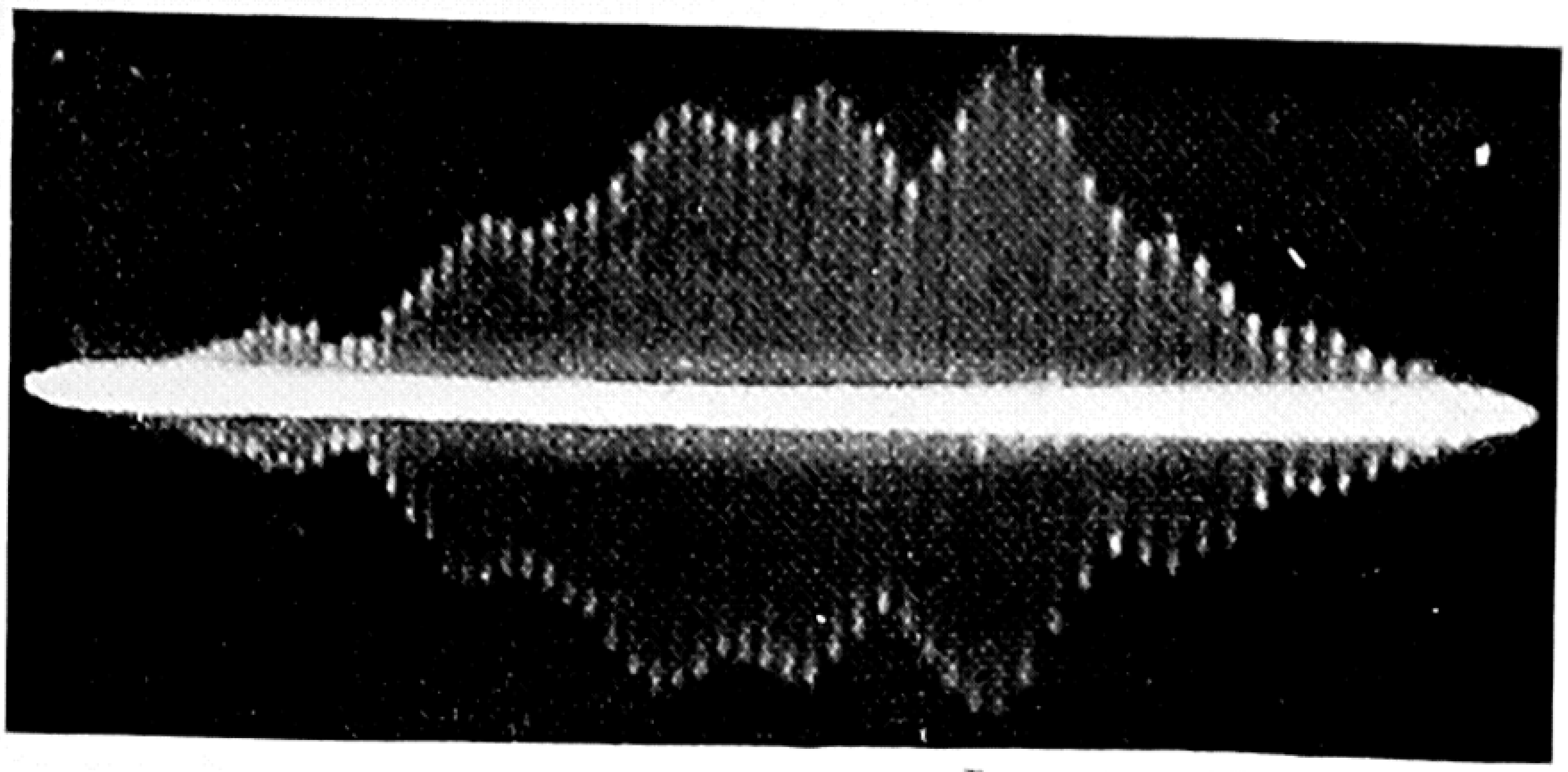
$$\sin \theta_c = \frac{V_1}{V_2}.$$

To prove this, we may put $\theta_2 = 90^\circ$ in the equation

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{V_1}{V_2}.$$

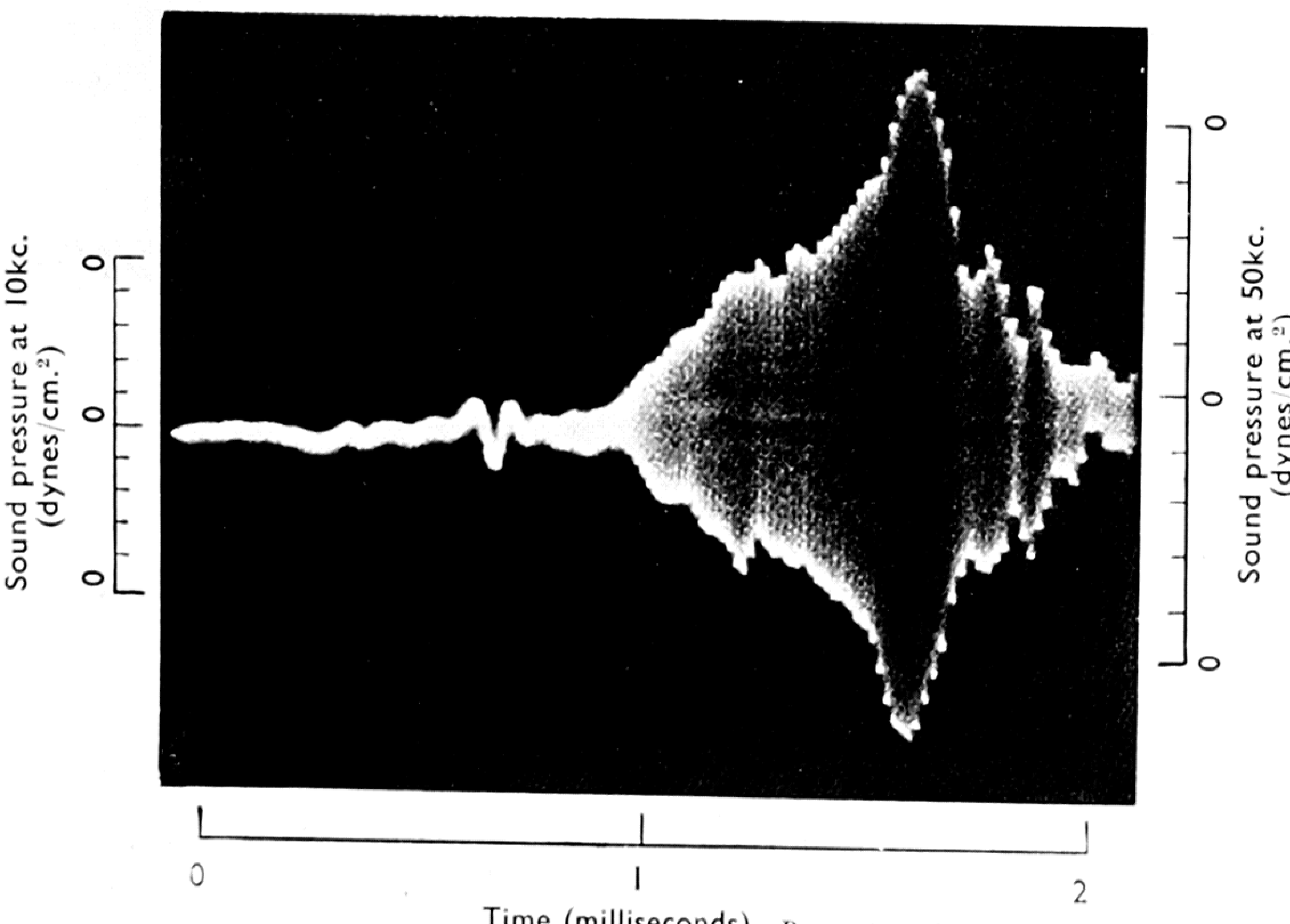
The relationship can also be derived from the equation given on p. 54 concerning the ratio between the reflected and incident amplitudes, namely :

$$\frac{r}{a} = \frac{\dfrac{\rho_2}{\rho_1} - \dfrac{\cot \theta_2}{\cot \theta_1}}{\dfrac{\rho_2}{\rho_1} + \dfrac{\cot \theta_2}{\cot \theta_1}}.$$



By courtesy of Dr. D. R. Griffin.

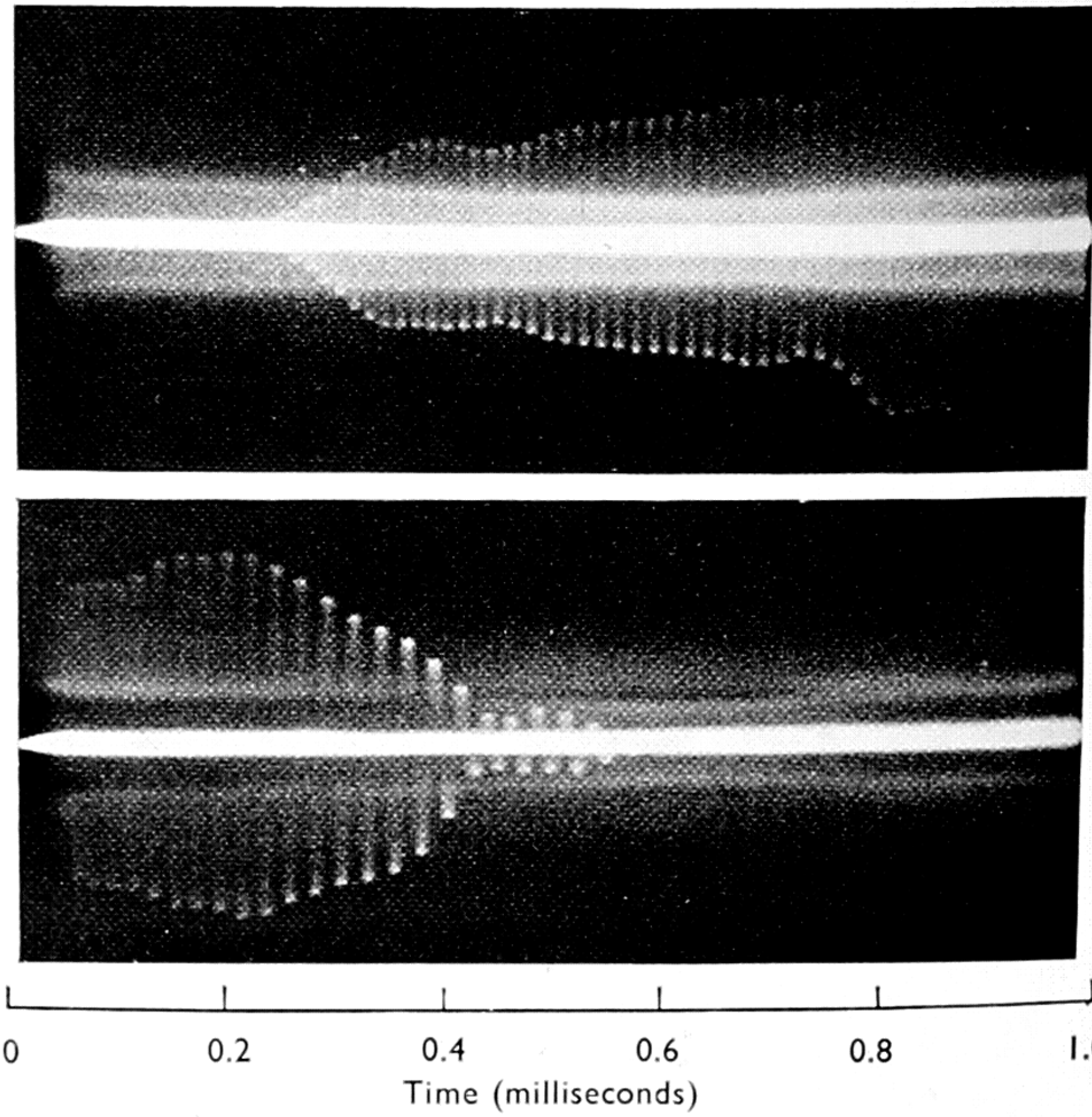
Typical ultrasonic pulse emitted by a bat showing individual waves and complete envelope.



By courtesy of Dr. D. R. Griffin.

Initial part of a pulse showing low-amplitude and low-frequency waves at the beginning. It is considered that these waves account for the faint audible click which accompanies the ultrasonic pulse.

PLATE 3



By courtesy of Dr. D. R. Griffin.

Two pulses emitted by the same bat within $\frac{1}{8}$ second. The upper photograph shows the beginning on one pulse. and the lower the end of the other pulse, and it will be noticed that frequency during a pulse is not constant. The initial frequency in the above photographs was 74 kc., and that at the end of the lower pulse was 43 kc., a typical drop of nearly an octave.

PLATE 4.

The photographs for Plates 3 and 4 were kindly provided by Dr. D. R. Griffin, of Cornell University, U.S.A. The top one in Plate 3, which seems to indicate that no low-frequency waves are recorded, was an early photograph. The lower one in Plate 3 which definitely shows the presence of low-amplitude and low-frequency waves at the beginning of the pulse, and the one in Plate 4 are much more recent.

For total reflection to occur, θ_2 must be zero and the value of $\frac{\cot \theta_2}{\cot \theta_1}$ is imaginary.

$$\text{Now} \quad \frac{\cot \theta_2}{\cot \theta_1} = \frac{\dfrac{\cos \theta_2}{\sin \theta_2}}{\dfrac{\cos \theta_1}{\sin \theta_1}} = \frac{\sin \theta_1}{\sin \theta_2} \cdot \frac{\cos \theta_2}{\cos \theta_1}$$

$$= \frac{\sin \theta_1}{\sin \theta_2} \sqrt{\frac{1 - \sin^2 \theta_2}{\cos^2 \theta_1}} = \frac{\sin \theta_1}{\sin \theta_2} \sqrt{\frac{1 - \sin^2 \theta_2 \times \dfrac{\sin^2 \theta_1}{\sin^2 \theta_1}}{\cos^2 \theta_1}}.$$

But from the law of refraction,

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{V_1}{V_2}.$$

$$\therefore \quad \frac{\cot \theta_2}{\cot \theta_1} = \frac{V_1}{V_2} \sqrt{\frac{1 - \dfrac{V_2^2}{V_1^2} \cdot \sin^2 \theta_1 .}{\cos^2 \theta_1}}$$

This ratio will be imaginary if

$$\sin \theta_1 > \frac{V_1}{V_2}.$$

Hence the critical angle $= \theta_c = \sin^{-1} V_1/V_2$.

Also, since $\quad V_1 = \sqrt{\dfrac{p}{\rho_1}}$ and $V_2 = \sqrt{\dfrac{p}{\rho_2}}$, for gaseous media

we have $\quad \theta_c = \sin^{-1} \sqrt{\dfrac{\rho_2}{\rho_1}}$ approximately.

Since V_1 for air may be taken as 1,100 ft. per sec. and V_2 for water 4,760 ft. per sec., the value of the critical angle from air to water is $13\frac{1}{2}°$ approximately, and above this angle no sound whatever can enter the water ; (it was shown on p. 55 that even at normal incidence the reflected amplitude is 0·99943 of the incident amplitude). From air to solid materials (such as brass, glass, etc.) $V_1/V_2 = 0{\cdot}065$ approximately ; whence the critical angle is about $3\frac{1}{2}°$.

In connection with the above, it is of interest to notice what happens when a plane wave of sound is incident from *dry* air on

to a fog bank or cloud in which the density is roughly 1 per cent less than that of dry air. If the incidence is normal we may calculate the reflection-incidence ratio from the equation

$$\frac{r}{a}=\frac{\sqrt{\rho_2}-\sqrt{\rho_1}}{\sqrt{\rho_2}+\sqrt{\rho_1}} \quad \text{(see p. 55)},$$

and we shall find that the amount reflected is exceedingly small. But here we have excluded the possibility of total reflection which may occur. Now, since $\theta_c=\sin^{-1}\sqrt{\rho_2/\rho_1}$ and taking the values of ρ_2 and ρ_1 to be 0·99 and 1·00 respectively we find that the value of the critical angle is about 84°. Therefore, although reflection is small at angles near normal incidence, it is, at any rate, possible for total reflection to occur; and when it does it will certainly change the direction of the sound.

Speaking tube. The boundary of a speaking tube always consists of material in which the velocity of sound is greater than in air; consequently, total reflection will occur unless the sound strikes the surface nearly normally. Thus there is very little loss in intensity, for the only way in which energy can be lost is by friction between the moving air and the tube. The sounding board sometimes placed over a pulpit in a church acts in a similar way. The sound is prevented from spreading out in an upward direction, so that the whole of the energy is given out in an approximately horizontal direction. The falling off in intensity is then more nearly inversely as the distance than as the square of the distance.

Amount of energy transmitted. If we consider the relationship between r and a for normal incidence, namely

$$\frac{r}{a}=\frac{\dfrac{\rho_2}{\rho_1}-\dfrac{V_1}{V_2}}{\dfrac{\rho_2}{\rho_1}+\dfrac{V_1}{V_2}} \quad \text{(p. 55)},$$

we see that if $\rho_1 V_1=\rho_2 V_2$, there is *no* reflected wave and the energy is transmitted. If we take the more general case of oblique incidence, then, for an angle θ_1 such that

$$\frac{\cot\theta_2}{\cot\theta_1}=\frac{\rho_2}{\rho_1},$$

we also have no reflected beam. Let us find the condition for θ_1 to exist. We have

$$\cot^2 \theta_1 = \frac{\cos^2 \theta_1}{\sin^2 \theta_1}\left(\frac{\cos^2 \theta_2 - \cos^2 \theta_1}{\cos^2 \theta_2 - \cos^2 \theta_1}\right) = \frac{\cos^2 \theta_1}{\sin^2 \theta_1}\left(\frac{\sin^2 \theta_1 - \sin^2 \theta_2}{\cos^2 \theta_2 - \cos^2 \theta_1}\right)$$

$$= \frac{\sin^2 \theta_2}{\sin^2 \theta_1}\left(\frac{\dfrac{\sin^2 \theta_1}{\sin^2 \theta_2} - 1}{\dfrac{\cos^2 \theta_2}{\cos^2 \theta_1} - 1}\right) = \frac{\left(\dfrac{\sin^2 \theta_1}{\sin^2 \theta_2} - 1\right)}{\dfrac{\sin^2 \theta_1}{\sin^2 \theta_2}\left(\dfrac{\cos^2 \theta_2}{\cos^2 \theta_1} - 1\right)}$$

$$= \frac{\left(\dfrac{\sin^2 \theta_1}{\sin^2 \theta_2} - 1\right)}{\left(\dfrac{\cot^2 \theta_2}{\cot^2 \theta_1} - \dfrac{\sin^2 \theta_1}{\sin^2 \theta_2}\right)} = \frac{\left(\dfrac{V_1^2}{V_2^2} - 1\right)}{\left(\dfrac{\rho_2^2}{\rho_1^2} - \dfrac{V_1^2}{V_2^2}\right)}.$$

In order then that such an angle a θ_1 may exist, it is necessary that $\rho_2/\rho_1 > V_1/V_2 > 1$ or that $\rho_2/\rho_1 < V_1/V_2 < 1$; that is, that V_1/V_2 is intermediate in value between 1 and ρ_2/ρ_1.

The amount of energy transmitted under ordinary conditions can be increased by using a suitably constructed apparatus in the form of a stethoscope with a very thin air chamber, and it can be shown that if A_2/A_1 is the ratio of the cross-section of the stethoscope tube to that of the air chamber, the transmission ratio is equal to

$$\frac{4\dfrac{A_2}{A_1} \times \dfrac{R_2}{R_1}}{\left(\dfrac{A_2}{A_1} + \dfrac{R_2}{R_1}\right)^2},$$

where $R_2 = \rho_2 V_2$ and $R_1 = \rho_1 V_1$, the specific acoustic resistances of the two media. It is obvious that this is equal to unity if

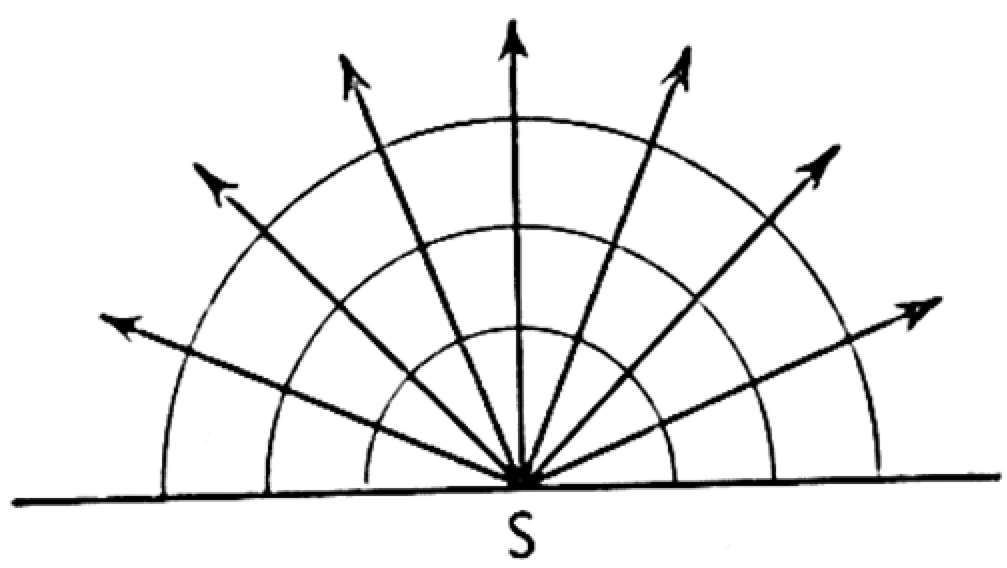
$$A_2/A_1 = R_2/R_1.$$

If some intervening medium is used between the air chamber of the stethoscope and a second medium, say water, the transmission ratio will be unity if the medium has a specific acoustic resistance which is the geometric mean between the values for air and water, and if the thickness of the medium is one-quarter of the wave-length of the sound in the medium. These conditions obviously mean that the arrangement is highly selective as far as transmission is concerned, and it would be of little advantage where a broad range of frequencies is used. It is interesting to note in this connection that if the intervening medium is rubber, the first condition given above is approximately fulfilled.

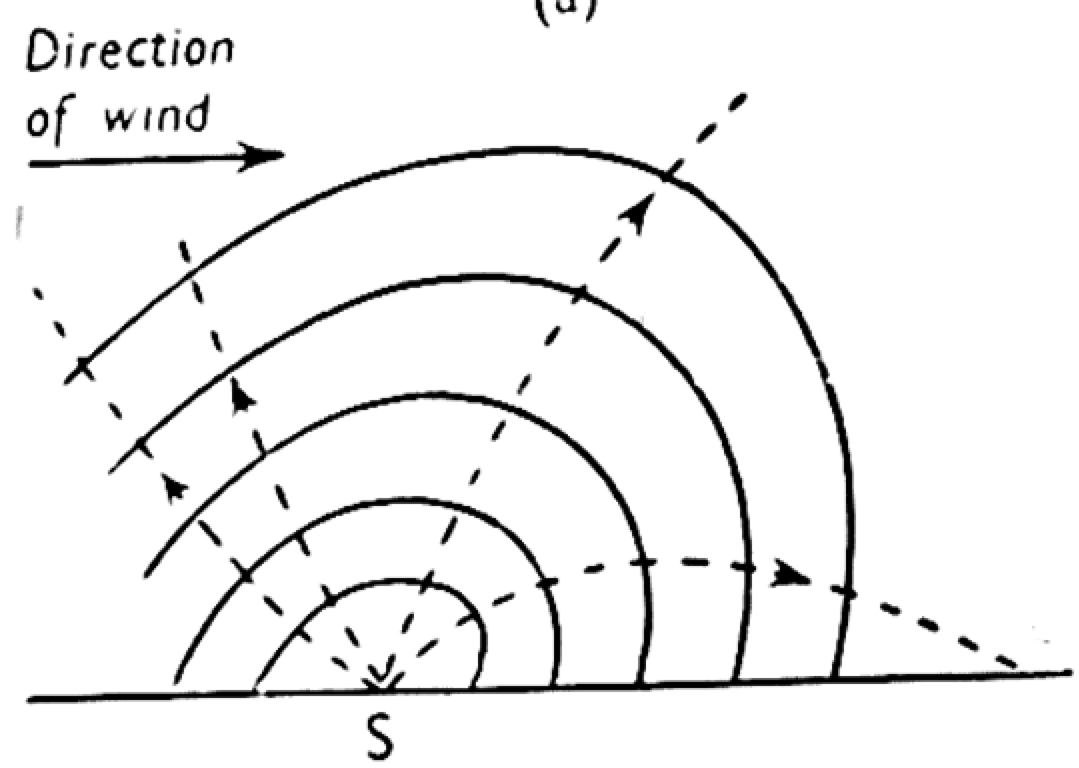
ATMOSPHERIC REFRACTION

The refraction of sound in a single medium like the atmosphere can take place in two ways, namely, by the effect of wind, and by the effect of temperature variations from place to place.

Effect of wind. It is well known that sounds are much more easily audible to a person on the ground when they are coming with the wind than when they come against it.



(a)

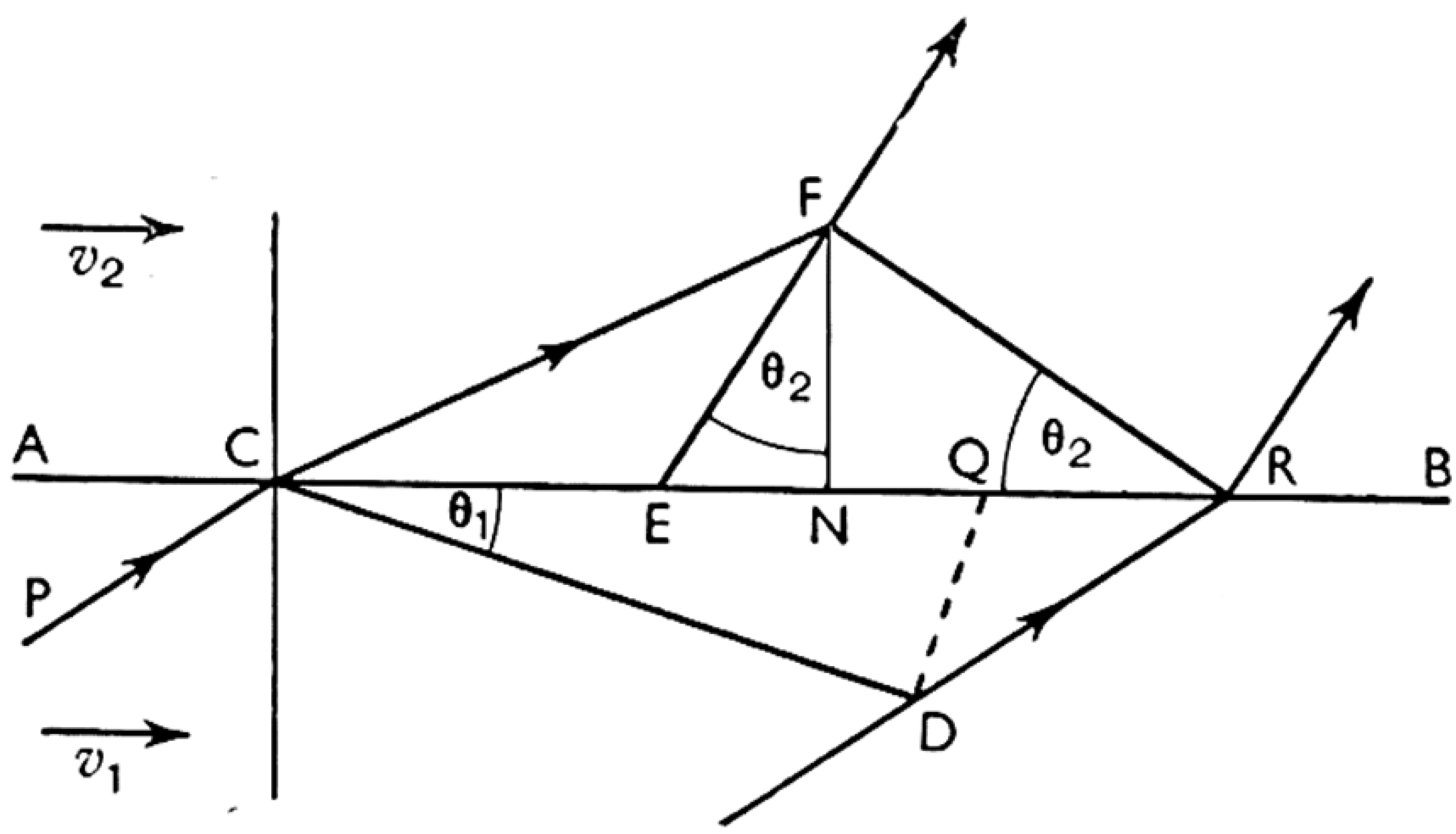


(b)

When a wind is blowing, the air near the ground is slowed down by friction at the surface ; hence the wave fronts are distorted. Consider a source of sound S near the ground. If there is no wind the successive portions of a wave-front would be a series of hemispheres as shown in diagram (a). But if there is a wind blowing, the tops of the waves move more rapidly than the lower parts. Hence the wave-fronts are no longer spherical, but are distorted as indicated in (b). Since the direction of propagation at any particular point on a wave-front is at right angles to the wave-front, it will be seen that the sound rays (indicated by the dotted lines) travelling in the direction of the wind, bend downwards and those travelling against the wind bend upwards. Hence a listener on the ground to the right of S would hear the sound much better than if he were to the left, where it is possible he might not hear the sound at all.

General case. The general case of refraction by the wind when sound is incident on the boundary between two regions where the wind velocity is different will now be briefly discussed. It must be borne in mind, however, that the condition of a definite bounding line between the two regions is most improbable in practice.

Let AB be the boundary of two adjacent regions of air, in the lower of which the wind velocity is v_1 and in the upper in which it is v_2 in the direction indicated. Suppose that v_2 is greater than v_1. Let CD be the wave-front of a plane wave which has just reached the boundary at C, making an angle θ_1 with the boundary,



The direction of propagation is represented by PC, and on account of the motion of the medium this is not normal to the wave-front. If there were no wind, the direction in the lower region would be DQ normal to CD. But on account of the wind, DQ is displaced to DR, and $RQ/DQ = v_1/V$, V being the velocity of sound in air. If the time taken for the sound to travel from D to R is t, then

$$t = \frac{DQ}{V} = \frac{QR}{v_1}.$$

In the same time the upper medium moves through the distance $CE = v_2 t$, and so far as the sound is concerned, the position of the wave is therefore the same as if it had originated at E. In time t the disturbance originally at C is at F, where $EF = Vt$. Hence the new wave-front is FR, making $\angle \theta_2$ with AB, and EF is the new direction of propagation. The relation between θ_1 and θ_2 is found as follows :

$$\frac{ER}{EF}=\frac{CQ+QR-CE}{Vt}=\frac{CQ}{Vt}+\frac{v_1-v_2}{V}.$$

$$\therefore\ \frac{ER}{EF}-\frac{CQ}{Vt}=\frac{v_1-v_2}{V},$$

or
$$\sec\theta_2-\sec\theta_1=\frac{v_1-v_2}{V},$$

since $DQ=Vt$. If the values are such that

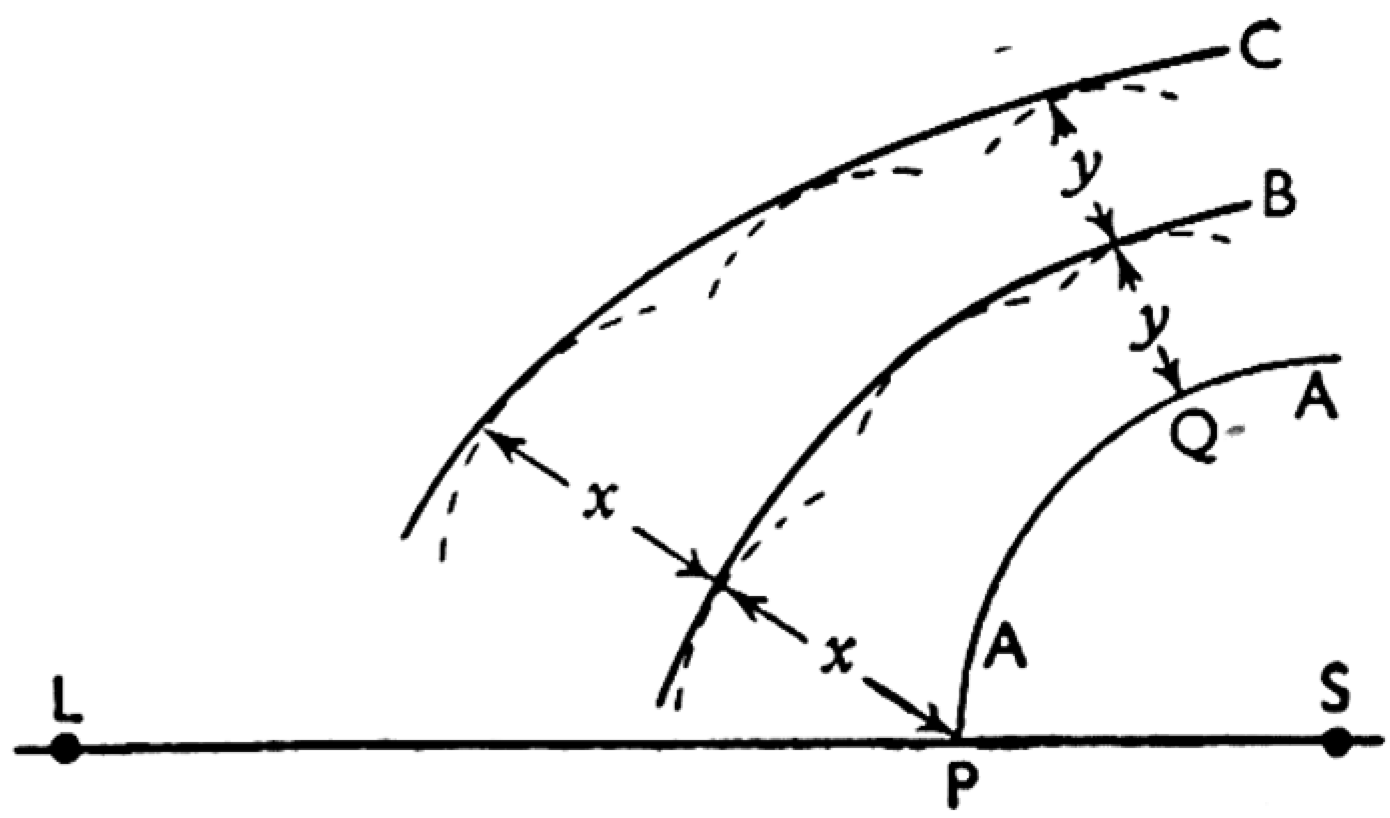
$$\sec\theta_1+\frac{v_1-v_2}{V}<1,$$

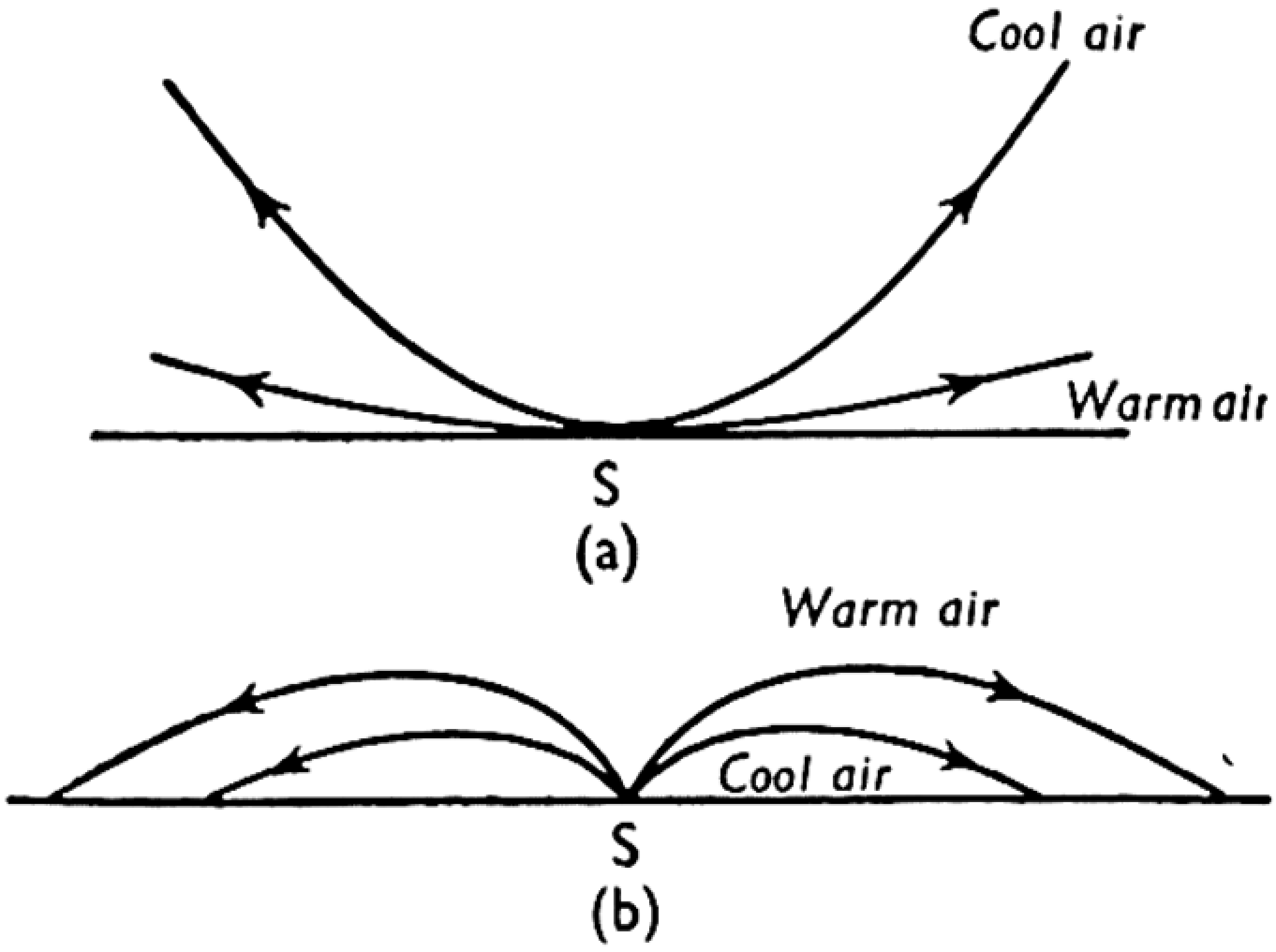
we have $\sec\theta_2<1$, which is impossible.

Therefore in such a case there must be total reflection and the value of the critical angle is $\theta_2=\sec^{-1}1$.

From the discussion in this section, it should be easy to understand why sound in air will often pass more readily in one direction between two points than in the other; also why it is so advantageous to put such things as bells and chimes in an elevated position.

Effect of temperature variation. As sound travels more rapidly in warm air than in cold, refraction takes place in the atmosphere wherever it is not at a uniform temperature. Consider the case, which often happens on a still day in summer, when the ground is warmer than the upper atmosphere. Let AA be the position of the wave-front at any moment of a sound emitted at S. The disturbance started at a position P near the ground will travel a distance x in the same time as a disturbance from Q travels a shorter distance y. If we draw the Huyghens wavelets, we see that the resulting wave-front is represented by the curve B, and it is tilted backwards so that its direction has been altered; it is





no longer travelling parallel to the ground. The next stage is C, and the wave-front is still more tilted. Hence, in this case the sound tends to pass *upwards* into the upper air, and a listener at L will probably not hear much. This is an example of a *positive* temperature gradient. It often happens, however, that the temperature gradient is a *negative* one, in which the coldest air is nearest the ground. Such a condition is likely to obtain on a summer evening in temperate regions, and then the wave-fronts are tilted so that the sound tends to pass downwards towards the ground ; hence audibility at ground level is better on a summer evening than during the day. The diagrams given above indicate in a simple way how the sound is refracted in both cases.

It must be emphasized that atmospheric refraction is much more complicated than is suggested in the above treatment, for the boundary between the different refracting media is not in practice a straight line ; in fact, it may vary from minute to minute. Also both wind and temperature variations probably occur simultaneously.

Refraction in water. We mentioned on p. 43 that the range of transmission of sound in water is influenced by the presence of a temperature gradient, and in the light of our knowledge of temperature gradients now, this is quite understandable.

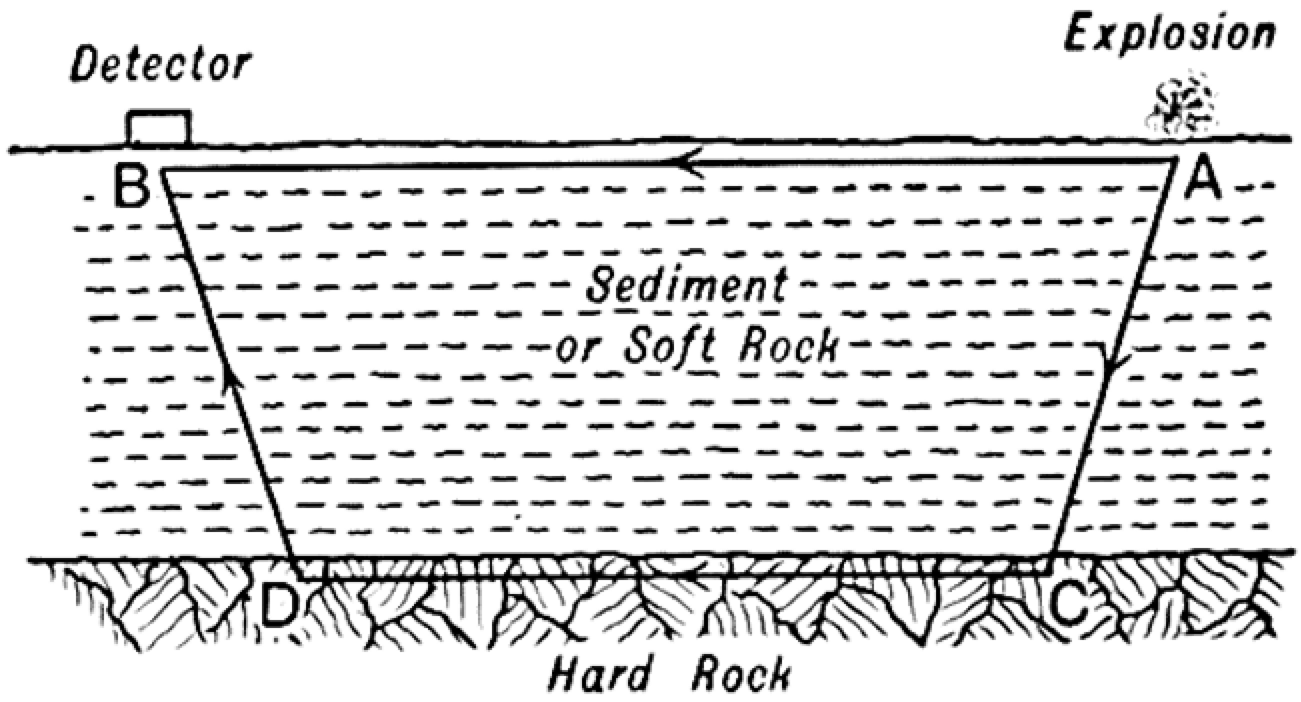
In summer the surface water is warmer than the lower depths and the waves bend downwards. Hence this must decrease the range, especially in very deep water, though in shallow water a certain amount of reflection may take place at the bottom.

In winter the temperature gradient is much less marked and may even be reversed. Hence the waves are bent upwards towards the surface where they are perhaps reflected down again, only to rise once more, thereby extending the range. There is no

doubt that surface and bottom reflections play an important part in the long-range transmission of sound in the sea.

In 1920 Lichte and Barkhausen noted a change in the range from 10 km. in summer to 20 km. in winter in the Baltic Sea.

Submarine geology. In recent years various methods have been developed for finding out the form, constitution and structure of the rocks hidden beneath the sea, and to obtain such information about the rocks which are deep below the bottom of the sea the most promising method appears to be the seismic method. In this method, charges of explosive are detonated on the sea-bottom, and the waves produced are recorded by seismographs also placed on the bottom. The waves travel in all directions, and



from the time taken for them to travel a known distance, their velocity may be found and some indication of the rock through which they have travelled obtained. Reflections of waves from the interfaces between hard and soft rock may also be observed. If a layer of a soft rock in which the velocity of the waves is relatively slow overlies a hard rock in which the velocity is high, it is possible for a wave to travel down to the hard rock, along in it and up again in a shorter time than it can traverse the direct path through the soft rock. It is on the observation of such refracted waves that the work done at sea depends.

INTERFERENCE

The passage of sound-waves through a medium is not affected by the passage of other sound-waves through the same part of the medium ; this principle, known as the *Principle of Superposition*, was first propounded by Huyghens in dealing with light-waves.

It is found that under certain conditions two trains of sound-waves may, at certain points in the medium, neutralise each other's effects and produce silence where previously there was a definite sound due to either of the trains of waves. In such a case, where the observed sound-distribution is not found to be

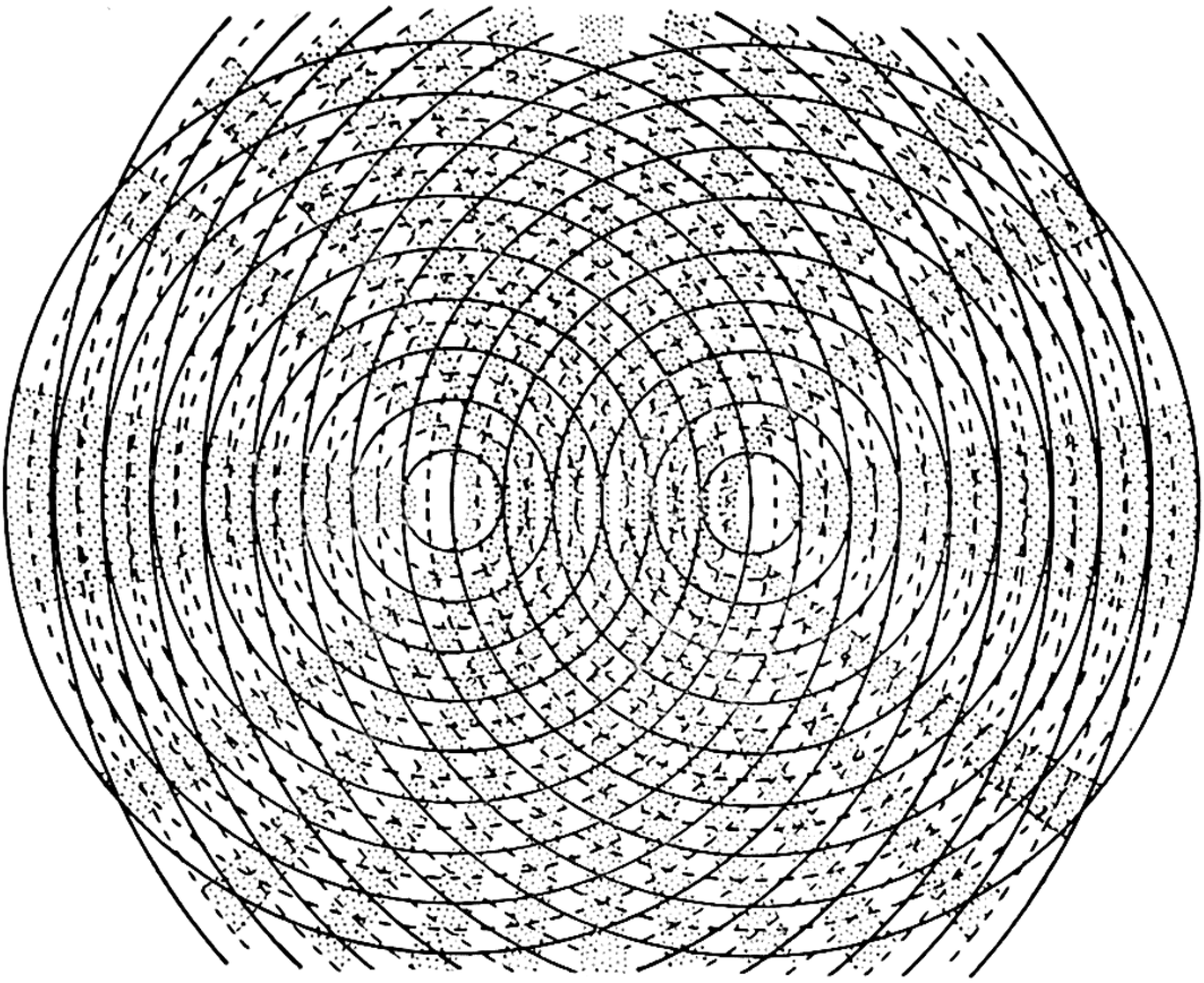
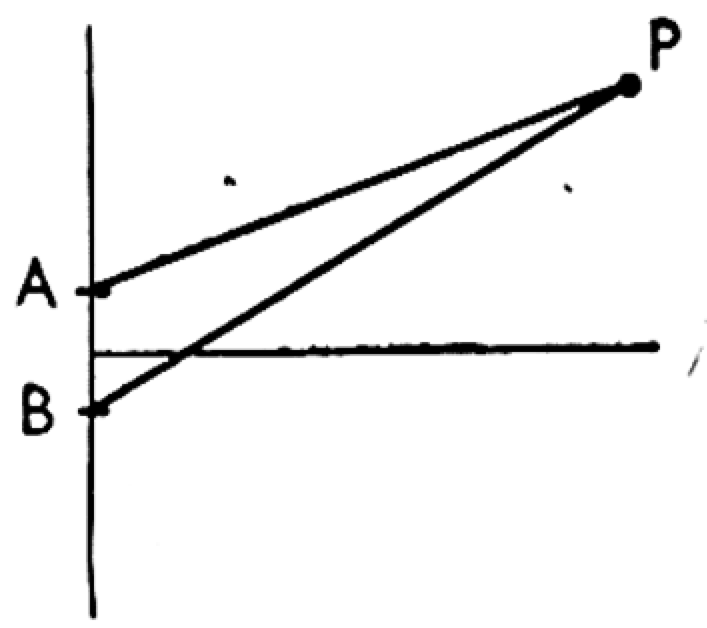


Diagram of interference pattern set up in water by two trains of waves

equal to the sum of the separate train of waves, the latter are said to have interfered with each other, and the phenomenon is described as **interference.** When this phenomenon does occur, the resultant effect at each point in the medium is the algebraic sum of the effects of the two waves.

Interference can take place, of course, with other forms of energy besides sound, provided such energy is propagated by wave-motion. It is well known that it can take place in water and also in light, though in the latter case it is imperative that the two trains of waves should have their origin in the same source.

The accompanying diagram indicates in a general way what happens when two different wave-trains are started simultaneously in water. The plain circles represent the crests of the waves and the dotted ones the troughs. Where two plain circles cross, it is clear that two crests come together, and the result is that a larger crest is formed. Similarly, where two dotted circles cross, a larger trough is formed. Where a plain circle crosses a dotted one. a crest is neutralised by a trough and the surface of the



water is level at these places; this occurs in the unshaded portions of the diagram.

Let A and B be two sources emitting sound-waves of the *same amplitude* and *frequency*. At any point P the two waves will reinforce each other if they are in phase, and a listener will hear a sound of twice the amplitude. If, however, the two waves differ in phase by $\lambda/2$, or, in general, by $(2n+1)\lambda/2$, where n is any integer, the displacements due to each set of waves are equal and opposite, and P will be in a position of silence.

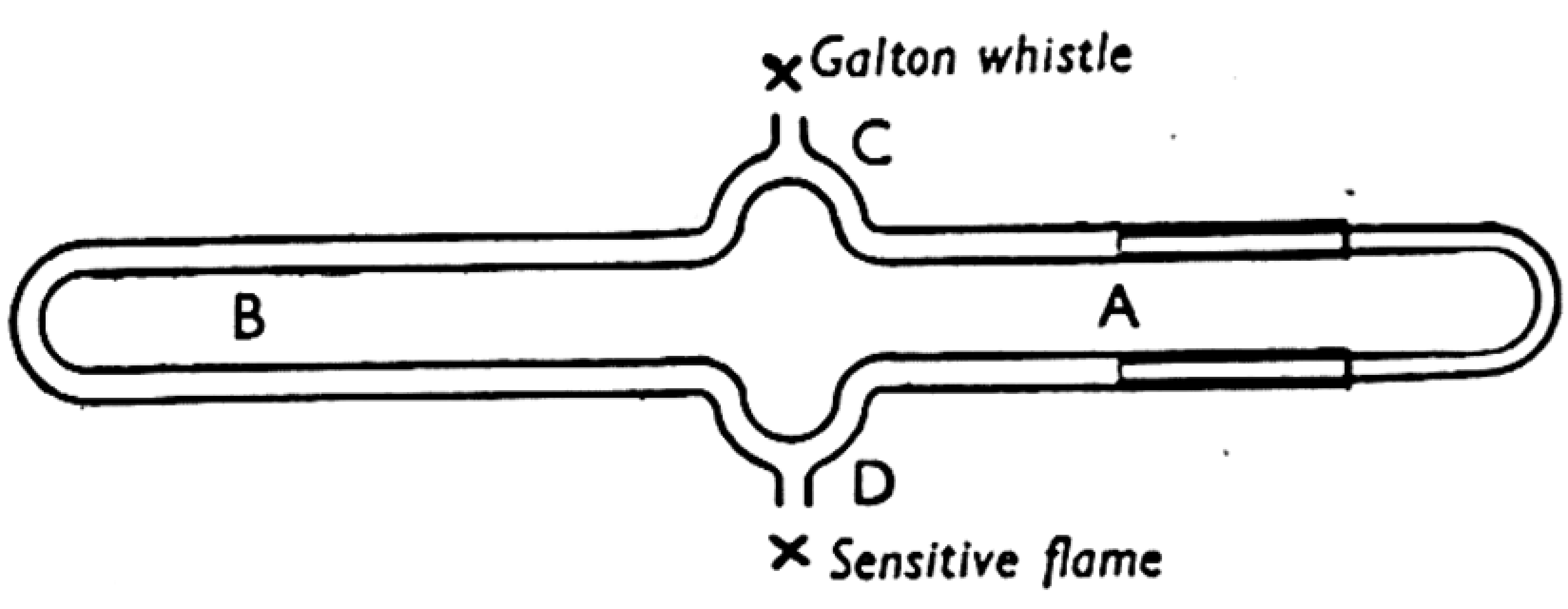
For interference between the two waves to occur, the following conditions must be fulfilled:

(i) The frequencies of the waves must be the same, otherwise any difference in phase at any particular point would not be maintained.

(ii) The amplitudes must be the same, for if they are not, the positions in which the phase difference is $(2n+1)\lambda/2$ will not be positions where the resultant displacement is zero.

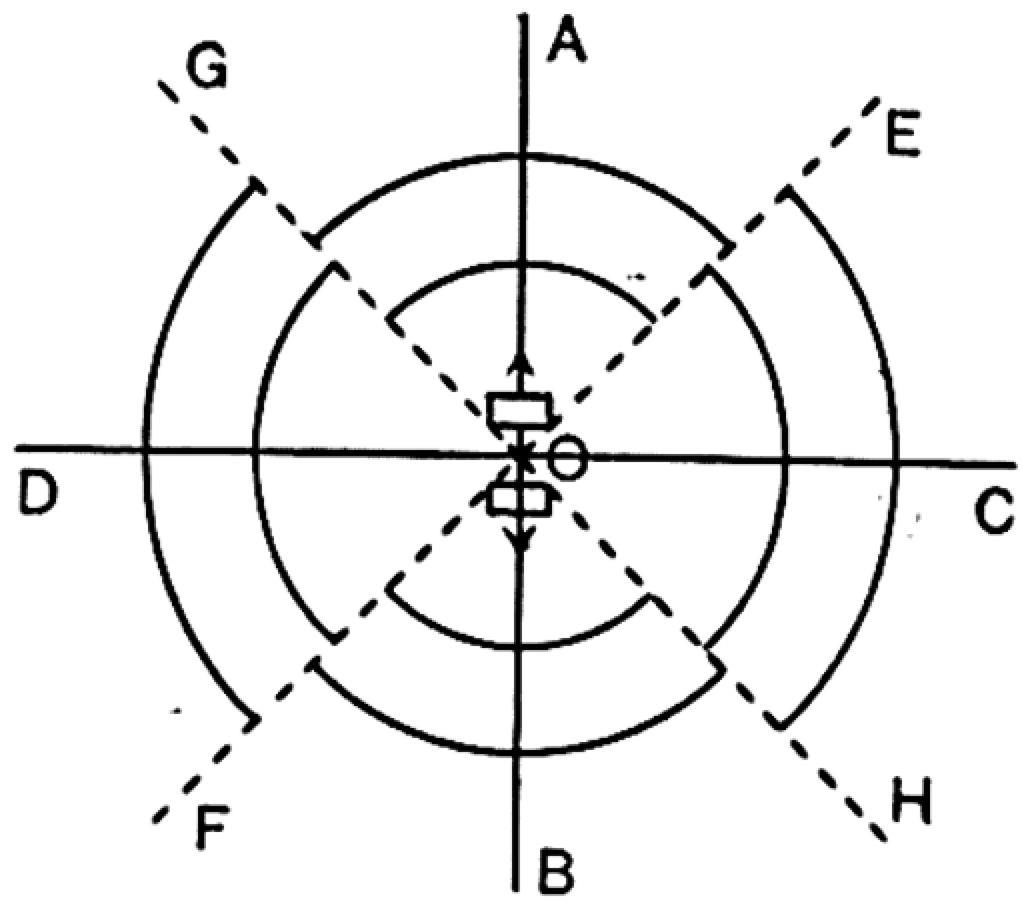
(iii) The displacements should be collinear.

Quincke's tube. The phenomenon of interference can be used to find the wave-length of a high-frequency sound with the aid of Quincke's tube. This consists essentially of two tubes, A and B, about 3 cm. in diameter and bent as shown in the diagram. The effective length of tube A may be altered by the sliding tube at the end. The source of sound, say, a Galton whistle, is put near the opening C, and the detector, which may be a sensitive flame, is arranged at the opposite opening D. The sound-waves from C may travel to D via the paths CAD or CBD. If these are equal, the two sets of waves will be in phase when they reach D and the flame will be violently disturbed. If, however, the path CAD is altered by moving the sliding tube, a position will be

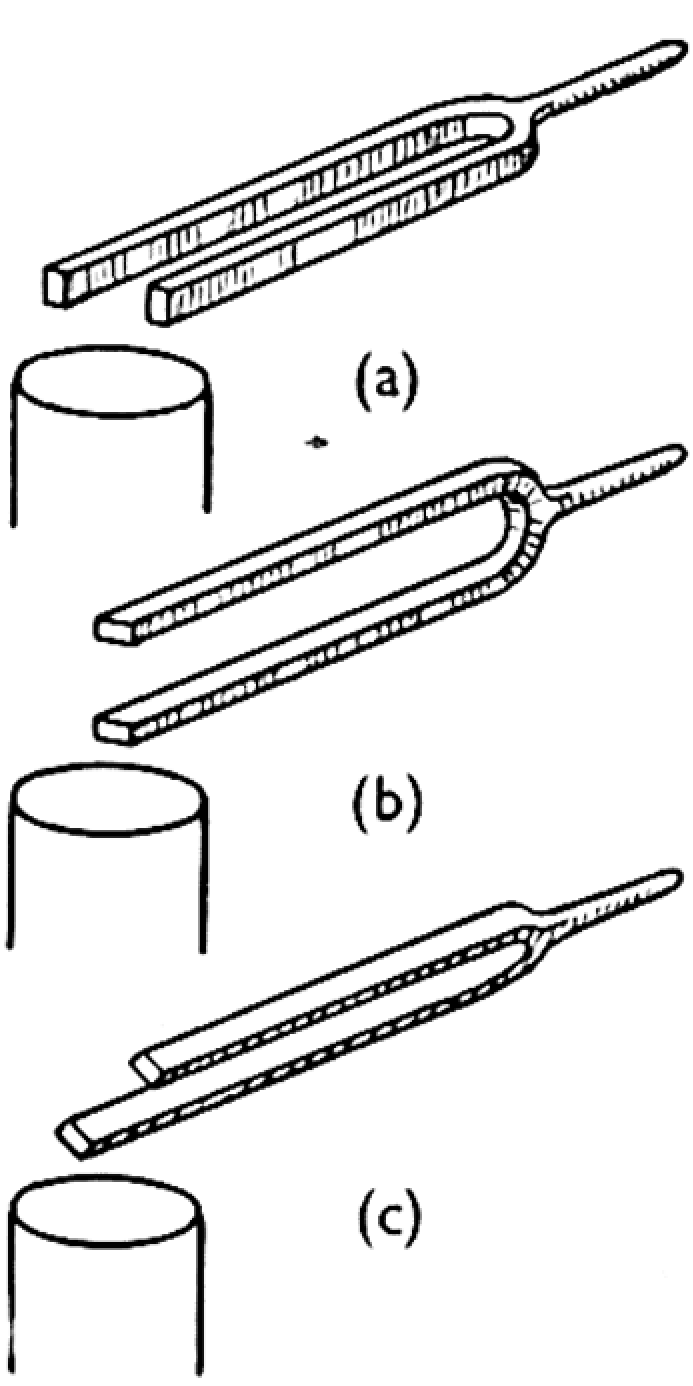


reached when the phase difference at D is $\lambda/2$. When this occurs, the flame will not be disturbed and the sliding tube will have been moved through a distance $\lambda/4$, which can be measured by means of the scale attached. When the tube is moved through a distance $\lambda/2$, the path difference will be λ so that the waves at D will be in phase again. Proceeding in this way, a series of readings can be taken and a mean value of $\lambda/2$ obtained, so that if the velocity is known the frequency can be determined.

Interference with a tuning fork. Interference in sound waves may be demonstrated very simply with a vibrating tuning fork. When the prongs move outwards, they cause a compression to be sent out in the directions OA and OB, and simultaneously a rarefaction in the directions OC and OD; on moving inwards they send out a rarefaction in the directions OA and OB and a compression in the directions OC and OD. Hence the two sets of waves are always in opposite phases, and may be represented by the circles in the diagram. Along the directions OE, OF, OG, OH, the compressions of one set of waves and the rarefactions of the other will almost exactly coincide, and there will be nearly silence. Thus, if a vibrating fork is slowly rotated on its stem as axis, alternations of loudness and softness will be heard, four of each occurring in a complete rotation.



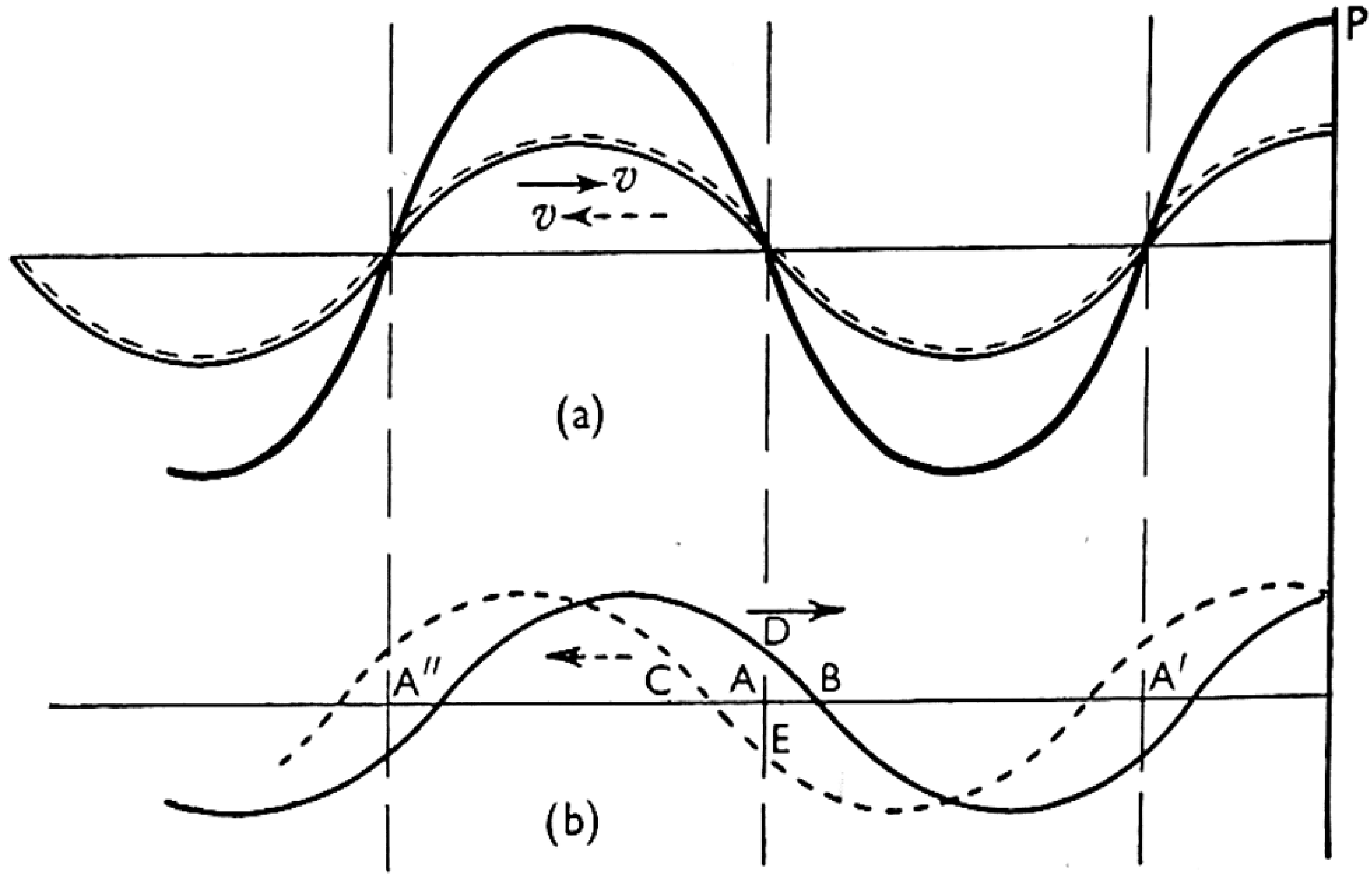
The effect can be more readily shown by using a resonance tube. The tube is adjusted for resonance, and it will then be found that if the fork is held in the position (a) or (b) in the diagram a loud sound is heard, but if it is held in position (c) no sound can be detected.



Stationary waves. A very important example of interference is seen when two waves of equal amplitude and frequency travel through a medium in

opposite directions. This occurs, for example, when a sound wave is propagated through a pipe and is then reflected at the other end. The forward incident wave and the returning reflected wave interfere with each other.

Simple physical treatment. In diagram (*a*) let the sine curve represent the forward wave travelling to the right with velocity v and incident on the reflecting surface P. The reflected wave travelling to the left with the same velocity and of the same amplitude and frequency is represented by the dotted curve. These two waves interfere with each other, and the resultant



effect is represented by the thick black curve. Now an instant later, diagram (*b*), the two waves have moved from A to B and from A to C (diagram *b*), and as the velocities are equal, $AB=AC$; also as the amplitudes are equal, $AD=AE$. It will be seen that the combination of the two waves will still produce zero amplitude at points A, A' etc.; and these positions will be fixed because AD and AE will always be equal. Hence the medium is set into a steady vibration, with fixed positions of zero amplitude (nodes), and fixed positions of maximum amplitude (antinodes). These steady vibrations are the so-called **stationary waves**, though they are not really waves at all, since a wave motion is essentially a phenomenon in which some state of disturbance travels from one place to another.

Mathematical treatment. The equation of the forward incident wave is

$$y_1 = a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right),$$

and that of the reflected wave is

$$y_2 = a \sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right).$$

The resultant is the sum of these two.

Hence $$y = y_1 + y_2 = a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + a \sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)$$

$$= 2a \sin 2\pi \frac{t}{T} \,.\, \cos 2\pi \frac{x}{\lambda}.$$

It will be noticed that this expression has no term of the character

$$\left(t - \frac{x}{V}\right) \quad \text{or} \quad \left(\frac{t}{T} - \frac{x}{\lambda}\right) \quad \text{(see p. 6)};$$

hence there can be no movement as in an ordinary wave-motion and the above equation characterises a steady vibration.

At any given position in the medium, the particle executes a vibration of

$$\sin 2\pi \frac{t}{T}$$

with an amplitude

$$2a \cos 2\pi \frac{x}{\lambda},$$

where x can be taken as the distance of the position from the point of reflection. If $x = 0$, $\lambda/2$, λ, etc.,

$$\cos 2\pi \frac{x}{\lambda} = 1.$$

Therefore at these points there is a maximum amplitude equal to $2a$. If $x = \lambda/4$, etc., the amplitude is zero, and at these points there is no movement of the medium.

The above treatment is satisfactory in the case of a closed pipe, where the amplitudes of the two waves are equal. But it becomes a little more complicated for an open pipe, because only

part of the incident wave is reflected and the rest transmitted; hence the two waves involved have different amplitudes.

The incident wave is represented by

$$y_1 = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

as before, and the reflected wave by

$$y_2 = -a' \sin 2\pi \left(\frac{t}{T} + \frac{x}{\lambda}\right).$$

The first equation can be written

$$y_1 = (a - a') \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right) + a' \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right),$$

and combining the two equations, we have:

$$\begin{aligned} y &= y_1 + y_2 \\ &= (a - a') \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right) + a' \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right) - a' \sin 2\pi \left(\frac{t}{T} + \frac{x}{\lambda}\right) \\ &= (a - a') \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right) - 2a' \sin 2\pi \frac{x}{\lambda} \cdot \cos 2\pi \frac{t}{T}. \end{aligned}$$

The term

$$(a - a') \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

represents the emerging or transmitted part of the incident wave, and it is of course the resultant of the incident wave and the emerging twin wave. For the incident wave is represented by

$y_1 = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$ and the twin wave by $y_2 = a' \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$.

$$\therefore \; y = y_1 + y_2 = (a - a') \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right).$$

The term

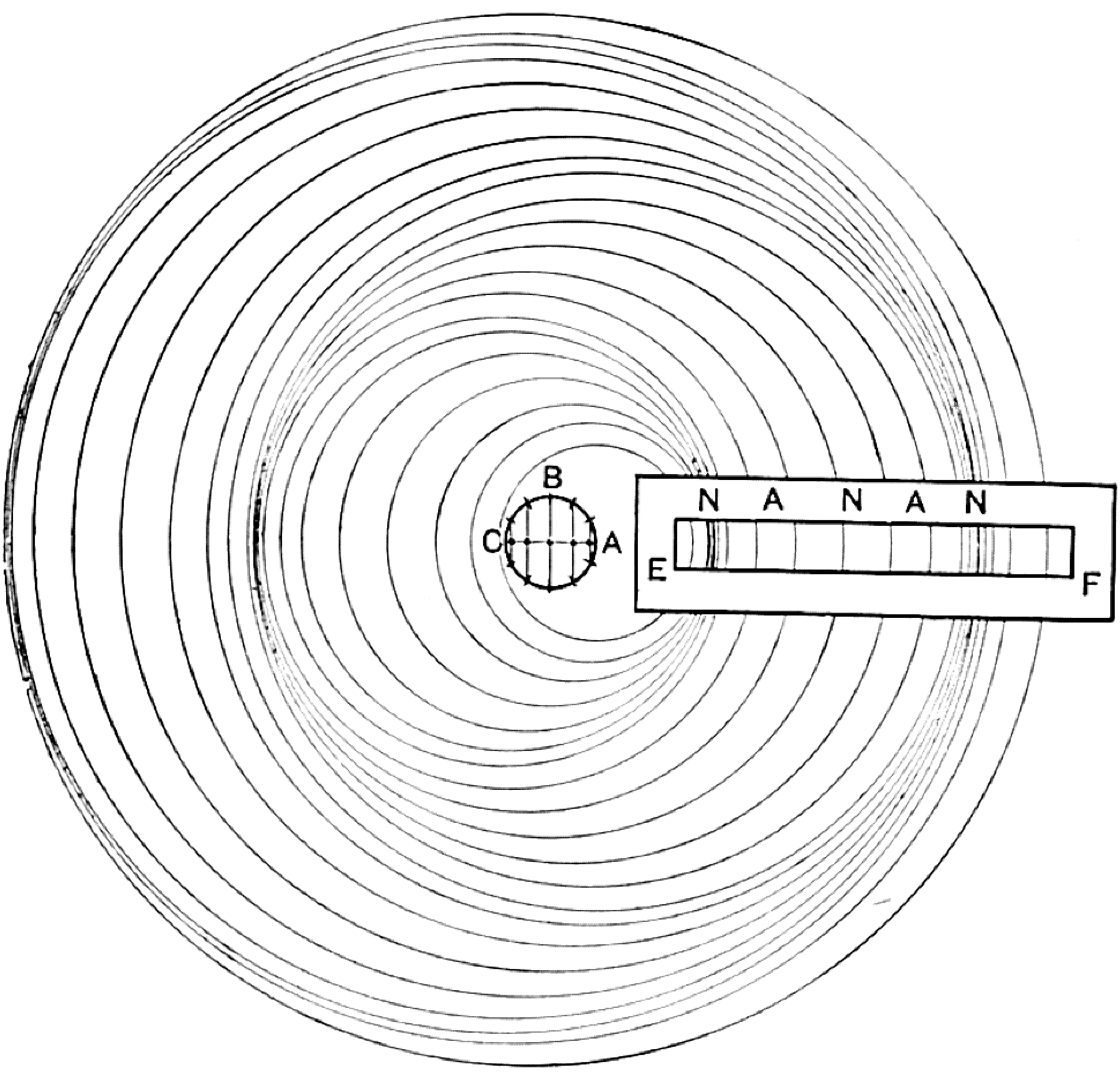
$$2a' \sin 2\pi \frac{x}{\lambda} \cdot \cos 2\pi \frac{t}{T}$$

represents the steady vibration or stationary wave.

Therefore, in the case of an open tube, we may consider that there is a progressive wave travelling along it, superimposed upon

the steady vibration, and that this progressive wave emerges at the end of the tube.

Cheshire's disc. To illustrate a stationary longitudinal wave, an apparatus known as Cheshire's disc may be used. A small circle ABC is drawn on a piece of cardboard, and the circumference is divided into twelve equal parts. Perpendiculars are dropped from these points upon the diameter AC, and the feet of these perpendiculars are taken successively as the centres of circles of gradually increasing radius. A is the first centre, then the next point to A, and so on until C is reached, and the diameter then retraversed until A is reached. On covering up all but a strip EF, and rotating the disc, the lines will vibrate longitudinally, N, N, N being nodes and A, A antinodes. It will be seen that the lines move from both sides towards a node for half a vibration, and away from it for the other half.

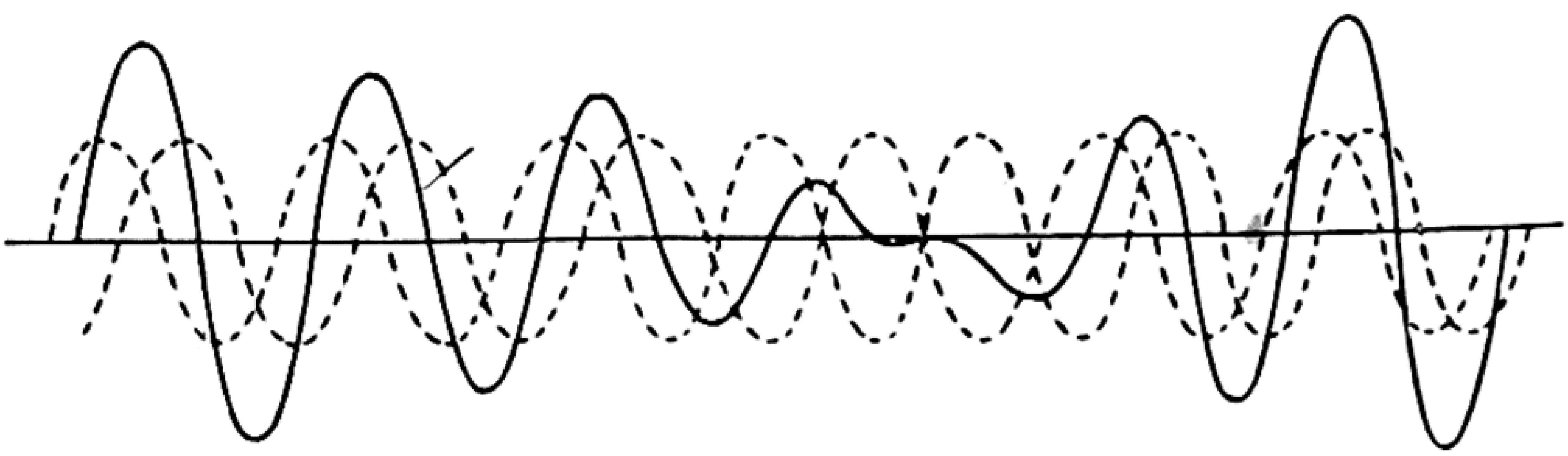


Cheshire's Disc

Beats. Interference may take place between two sounds even though they may not be of the same frequency; in this case the condition at any fixed point in the medium does not remain constant but is continually changing. At one moment the compressions from both sounds arrive at the point simultaneously, as do the rarefactions; hence there is great disturbance at the point. A short time later, however, the more rapidly vibrating object is half a vibration ahead of the other, and the compression from one will arrive at the same time as the rarefactions from the other, thus producing a minimum of sound. Thus a throbbing or pulsating effect is produced and the phenomenon is known as **beats.**

On a large scale, the effect can often be noticed when a twin-engined aeroplane is flying in the neighbourhood; in the laboratory, beats may be produced by obtaining two tuning forks of the same frequency and slightly reducing the frequency of one of them by attaching to one prong a small piece of wax. On sounding the two forks, beats will be distinctly heard, the effect being more pronounced if the two forks are fixed on resonating boxes.

In the diagram the two dotted curves represent two sets of waves emitted by two sources, for example, two forks of slightly different frequencies. The thick continuous line represents the resultant sound, obtained by adding algebraically the displacements in the two waves. At first, the two waves are in step and reinforce each other, but they gradually get more and more out of step until they practically cancel each other out. Then they get more in step, until they are once more together and reinforce each other. The resultant curve therefore represents the alternations in loudness and softness which periodically occur. It will be noticed that since a maximum occurs every time that one source has gained a complete vibration on the other, the number of maxima per second is equal to the difference between the two frequencies. Hence if two forks of frequencies 256 and 254 respectively are sounded together, two beats per second will be heard.



This can be shown as follows. Suppose the two components have equal amplitudes but slightly different frequencies, m and n, so that $m-n$ is small, and further suppose that when first observed they are in the same phase. The resultant amplitude at this moment is twice that of either component. After a time $1/n$ sec. the component of frequency n has completed one vibration, and the other component of frequency m has made m/n vibrations. Hence the latter is $(m-n)/n$ vibrations ahead of the former, and after $n/(m-n)$ such intervals the two will be separated by

$$\frac{m-n}{n} \times \frac{n}{m-n},$$

or one complete vibration, and they will be in phase again, reinforcing one another.

The time interval between these reinforcements is

$$\frac{n}{m-n} \times \frac{1}{n} = \frac{1}{m-n} \quad \text{sec.},$$

so there will be $(m-n)$ reinforcements per sec.; that is, the number of beats per second is equal to the difference of the frequencies. The resultant motion is still harmonic, but with the amplitude varying from zero to twice that of either component.

It is left as an exercise for the mathematical student to obtain the same result by considering the equations representing the two harmonic motions and finding their resultant. The equations concerned are $y_1 = a \sin (p_1 t - q_1 x)$ and $y_2 = a \sin (p_2 t - q_2 x)$, where

$$p = 2\pi n \quad \text{or} \quad 2\pi/T \qquad \text{and} \qquad q = 2\pi/\lambda.$$

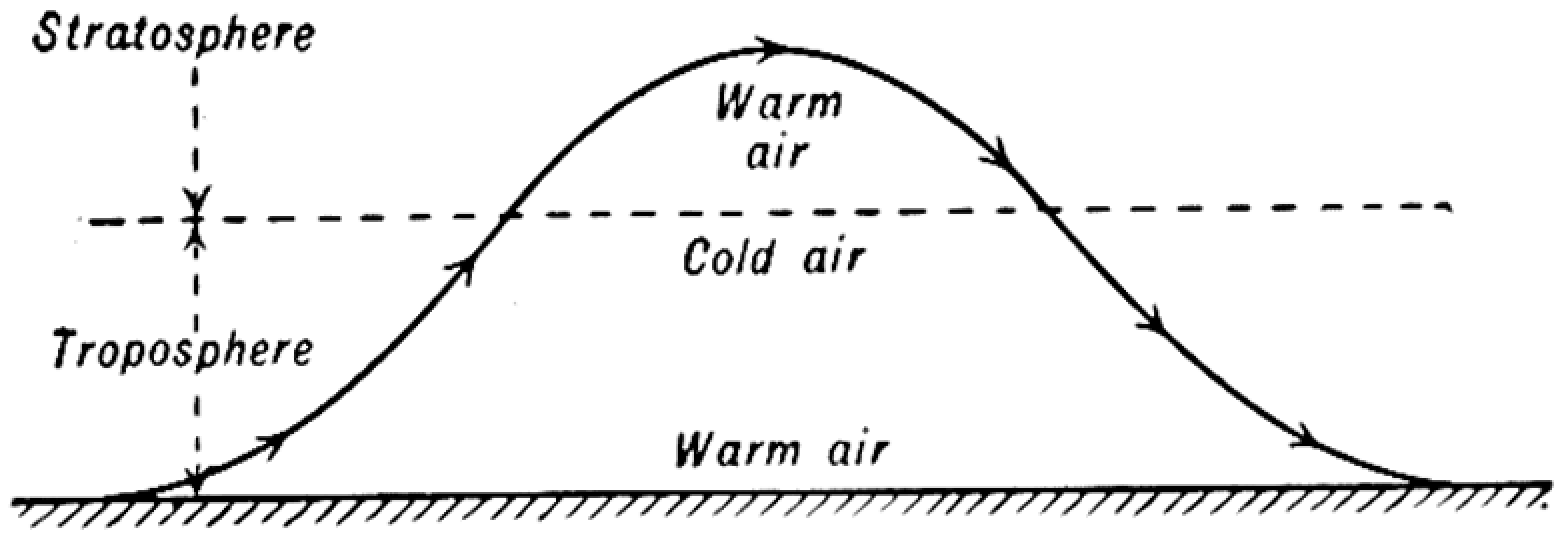
Beats, therefore, provide a very sensitive means of determining whether two notes are in tune or not, and this test is used by tuners of pianos and organs. Beats are used for the production of certain effects in organs. In the *vox-humana* and *vox-angelica* stops, two pipes having nearly the same frequency are used. The beating between the two gives the tremulous effect which is intended to imitate the human voice. Organ builders sometimes utilise beat-notes to obtain the low notes of an organ where space is limited. Two 16-foot pipes tuned to sound a *fifth*, give the effect of a single 32-foot pipe.

An apparatus sometimes used to detect dangerous gases in mines depends on the phenomenon of beats. It consists of two small and exactly similar pipes carried about and blown together,

one by pure air from a reservoir and the other by air from the mine. So long as the air is pure, the pipes remain in tone ; but when " fire-damp " is present, the air becomes less dense, with a consequent variation in the velocity of sound, and the pitch of the note emitted by the pipe is slightly altered. Thus beats are produced, and can be heard long before the mine air is bad enough to be dangerous ; the method is very sensitive.

Discords in music are due to beats. When less frequent than about 10 per second, beats can be distinguished separately, but when more frequent than this they give rise to a discord ; this will be referred to again in Chapter VII.

Zones of silence. The sound from a big explosion travels a great distance, but it has been noticed that whereas the noise has been heard near the scene of the explosion and also at places far



distant, yet there are places between these two where the sound has not been heard at all ; this area of silence is known as the **silent zone.** There may even be several silent zones between successive places in the same direction from the source.

The cause of such areas of silence may be traced to meterological conditions of wind and temperature, though a theory was developed by Wiechert in 1926 suggesting that there is a reflecting layer some 50 km. above the earth's surface (analogous to the Heaviside layer in radio-wave transmission), and the direct incident wave and the reflected wave interfere with each other. This explanation does not seem to be generally accepted, and the most probable single explanation seems to be found in the reversal of the temperature gradient which occurs in the stratosphere, where temperatures may be of the same order as those at the surface. Such a state of affairs would be effective in producing the necessary bending of the wave-fronts to account both for the zone of silence and also the distant audible zone, as indicated in the diagram.

The above is probably only a partial explanation of the phenomenon, since the presence of a wind is bound to affect the path taken by the sound.

In connection with the temperature in the stratosphere referred to above, it is of interest to note that such temperatures are believed to be maintained by the absorption of the solar energy by the ozone layers.

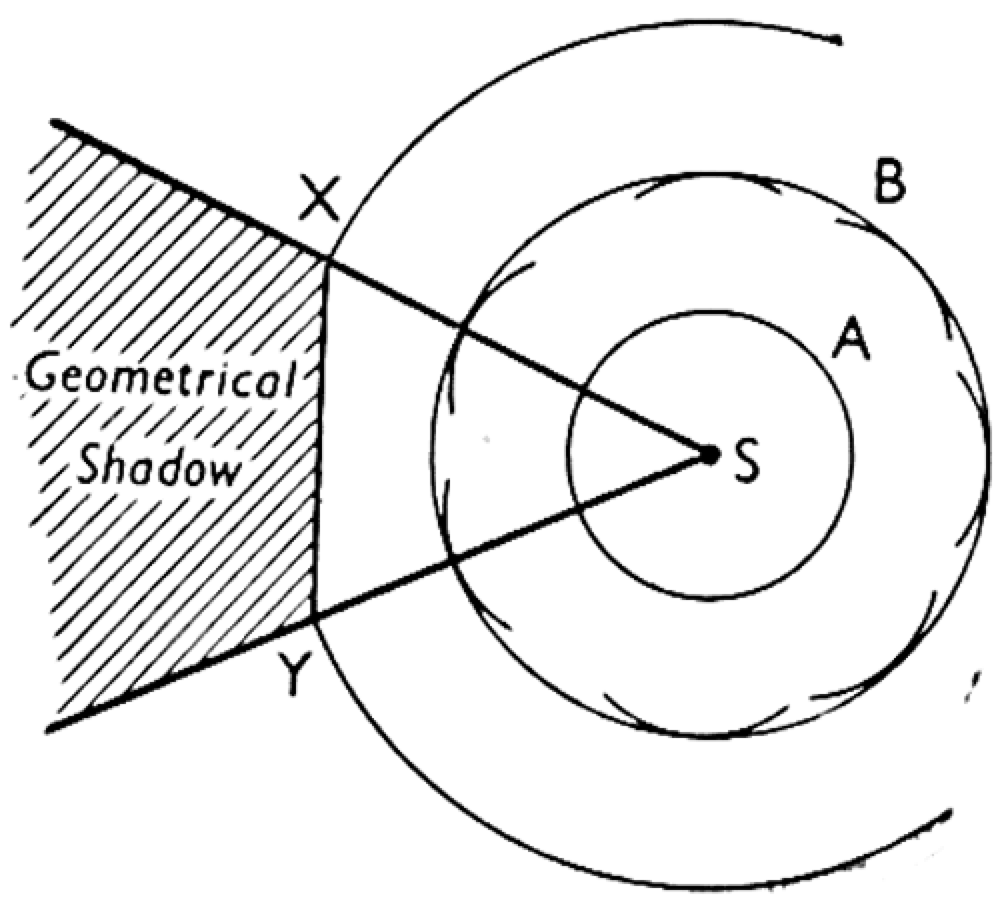
In certain cases, zones of silence are due to the interference between sound-waves reaching the listener by different paths. Tyndall observed such zones when listening on a ship to the sound of a fog signal on a neighbouring cliff, and they are ascribed to the interference between the direct sound and that reflected from the surface of the sea. If these paths differ by an odd multiple of $\lambda/2$, the two trains of waves neutralise each other and no sound is heard.

Wood and Young in 1921 observed interference zones *under* water, and such effects are of considerable importance in the case of long-distance transmission in the sea.

DIFFRACTION

When refraction occurs, it is necessary to have two different media, but the direction of sound can be altered even though the energy is travelling in one uniform medium. Indeed, if it were not so, we should all be at a great disadvantage, for it would be impossible to hear sounds when the source is screened. For example, we can hear the sound of approaching traffic round a corner, and again, if two men are standing on opposite sides of a fairly high wall, they can talk to each other. The sound waves emitted by the speaker spread out over the top of the wall and reach the other man ; hence the sound shadow and the geometric shadow are not coincident.

This spreading of waves round the edge of an obstacle is known as **diffraction**, and it can be explained, as in light, by the application of the principle of Huyghens' secondary wavelets and the mutual interference between them. If S is a source of sound, the successive wave-fronts can be represented by the circles such as A, B, etc., these being the envelopes of the wavelets. When the wave-front reaches the edge X of the obstacle XY, there will be a

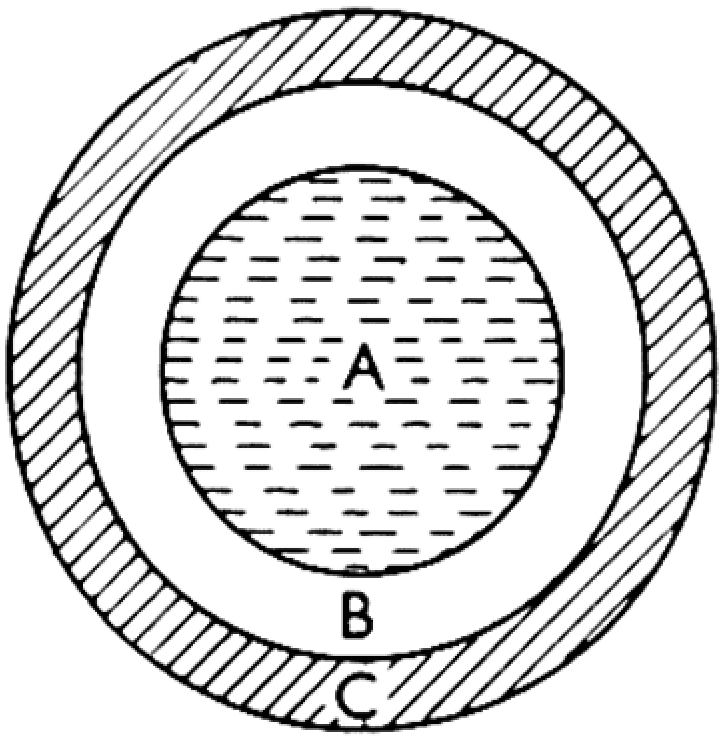


geometrical shadow represented by the shaded portion; but some of the energy will get inside this shadow because the interference between the secondary wavelets is not now complete. If the effect at any point in the geometrical shadow is to be computed, due allowance must be made for the contribution of each surface element of the wave-front to the amplitude at the point under consideration, and the estimation of this amplitude will involve the use of Fresnel's " half-wave zones ".

It is worth recording here that the rectilinear propagation of energy which travels in a wave-motion really depends on the destructive interference within the geometrical shadow of the secondary wavelets originating from the wave-front. That the destruction in light is as complete as it is depends on the shortness of the wave-length compared with the size of the obstacle, and it can be shown that the longer the wave-length the more likely is diffraction to occur. In sound, as the wave-lengths are longer, it requires a very big obstacle to screen the sound effectively.

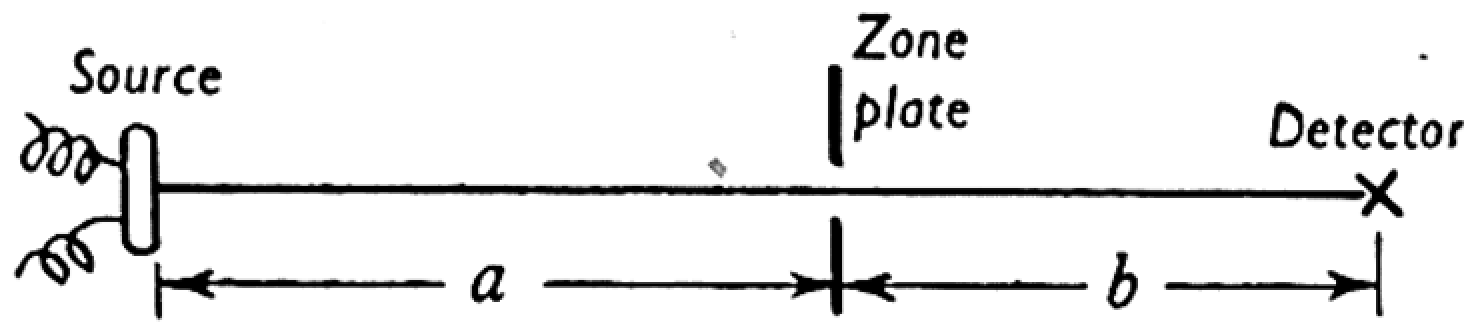
In the case of a straight edge as the obstacle, there is a fluctuation of intensity *outside* the geometrical shadow which soon settles down to the normal full intensity as the distance from the edge increases. *Inside* the geometrical shadow the intensity steadily falls off from one-quarter its normal value at the edge to zero at some distance inside.

Zone plates. If circles are drawn on a plane reflector with radii r_1, r_2, etc. so that $r_n^2 = n\lambda d$, where $n = 1, 2, 3$, etc.; and d is the distance of the centre O from a point P on the axis normal to the reflector, then these circles will divide the surface into Fresnel's half-wave zones. The annular zones thus formed are of equal area, and if alternate zones are cut away a plane sound-wave falling on the surface and passing through the annular openings will arrive in phase at P, resulting in a considerable increase of intensity at the point. Such a surface is known as a **zone plate** and it has focusing powers like a convex lens of focal length $r_n^2/n\lambda$, r_n being the radius of the nth zone.



Zone plate.

A zone plate suitable for experimental work can be made by cutting a series of annuli out of a large sheet of cardboard. The radii of the circles should be in the ratio $1 : \sqrt{2} : \sqrt{3}$, etc., so that the area of the middle disc A is equal to the areas of the portions B, C, etc.



The whole plate can be suspended by tapes from the outer sheet so that any zone may be removed at will.

A valve oscillator as source and a sensitive flame as detector are set up on the bench, and the zone plate with the central disc removed is placed between them. The zone plate is then moved until a position is found when the flame roars. It can be shown that if the distances of the source and the detector from the zone plate are a and b respectively, the flame will respond when

$$1/a + 1/b = n \,.\, \lambda/2,$$

where n is a whole number. If the second zone B is now removed, leaving everything else undisturbed, the flame becomes silent, showing that the disturbances due to adjacent zones neutralise each other. If the third zone is also removed, the flame responds once again, while if the second zone is now replaced, the flame is still further affected, showing that disturbances from alternate zones assist one another.

Using a circular piece of cardboard, say 12 in. radius, a similar experiment can be carried out to show the diffraction effects caused by such an obstacle, and the positions of the various maximum and minimum intensities established. The series of experiments described above certainly shows that precautions to eliminate diffraction effects as far as possible should be taken in all experiments on refraction of sound.

Diffraction gratings. We have already referred (p. 64) to the fact that an echelon structure such as a row of palings can act as a sound " diffraction grating ", for the reflected waves may assist or neutralise each other in certain directions, depending on the wave-length of the incident sound and the spacing of the reflectors. The diffracted waves have maxima in directions θ given by

$$\sin \theta = \pm n\lambda/d,$$

where d is the distance between the successive reflectors, and $n = 1, 2, 3,$ etc. When d is smaller than λ, there are no diffracted waves, and the incident beam is reflected in the ordinary way. Thus sounds of moderate frequency are reflected, and little sound is returned to the source except at normal incidence. But high-frequency sounds, where $\lambda < d$, are thrown back in all directions,

causing reinforcement in certain directions and neutralisation in others.

In 1907 Altberg demonstrated a grating by means of glass rods 1 cm. apart, using a concave reflector to produce plane waves incident on the grating ; the sound was produced by means of a high-frequency spark. A second concave mirror received the diffracted sound and brought it to a focus at a sensitive detector, and the sound spectrum was obtained by the rotation of the grating with respect to the source and the receiver. Wave-lengths of the order of 0·2 mm. were measured in this way.

CHAPTER V

VIBRATIONS OF STRINGS AND RODS

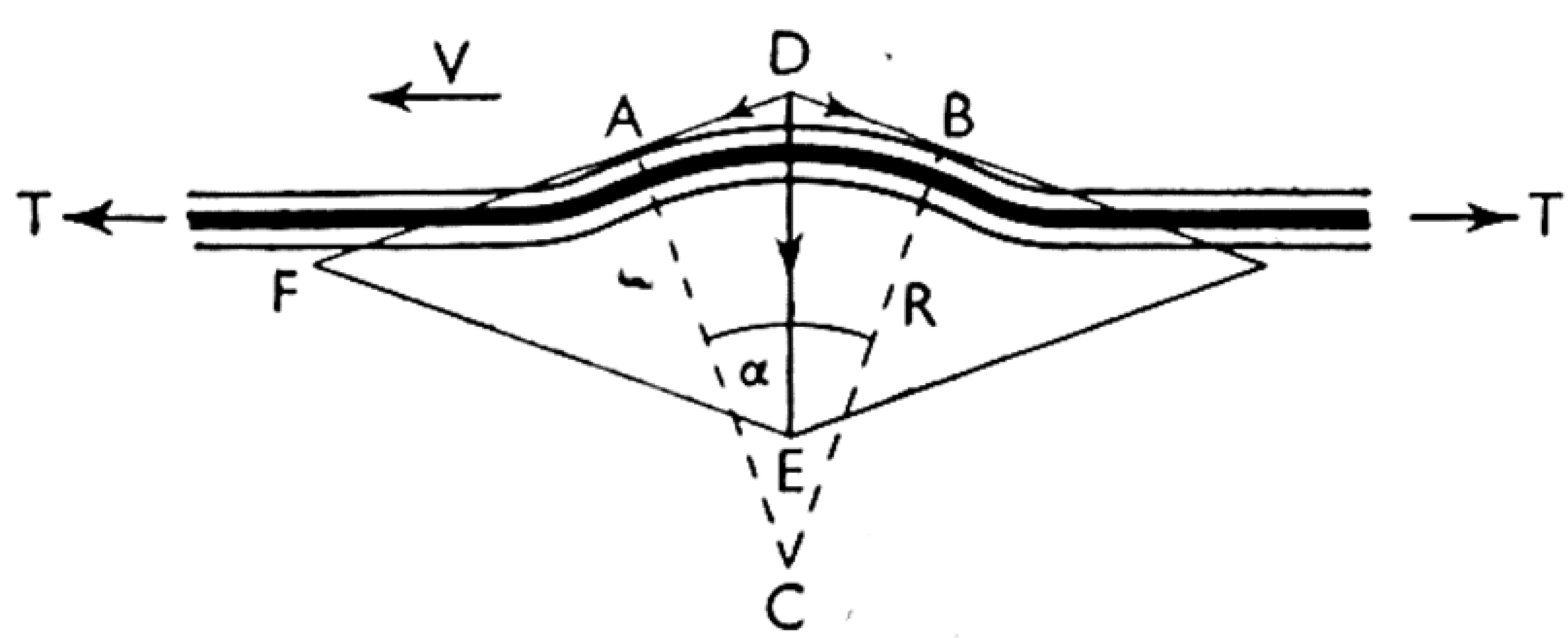
STRINGS and rods can both vibrate in two distinct ways, namely, transversely and longitudinally. In the case of a string we must assume that it is perfectly flexible, though of course in practice any string will possess a certain amount of rigidity. If, however, the length of the string is great compared with the thickness, the effects of rigidity are very small and can be disregarded.

We shall first consider transverse vibrations in strings and rods, and then proceed to a consideration of longitudinal vibrations.

TRANSVERSE VIBRATIONS IN STRINGS

Before we can deduce the modes of vibration of a stretched string, it is necessary to know the velocity with which a displacement wave travels along the string, and also the manner in which stationary waves are produced in the string when reflection takes place.

Velocity of propagation. Suppose that a perfectly flexible string of mass m per unit length is stretched by a force of T units. To deduce the velocity of propagation of a wave started in the string, we shall use the method originated by Tait. He imagined that the string is passed through a smooth tube with a velocity V, and the tube is straight except for the isolated portion which represents the wave in the string. The tension in the string gives rise to a force tending to straighten the tube and the string; this



is opposed by the centrifugal force due to the velocity V. Consider a portion AB of the string and let its length be l. The resultant force due to the two tensions is represented by DE, which is equal to $2DF \sin \alpha/2$ or $2T \sin \alpha/2$. If α is small, this expression can be written $T\alpha$, and, as $\alpha = l/R$, where R is the radius of curvature, we have the resultant force due to tension is equal to

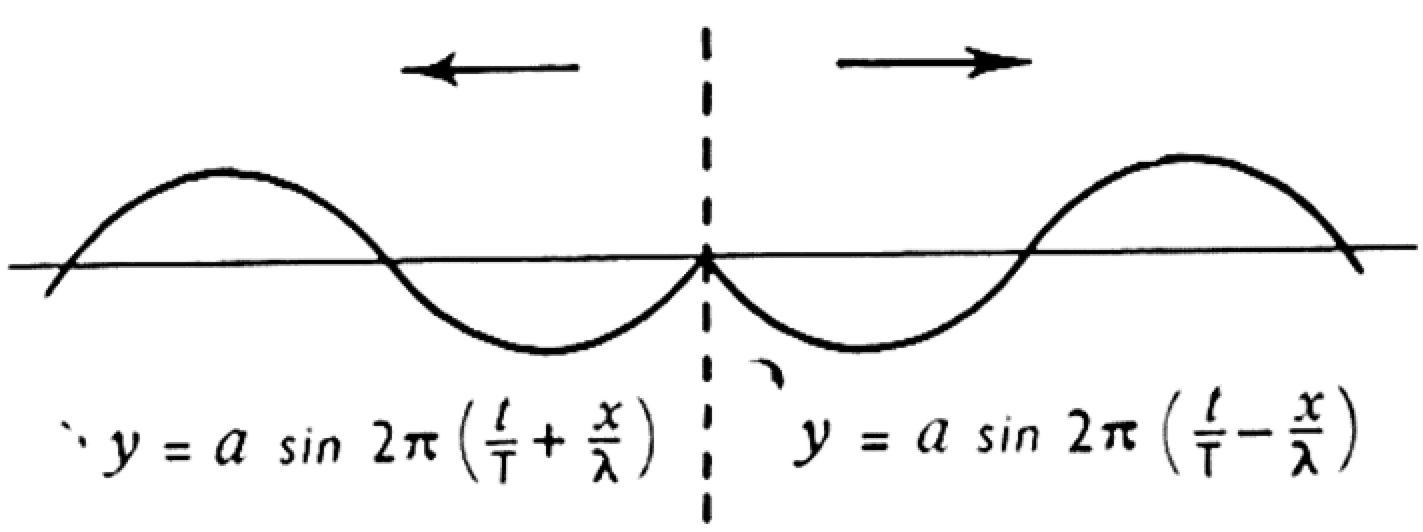
$$T \cdot \frac{l}{R}.$$

Now the centrifugal force due to a mass ml (m being the mass per unit length) moving with velocity V is mlV^2/R.

$$\therefore \quad \frac{mlV^2}{R} = T \cdot \frac{l}{R};$$

whence
$$V = \sqrt{\frac{T}{m}}.$$

Hence the velocity of propagation of a transverse wave in a string depends only on the tension of the string and the mass per unit length, and is quite independent of the wave-length.



Reflection of waves in strings. When a point in the middle of a string is given a simple harmonic motion, waves will start in both directions from the point, just as waves spread out in all directions when a stone is thrown into a pond of water. In the diagram, the wave travelling to the right of the disturbance is regarded as in the positive direction of x, and the one to the left as in the negative direction. The equation of the former wave is

$$y = a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right),$$

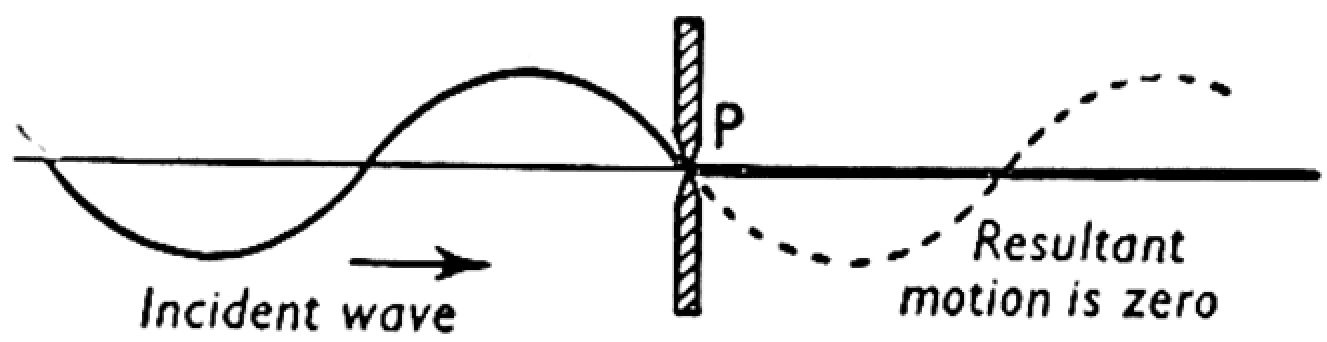
and that of the wave travelling to the left is

$$y = a \sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right).$$

When $x=0$, both equations reduce to the form

$$y=a \sin 2\pi \frac{t}{T}.$$

Now, when the waves in a stretched string reach a point at which the string is clamped, reflection occurs, and so a reflected wave is produced. To understand this, refer to the diagram,

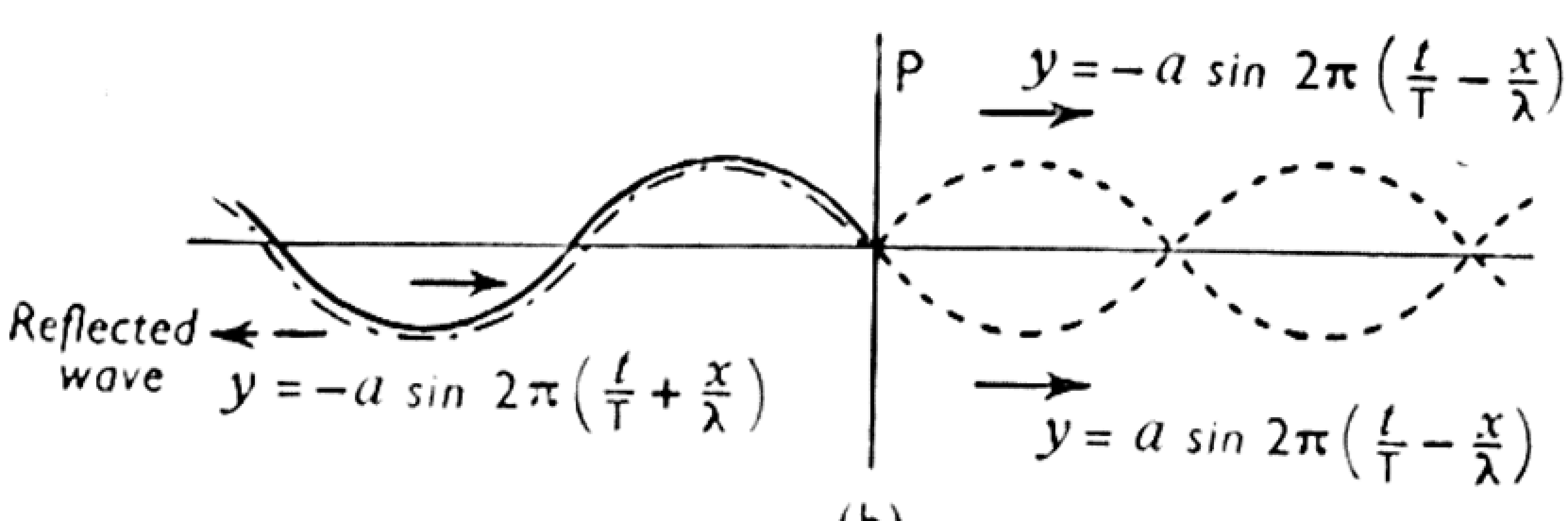


(a)

which depicts a string clamped at P and a wave travelling along the string towards P. If the clamp were absent, P would execute a simple harmonic motion, but this is prevented and the clamp exerts a simple harmonic force upon the string. Therefore from the last paragraph we see that two waves are set up at P, travelling in opposite directions. Since the resultant motion of all points to the right of P is zero, the wave started at P and moving to the right must be exactly equal and opposite to the continuation of the incident wave beyond P. The wave started at P which travels to the left is the reflected wave, and the phase of this wave is always such that the resultant displacement of P due to the incident and reflected waves is zero. In diagram (*b*) the various waves are shown. The incident wave is represented by the firm line; its continuation past the point P is given by the dotted line and the reflected wave started at P is shown by the chain line.

The equation of the incident wave is

$$y=a \sin 2\pi \left(\frac{t}{T}-\frac{x}{\lambda}\right).$$



(b)

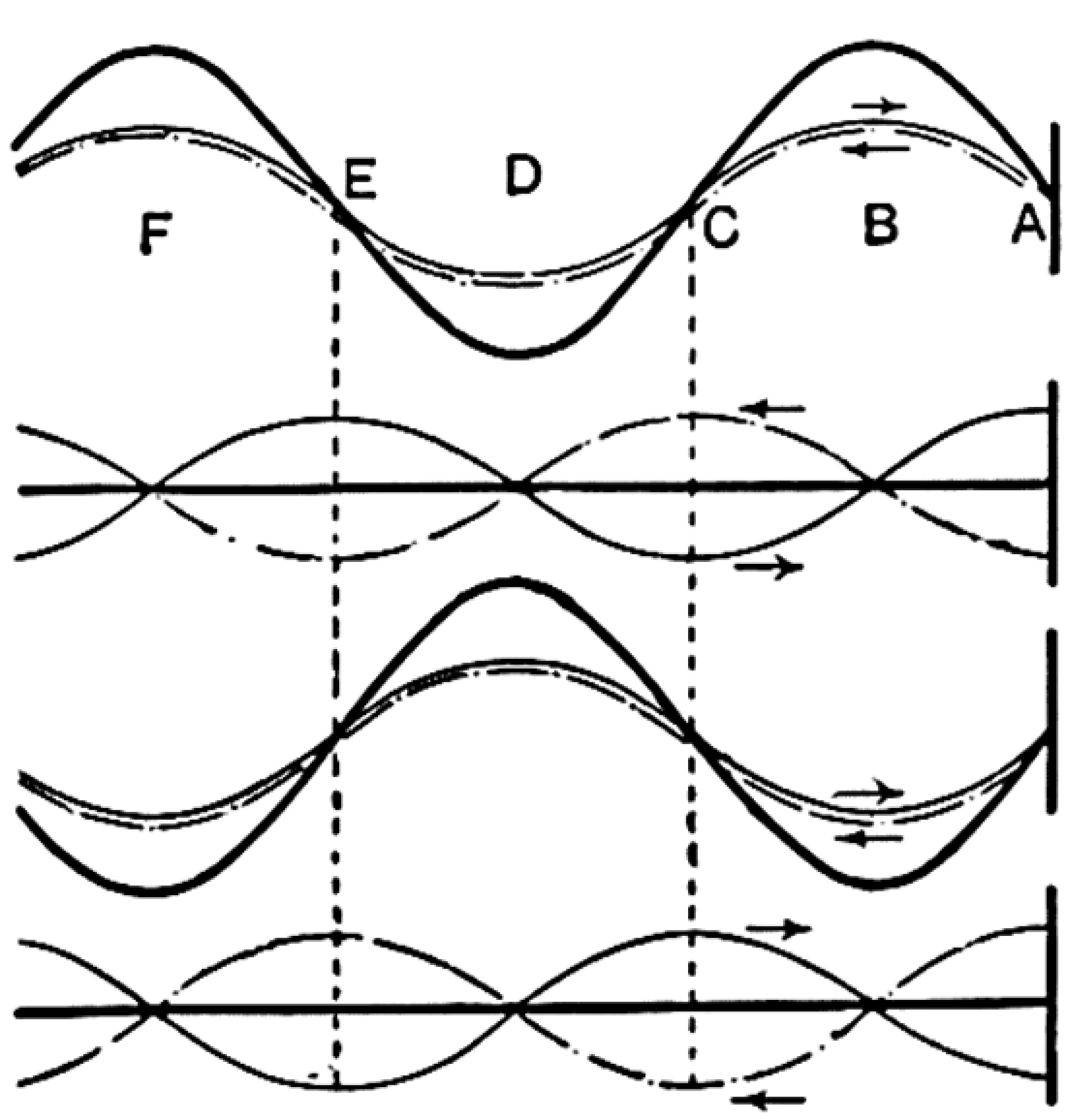
Since the continuation of this wave is cancelled out, the wave started at P and moving to the right must be represented by

$$y = -a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right).$$

The companion wave to this moving to the left, which is the reflected wave, is represented by

$$y = -a \sin 2\pi \left(\frac{t}{T} + \frac{x}{\lambda}\right).$$

Stationary waves in strings. In the above discussion, it will be noticed that, when a string is fixed at one end, we have an incident wave and a reflected wave of the same frequency. Hence interference occurs and the two sets of waves give rise to a state of steady vibration, the so-called **stationary waves.** The diagram shows the state of the string during one whole period of vibration, at quarter period intervals. In each case the firm lines represent the forward incident wave, and the dotted line the reflected wave, while the resultant is shown by the thick black line, and this, of course, is the actual shape of the vibrating string. It will be noticed that at certain points such as C, E, etc., there is no movement of the string; these positions are the **nodes.** At points B, D, etc., there is maximum disturbance and these are the **antinodes.** The wave-length is equal to the distance between alternate nodes or alternate antinodes.



Any segment of the string such as AC, CE, etc., vibrates from side to side, and on account of the rapidity of motion the observer sees the length of the wire divided into loops. Each part of the string executes a simple harmonic motion, and opposite segments are in opposite phases, although all points of any segment are in the same phase.

Mathematical treatment. The equation of the incident wave is

$$y_1 = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right),$$

and that of the reflected wave is

$$y_2 = -a \sin 2\pi \left(\frac{t}{T} + \frac{x}{\lambda}\right).$$

Therefore, the resultant is

$$\begin{aligned} y = y_1 + y_2 &= a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right) - a \sin 2\pi \left(\frac{t}{T} + \frac{x}{\lambda}\right) \\ &= -2a \sin 2\pi \frac{x}{\lambda} . \cos 2\pi \frac{t}{T}. \end{aligned}$$

Hence, each point on the string executes a simple harmonic motion of amplitude equal to

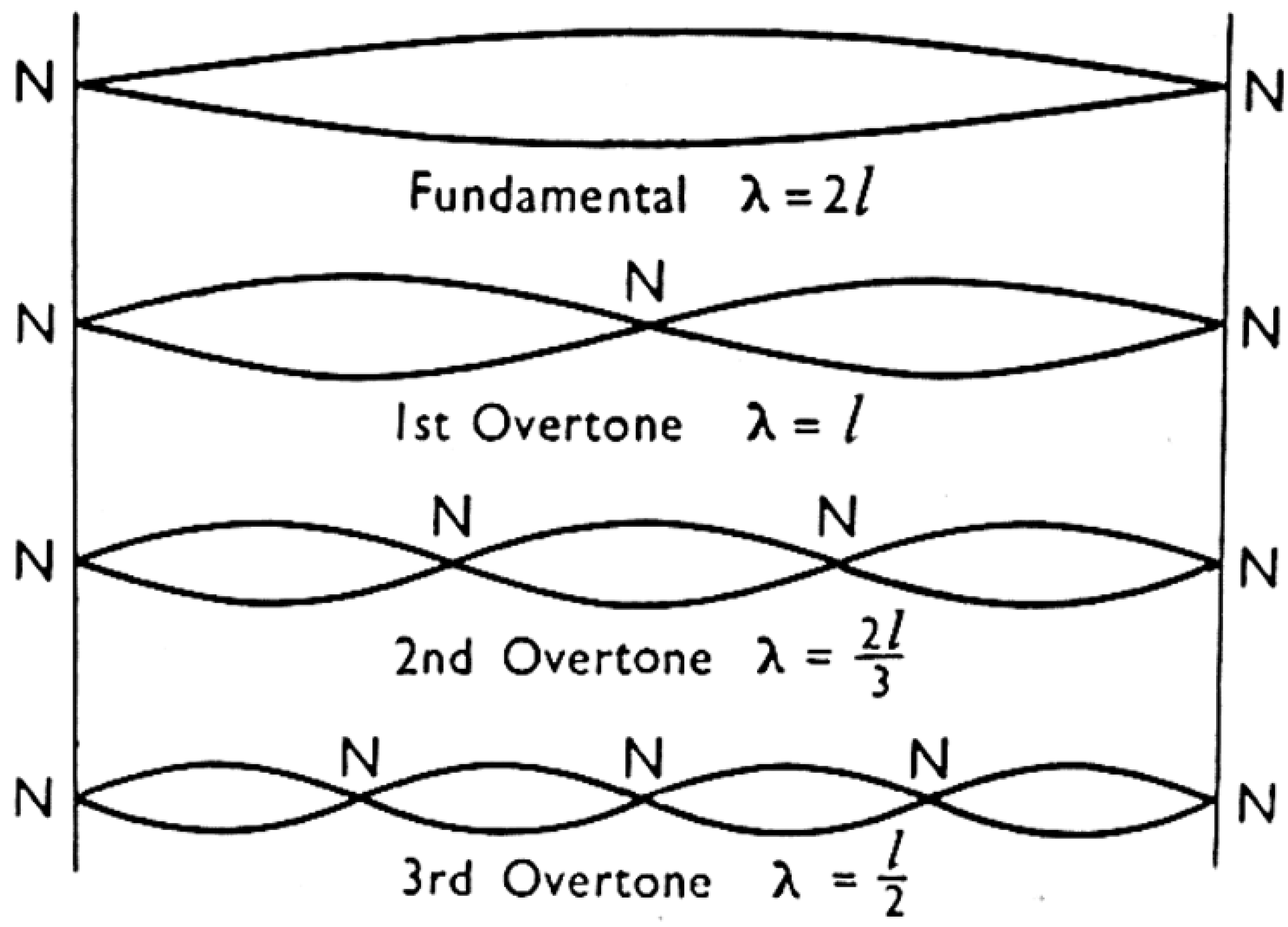
$$-2a \sin 2\pi \frac{x}{\lambda}.$$

If $t=0$, the equation becomes

$$y = -2a \sin 2\pi \frac{x}{\lambda},$$

and this gives the shape of the string at this instant. When $x=0$, $\lambda/2$, λ, etc., the amplitude is zero and these positions are the nodes. When $x=\lambda/4$, $3\lambda/4$, etc., the amplitude is $2a$ (or $-2a$) and these are the antinodes.

String fixed at both ends. In acoustics we are chiefly concerned with a string which is fixed at both ends. If a simple harmonic motion is started in such a string, the waves travel to both ends, are reflected to the opposite ends and are again reflected, returning to the original point after having travelled twice the length of the string. If these waves are exactly in phase (when they reach the point of the disturbance) with the disturbance then being produced, the waves will be reinforced and the process will



be continued with increasing amplitude. Thus the wave has travelled *twice* the length of the string, and for the simplest type of vibration, that is, when the string vibrates in one segment, the interval of time is that required for one complete vibration.

Hence, we have $2l = \lambda$, and since $V = n\lambda = \sqrt{T/m}$ (p. 94), we may write

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}.$$

The note emitted by the vibrating string under the above conditions is the **fundamental** note. But the string may and does vibrate in different modes at the same time as the fundamental note is sounding. For example, the string also vibrates in two segments, in which case the frequency is given by

$$n_1 = \frac{1}{l}\sqrt{\frac{T}{m}},$$

and the resulting note is the octave above the fundamental. There are many possible frequencies of vibration for a string, the frequencies being proportional to the numbers 1, 2, 3, 4, etc. The notes resulting from the frequencies after the fundamental are termed **overtones**. In the case of a stretched string, the frequencies form a harmonic series (see p. 159), and for this reason the overtones are sometimes called **harmonics**. But it must not be thought that the overtones produced by all musical instruments are harmonics ; for example, the first overtone of a tuning fork has a frequency of about 6·25 times that of the fundamental.

It should be noted that when the thickness of a stretched string becomes appreciable in relation to the length, the stiffness

may have a perceptible effect on the frequency, this effect becoming more and more important the greater the number of loops. For greater accuracy the equation

$$n=\frac{s}{2l}\sqrt{\frac{T}{m}}$$

must be modified to

$$n=\frac{s}{2l}\sqrt{\frac{T}{m}\left(1+\frac{\pi^3r^4s^2E}{8l^2T}\right)},$$

where r is the radius of the wire and E the modulus of elasticity.

The laws concerning the period of vibration of a stretched string were discovered experimentally by Mersenne in 1636, but it is to Bernoulli that we owe the mathematical solution of the problem. Mersenne's laws may be stated as follows :

(1) For a given string and a given tension, the time of vibration varies as the length. This fundamental principle of the sonometer appears to have been understood by the ancients, for Aristotle " knew that a pipe or a cord of double length produced a sound of which the vibrations occupied a double time ; and that the properties of concords depended on the proportions of the times occupied by the vibrations of the separate sounds."

(2) When the length of the string is given, the time varies inversely as the square root of the tension.

(3) Strings of the same length and tension vibrate in times which are proportional to the square roots of the linear density.

If it can be assumed that the period of vibration of a string depends only on length, mass per unit length and tension, the relation between these quantities can be obtained by the method of dimensions.

Let t (periodic time) $=k\,.\,l^xm^yT^z$, where k is a number. Dimensions of $t=(T)$, of $l=(L)$, of $m=(ML^{-1})$, of $T=(MLT^{-2})$.

$$\begin{aligned}\therefore\quad (T)&=k(L)^x\times(ML^{-1})^y\times(MLT^{-2})^z\\&=k(L)^{x-y+z}\times(M)^{y+z}\times(T)^{-2z}.\end{aligned}$$

Comparing both sides, we get $x-y+z=0$, $y+z=0$ and $-2z=1$. From which, $z=-\frac{1}{2}$, $y=\frac{1}{2}$, $x=1$.

Hence $$t=k(lm^{1/2}T^{-1/2}),$$

or, expressing it in the more usual form,

$$t=kl\sqrt{\frac{m}{T}}.$$

The only thing left undetermined is of course the value of k.

It must be noted that in using this method of dimensions, we have *assumed* only, and not proved, that there is a definite periodic time depending on *no other quantities* than those above mentioned; for example, we have not proved that t is independent of the amplitude of vibration, though of course we know both from experiment and theory that it is. It is important that the student should realise the limitations of the method of dimensions, as well as the fact that with proper care the method can be of great value.

EXPERIMENTAL WORK

Sonometer. The sonometer, with which the student will be familiar, affords a convenient method of proving the validity of the relationship $n=\frac{1}{2l}\sqrt{\frac{T}{m}}$.

(*a*) To show that $n\propto\frac{1}{l}$, a suitable load should be attached to the end of the string, and the movable bridge moved until, on plucking the string, the note emitted is in unison with that given by a tuning fork of known frequency, say 256. Measure the length of the wire. Now repeat the experiment with different forks, and measure the corresponding lengths in each case. A graph should now be plotted of log n against log l, and if a straight line results the relationship is true.

(*b*) A direct or an indirect method can be used to verify that $n\propto\sqrt{T}$. In the direct method, the length of vibrating string is kept constant (though not necessarily the whole length of the string) throughout the experiment. A suitable load should be put at the end of the string, so that the note emitted by the string is in unison with that of a tuning fork whose frequency is known; measure the load. Another fork of different frequency should now be used and weights added to the load until there is again unison. The experiment should be repeated with other forks, and the corresponding weights noted. On plotting a graph of log n against log T, a straight line of slope $\frac{1}{2}$ should result if the relationship is true. For indirect methods the student should refer to text-books on Practical Physics.

(*c*) To verify that $n\propto\sqrt{\frac{1}{m}}$, two wires may be used on the sono-

meter, one tuned to the frequency of a suitable fork ; this may be regarded as a standard wire. Apply a suitable tension to the second wire and find the length l_1 for unison between the two wires. Now replace the second wire by another one stretched with the same load, and find the length l_2 for unison with the standard wire. Repeat for several wires of different diameters or different material, and in each case find the mass (m) per unit length. Now plot $\log l$ against $\log m$. If the relationship is true, the resulting graph should be a straight line, the slope of which is $-\frac{1}{2}$.

The non-musical ear has a certain difficulty in deciding when two notes are in unison. In such a case, two methods may be used to indicate when the tuning is correct.

(i) By using the phenomenon of beats. The sonometer wire can be adjusted in length or tension so that beats can be distinctly heard when the wire and tuning fork are sounding together. Further adjustments are now made so that the beats get slower and slower. When they can no longer be distinguished, the two notes may be regarded as being in unison.

(ii) A small paper rider is placed on the middle of the string to be tuned. When the tuning fork is sounding and its stem resting on the sonometer board, the rider will flutter if the tuning is approximately correct, and when the tuning is exact, it will be thrown off. Thus, by altering the length of the wire so as to produce this effect, the wire may be correctly tuned. For success in this method, it is important that the rider should be small, and lightly placed on the middle of the string where the amplitude is greatest. Further, it should be noted that a shorter length of string will have a smaller amplitude than a longer length for a given tension.

Velocity of transverse waves in stretched strings. The sonometer may be used to find the velocity of a transverse wave in a string. We know that $V = n\lambda = \sqrt{\frac{T}{m}}$. Therefore, if we can find the wave-length and the frequency of the note emitted by a vibrating string, we can find the value of V. Put a suitable load at the end of the string so that the note produced when the string is set vibrating is in unison with a tuning fork of known frequency. The string will sound its fundamental note and the wave-length will be equal to $2l$. Hence the velocity can be determined, and this can be checked by the application of the relationship

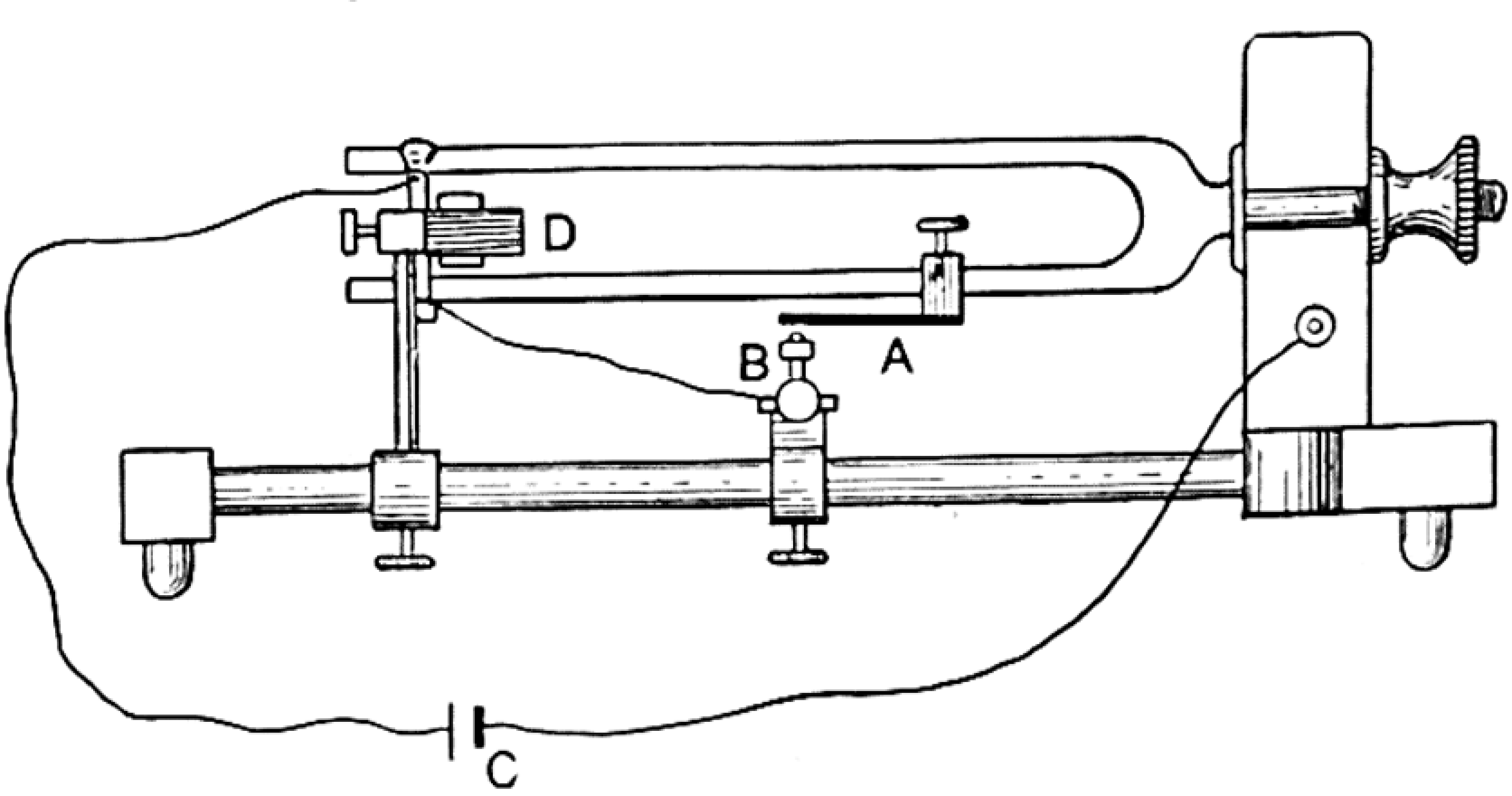
$$V = \sqrt{T/m}.$$

An alternative method is as follows. Fix a long piece of ordinary string or cord to a hook at one end of the laboratory and pass it over a pulley arranged at the other end ; suspend a suitable mass (M) from the free end. Near each end the string should pass over the edges of wooden bridges, which can be ordinary wooden metre scales. Strike the string sharply near one of the bridges. A pulse travels along the string and on reaching the other bridge it is reflected with a change of displacement. If the original displacement is downwards the reflected displacement is upwards. Reflection occurs again when the disturbance reaches the opposite bridge, and a pulse with a downward displacement passes towards the pulley, and so on. If a point near the bridge at the fixed end is observed, it will be seen to move down and then up again every time the pulse returns to the fixed end. The interval between the beginnings of successive upward movements of the point is the time in which the disturbance travels twice the length of the string between the bridges. The amplitude of the disturbance will of course decrease continuously, but it should be possible to count a satisfactory number of displacements.

The time t between a displacement and the nth following displacement, which is the time occupied by n double journeys, is found by means of a stop watch, and the length l of the string between the bridges is measured. The velocity of the wave is then given by $V=2nl/t$. The tension, given by Mg if we neglect the small effect due to sagging, should be calculated, and m should also be found. The velocity can then be obtained from

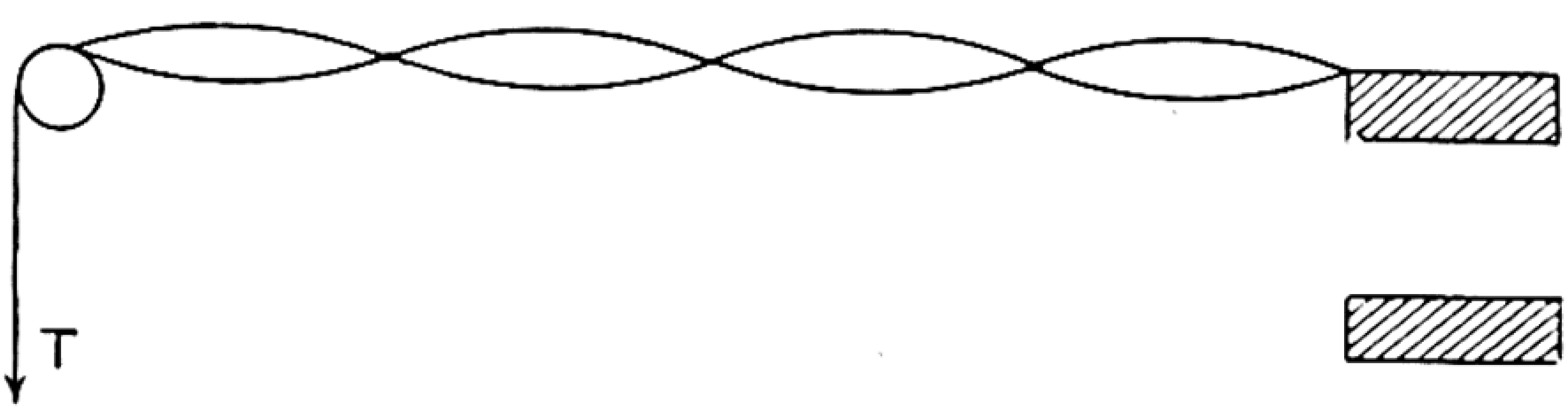
$$V=\sqrt{T/m}.$$

Melde's experiment. A very effective method of demonstrating the vibrations of strings is by Melde's experiment. For the purpose an electrically driven tuning fork should be used. In this,



the fork is clamped to a stand and is provided with a short metal bar A having a platinum contact which touches a similar one at B. An electric cell C is connected to the stand and then through the fork to A. B is connected to the coil of a small electro-magnet D situated between the prongs of the fork, and the other end of the coil is connected to the cell. When contact is made at B, the electrical circuit is closed and the magnet pulls the prongs together slightly, thus breaking the contact at B and so opening the circuit. The prongs now move apart and contact is again made at B and the process is repeated. Thus the prongs receive impulses which bring them nearer together at regular intervals determined by the frequency of the fork, and the fork is maintained in continuous vibration.

To perform Melde's experiment, place the fork so that the motion of the prongs is in the direction perpendicular to the string as indicated in the diagram. Attach one end of the thread to one prong of the fork, and the other end to a pan passing over a pulley which can be clamped in position at the end of the bench. Excite the fork and load the pan until the string is vibrating in one loop, that is, in its fundamental mode. The length may be adjusted as well as the tension in order to get this condition.



Now, keeping the length the same, reduce the tension sufficiently to obtain first two loops, then three loops, and so on, each time finding the value of the tension corresponding with each of the modes of vibration. Tabulate the results as under :

Tension (T)	No. of loops	λ	λ^2
	1		
	2		
	3		

Measure the distance between the first well-defined node on the right of the string and the last on the left. Let this be d, and suppose there are n loops between them; then $\lambda = 2d/n$.

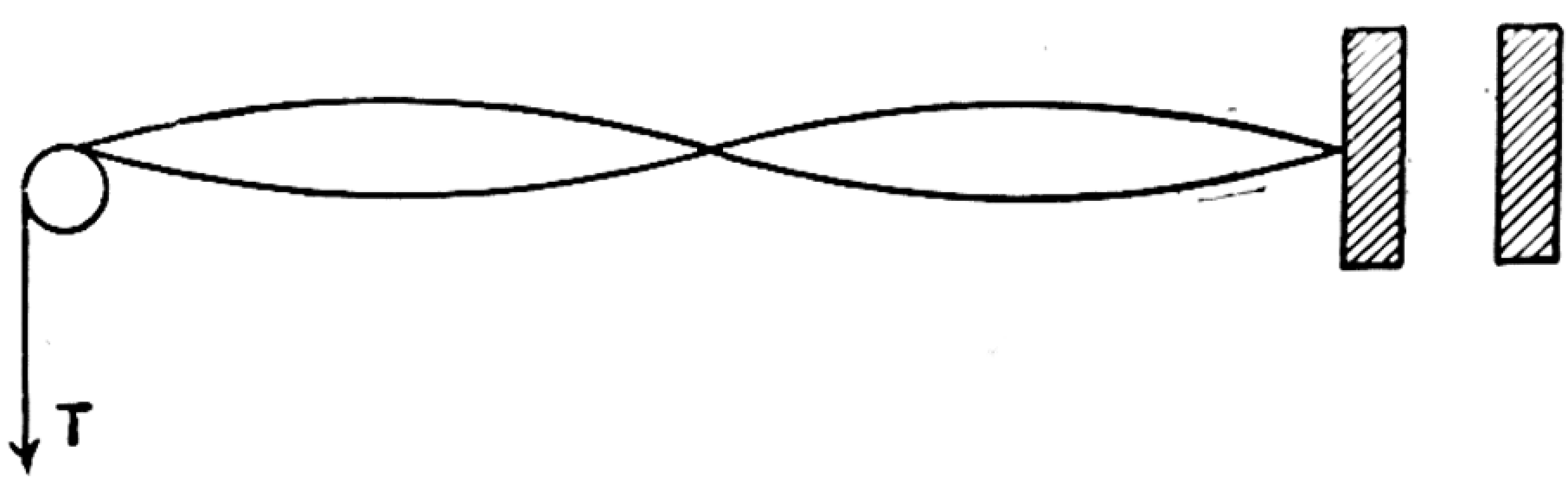
Now in the experiment, a disturbance travels along the string with a velocity V given by $V = \sqrt{T/m}$. When the string is vibrating in its fundamental mode, there is a node at each end and an antinode in the middle, and the wave-length is twice the length of the string. We have

$$n\lambda = V = \sqrt{\frac{T}{m}}$$

or

$$n = \frac{1}{\lambda} \cdot \sqrt{\frac{T}{m}}.$$

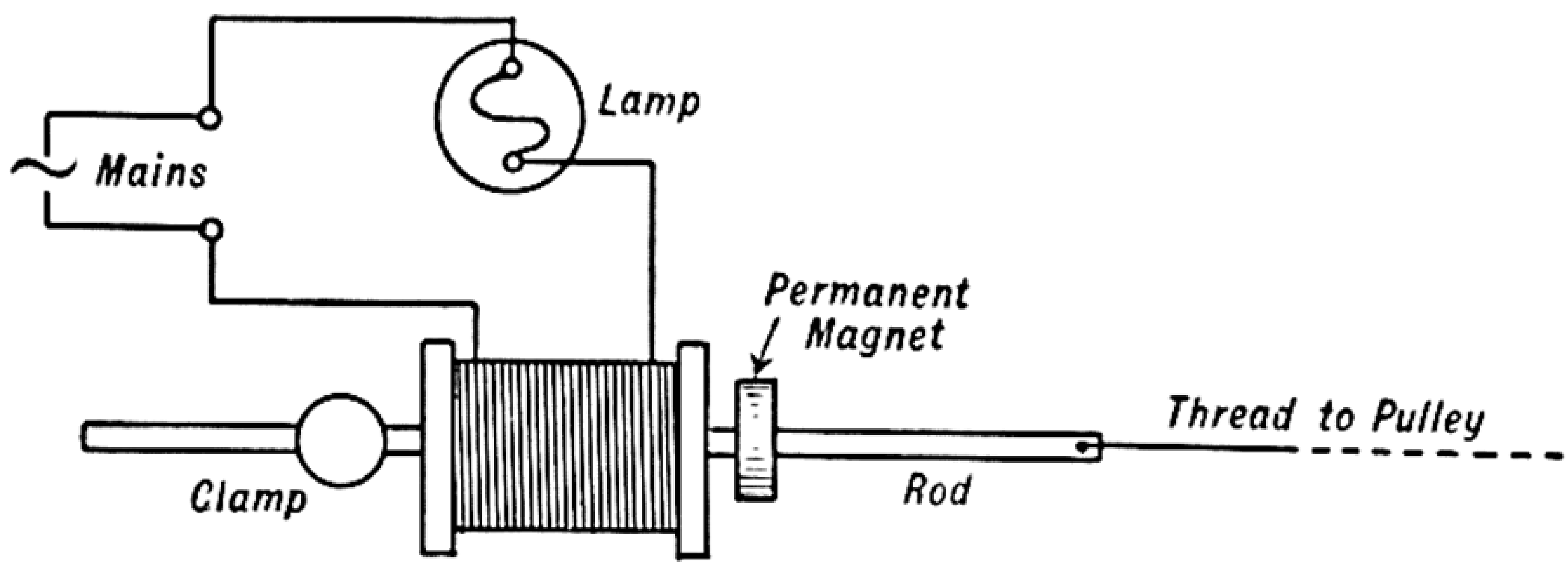
Therefore, if T is plotted against λ^2, the graph should be a straight line, since both n and m are constant, thus proving that $n \propto \sqrt{T}$.



The experiment should be repeated, only this time arrange the fork so that the motion of the prongs is in the direction along the string. For the same length of wire and similar values of T as in the first case, it will be found that the number of loops is halved, for in the first case the frequency of the string is the same as that of the fork, while in the second case it is only half as great.

This may be understood by noting that, when, in the second case, the prong is in the extreme position on the left, the string is slack in the first vibration, and when in the extreme position on the right, it is horizontal and tight. The inertia of the string carries it onwards, so that when the prong returns to the extreme left position and so completes one vibration, the string has completed one half-vibration only.

Electrical vibrator. Melde's experiment can also be performed by using as a vibrator a short rod which is excited by means of the A.C. mains supply. One type of vibrator consists of a steel rod about 9 in. long and $\frac{1}{16}$ in. diameter, used as the vibrator,



suitably clamped at one end. This rod passes through the centre of a solenoid and then between the poles of a strong magnet. The solenoid is wired in series with a suitable lamp so that direct connection may be made with the mains supply. The length of the vibrating portion of the rod can be varied by loosening the clamp and sliding the rod one way or the other through the solenoid. On switching on the current the vibrator is magnetised longitudinally with polarity which is reversed with the current. The interaction with the field of the permanent magnet sets up lateral forces which tend to make the rod vibrate with the frequency of the mains. If now the length of the rod is slowly adjusted, a position will be found when the frequency of its vibrations is the same as that for the supply ; when resonance is established in this way, a satisfactory amplitude of vibration of the string attached to the rod is maintained.

APPLICATIONS OF VIBRATIONS OF STRINGS

Frequency of the A.C. mains supply. It will be obvious from the above that an electrical vibrator can be used to find the frequency of the A.C. mains supply. Attach a long wire or string to the end of the vibrator and place a load in the pan at the other end, sufficient to produce a definite number of well-defined loops when the string is set in vibration. Since the end of the rod is vibrating, there will probably not be an exact node at this point, so it is advisable to neglect this first loop. Count the number of loops and the length of the wire involved ; also weigh the wire after the experiment to find the mass per unit length. Repeat with different loads and obtain as many results as possible. If S is the number of loops in the length l of the wire, we have $\lambda = 2l/S$. Therefore the frequency of the wire and of the A.C. supply is given by

$$n = \frac{V}{\lambda} = \frac{1}{\lambda}\sqrt{\frac{T}{m}} = \frac{S}{2l}\sqrt{\frac{T}{m}}\,.$$

A variation of the above experiment is to pass the alternating

current through the stretched wire and to cause the vibrations by arranging the poles of two bar magnets, one on either side of the middle of the string.

Acoustic strain gauge. The measurement of small mechanical strains in structures under tension and compression has engaged the attention of engineers for a number of years, and various gauges have been devised for the purpose. The most popular one at the present time is the electrical variable resistance gauge, but much useful work has been done by quasi-electrical types, one of which is the acoustic gauge.

In this type, the frequency of vibration of a stretched wire clamped to the structure at the point where the strain is to be measured is matched against the frequency of a second similar wire known as the reference gauge. The testing wire is stretched between two knife edges, one fixed and the other movable, and it is electrically maintained in vibration at its natural frequency. The wire passes between two pairs of pole-pieces of two small bar magnets, and around the north pole of each magnet is fixed a small coil. One coil acts as an exciter coil and the other as a pick-up coil; when the set is switched on, the small vibrations of the string generate small oscillating currents in the pick-up coil.

The second wire is also connected in the electrical circuit, but it is not attached to the structure. It is, however, fastened to a tensioning screw which incorporates a calibrated dial, and when it is vibrating, the frequency can be altered by adjusting the tensioning screw.

The vibrations of the first wire are communicated to one ear-piece of a headphone worn by the operator, and those of the second wire to the other ear-piece; thus the pitches of both notes can be compared. If the matching is not exact, the tensioning screw is used and the reading on the calibrated dial is a measure of the strain in the specimen.

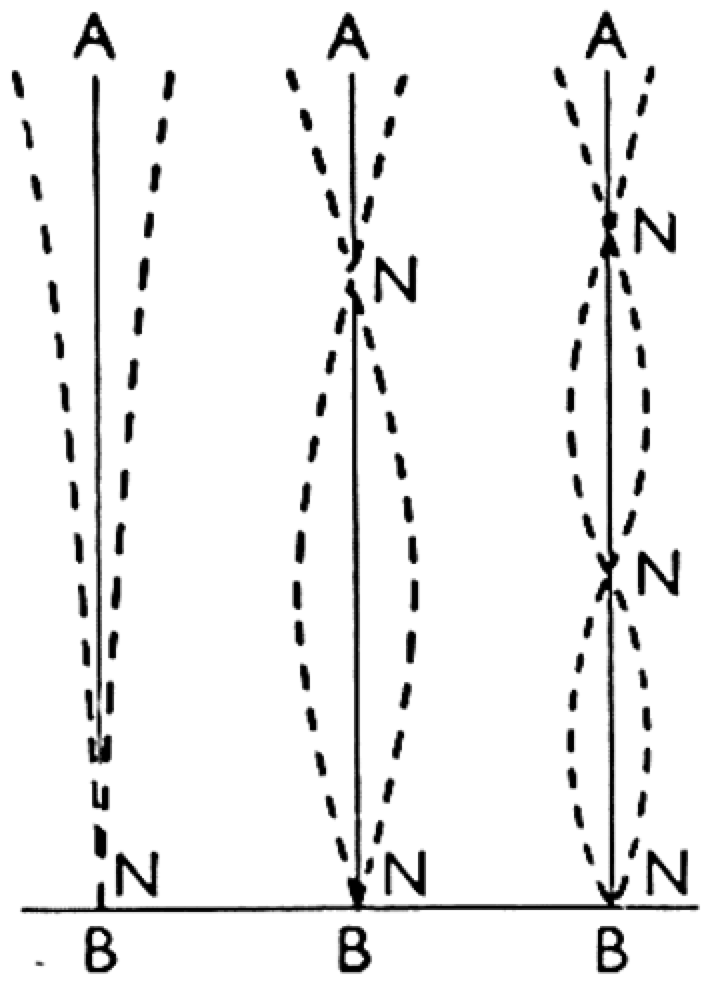
For further information on this and other forms of strain gauges the student should refer to the original literature on the subject.

TRANSVERSE VIBRATIONS OF RODS

In the case of a thick wire or rod, the stiffness may become the all-important factor and the tension may be disregarded. It can be shown that the velocity of a transverse wave in a rod is proportional to

$$t\sqrt{\frac{E}{\rho}}\,.\,\lambda,$$

where t is the thickness of the rod in the direction of displacement, E is Young's modulus and ρ is the density of the material. Hence in this case the velocity of propagation depends on λ. Further, it can be shown that the possible frequencies of a transverse vibration of a bar are given by

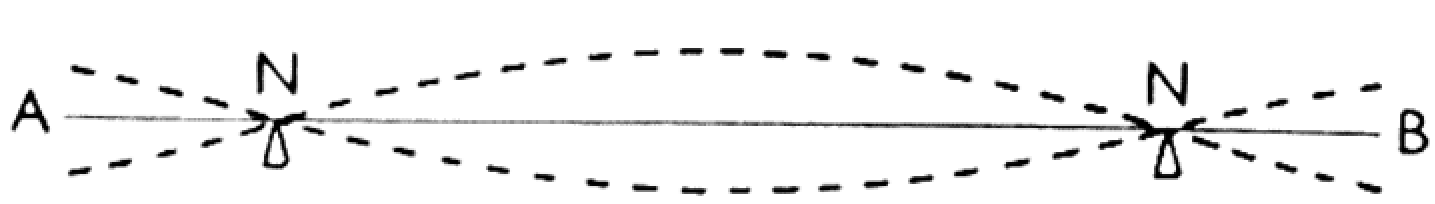


$$n = c\,\frac{k}{l^2}\sqrt{\frac{E}{\rho}},$$

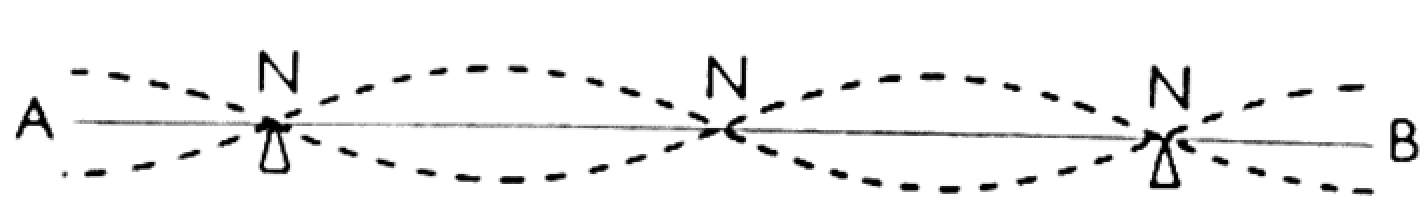
where k is the radius of gyration of the section of the bar about the neutral plane, l is the length of the bar and c is a constant depending on the method of supporting or clamping the bar and on the overtones to be excited. It is worthy of note here that n is proportional to $\sqrt{E/\rho}$, which is of course the velocity of a *longitudinal* wave in the rod.

If a rod is clamped at one end there is a node at this end, and the rod vibrates in its fundamental form as in (a); the modes of vibration when the rod is sounding the first and second overtones are given in (b) and (c). If the frequency of the fundamental is n, then the frequencies of the overtones are $6{\cdot}25n$, $17{\cdot}55n$, $34{\cdot}39n$, etc.

Now suppose the rod when vibrating as in (b) to be prolonged, and instead of being clamped at B to be simply supported at N and B. We should then have a rod free at both ends (a " free-free " rod) and the fundamental form of its vibration is shown in (d), while the mode of vibration for the first overtone is shown at (e), where there are three nodes. The relation between the frequencies of the fundamental and the overtones in this case are 1, 2·76, 5·40, 8·93, etc.



(d)

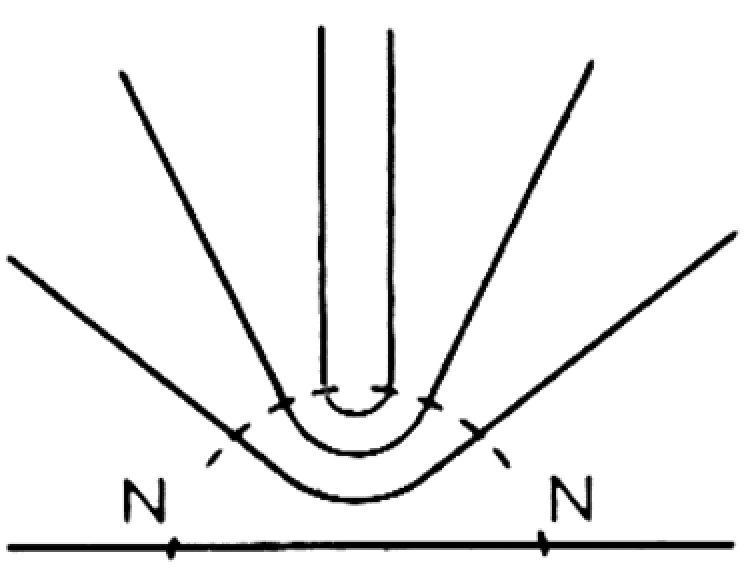


(e)

As in the case of strings, stationary waves can be set up in rods by the combined effects of the direct and reflected waves, and the possible forms of these will depend upon the method of supporting the rod.

The transverse vibrations of rods may be excited by means similar to those used in the case of strings, the overtones present depending on the method of excitation.

Tuning forks. A tuning fork may be regarded as a development of a " free-free " bar bent into the form of a *U*, or as a pair of " clamped-free " bars attached to a common block. If we consider it as a " free-free " bar, there are two nodes in the positions shown in the diagram. If the bar is gradually bent at the middle the nodes come nearer together, and when both limbs are parallel the nodes practically coincide and the arrangement forms a tuning fork. A stem is attached to the middle for convenience in holding, and since this point is a node the vibrations are not interfered with. It must be noted, however, that the stem *is* vibrating up and down at right angles to the prongs, and this motion can be communicated to a surface on which the stem rests. It will be clear from the above treatment why it is that a tuning fork vibrates in such a way that the ends alternately approach and recede from each other.



The period of vibration of forks of the same material and shape varies as the linear dimensions. The period will be approximately independent of the thickness *perpendicular* to the plane of bending, but will vary inversely with the thickness *in* the plane of bending. If the linear dimensions of a fork be doubled, its note will fall an octave, provided the material remains the same and the shape constant. The absolute frequency of a tuning fork depends on the velocity of sound in the material; hence a tuning fork made of brass would give a note about a fifth lower in pitch than one of the same dimensions made of steel.

To increase the frequency of a fork it is necessary to reduce the equivalent inertia of the systen. This can be done by filing away the ends of the prongs, either diminishing their thickness or actually shortening them. To lower the frequency, the material of the prongs near the bend may be reduced, the effect of which is to diminish the force of the spring, leaving the inertia practically unchanged. Another method is to increase the inertia by loading

the ends of the prongs with wax, or other material. Large forks are sometimes provided with movable weights which slide along the prongs and can be fixed in any position by screws. As these approach the ends, the equivalent inertia of the system increases, and in this way a considerable range of pitch can be obtained from one fork.

One reason why tuning forks are so important in acoustics is that the note given by a properly proportioned fork is practically a *pure* tone, free from overtones. Immediately after a fork is struck, high tones may indeed be heard, but these rapidly die away, and even while they exist, they do not blend with the proper tone of the fork, partly on account of their very high pitch and partly because they do not belong to its harmonic scale. The first and second overtones of a fork have frequencies about 6·25 and 17·6 times the frequency of the fundamental.

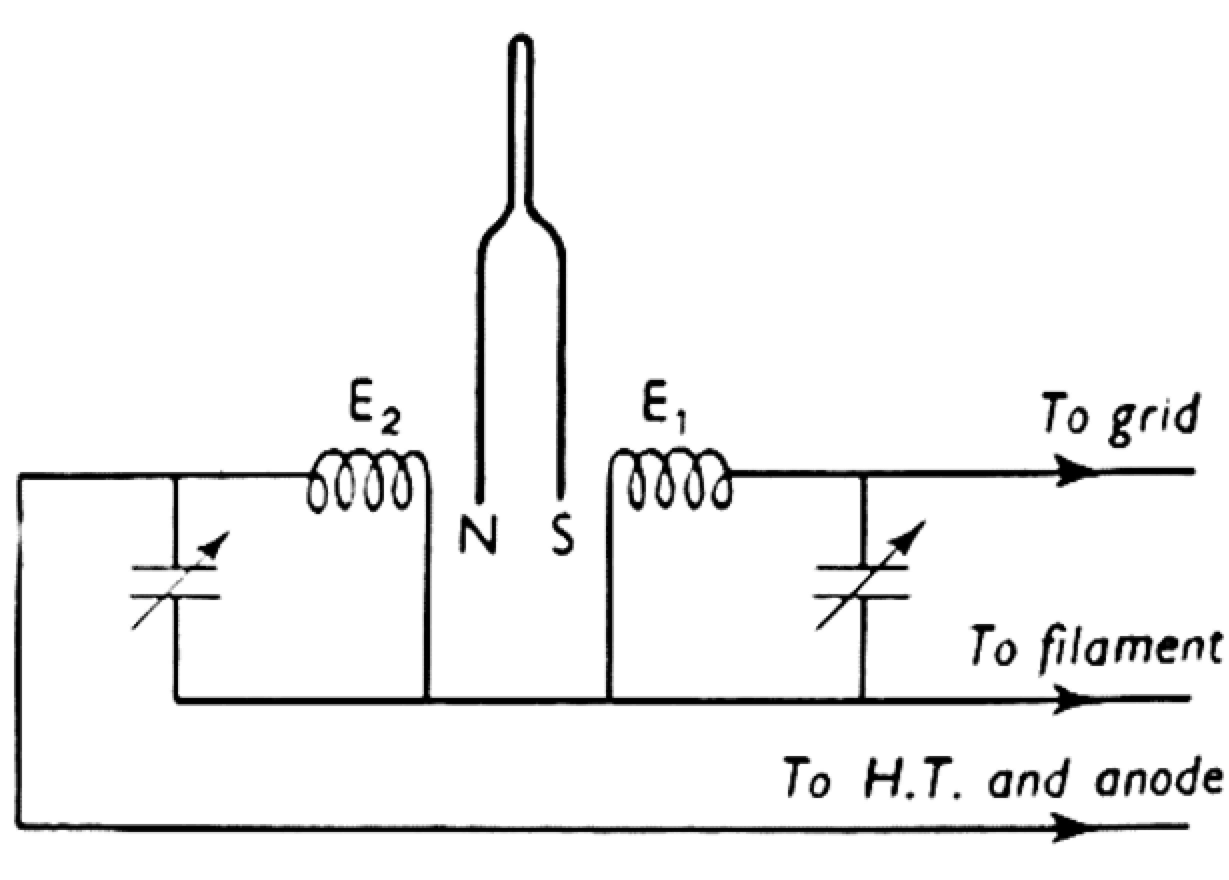
Tuning forks are sometimes provided with resonance boxes, and Koenig investigated the influence of resonators upon the pitch of forks. Without a resonator, a fork of frequency 256 sounded in a satisfactory manner for about 90 seconds. A resonator of adjustable pitch was then brought into proximity, and the pitch, originally much lower than that of the fork, was gradually raised. Even when the resonator was still a minor third below the fork, there was observed a slight diminution in the duration of the vibrations, and at the same time an increase in the frequency of about 0·005. As resonance between the two sounds was approached, the diminution in the time and the increase in frequency became more pronounced right up to the immediate neighbourhood of unison. But at the moment when unison was reached, the alteration of pitch suddenly disappeared, and the frequency became exactly the same as in the absence of the resonator. At the same time the sound was powerfully reinforced ; but this increase fell off rapidly and the vibration died away after 8 or 10 seconds. When the pitch of the resonator was again raised a little, the sound of the fork began to change in the opposite direction, being now as much too low as, before unison was reached, it had been too high. As the pitch of the resonator was further raised, the duration of the vibrations gradually recovered its original value of about 90 seconds. The maximum disturbance in the frequency observed by Koenig was 0·035 complete vibrations. By a mathematical analysis, Rayleigh showed that the effects described above, showing the instability of pitch accompanying a strong resonance, are to be expected. A similar example is found in the anomalous refraction of light.

The periodic time of a tuning fork is a very constant quantity; hence forks are in common use as standards of pitch and sub-standards of time. The pitch of organ pipes rapidly varies with temperature and with the pressure of the wind; that of strings with the tension, which can never be retained constant for long. But a tuning fork kept clean and not subjected to violent changes of temperature or magnetisation, preserves its pitch with great fidelity. It must not be supposed however that the frequency of a fork is altogether independent of temperature, for a change in temperature brings about a change in dimensions and in elasticity. According to the observations of McLeod and Clarke (1880), the frequency decreases by 0·00011 of its value for every degree centigrade rise, while the temperature coefficient found by Koenig is 0·00012. This means that a fork of frequency 256 falls 0·0286 for every degree centigrade rise of temperature.

Most modern forks are usually made of *elinvar*, which has an extremely small temperature coefficient.

Valve-maintained fork. For experimental work, especially in frequency measurements, it is useful to have a fork which can be maintained in vibration for comparatively long periods. One such fork was described on p. 103, and another one suitable for maintaining vibrations of rather higher frequency is the *valve-maintained fork* designed by Eccles.

The circuit is as shown in the diagram. The two prongs of the fork are magnetised, and two electromagnets, E_1 and E_2 are placed one on each side of the fork, one being connected to the grid and the other to the anode of the valve. Variable condensers are included in both the anode and grid circuits for tuning purposes. Briefly, the action is as follows. When the prongs are vibrating say outwards, a potential difference is induced in E_1 by S, and the winding of E_1 is in such a direction as to make the grid more



positive. This causes the anode current to increase, and if the direction of the winding of E_2 is suitably arranged, N is attracted and the vibration assisted.

It will be noticed that in this type of fork there are no attachments to interfere with the normal frequency.

LONGITUDINAL VIBRATIONS IN STRINGS AND RODS

In addition to transverse vibrations, it is also possible for strings and rods to vibrate longitudinally, and we have dealt with a practical case of this on p. 45 when discussing Kundt's tube. In the case of a stretched string, the frequency of the longitudinal vibrations is independent of the tension with which the string is stretched. For when a particle of the string is displaced from its normal position, the force with which it tends to return depends on the stress of the displacement from its normal position, and by Hooke's law this stress is quite independent of any previously existent stress. Thus the velocity of a longitudinal wave is independent of the tension and depends only on the elasticity and the density of the medium, that is, $V=\sqrt{E/\rho}$.

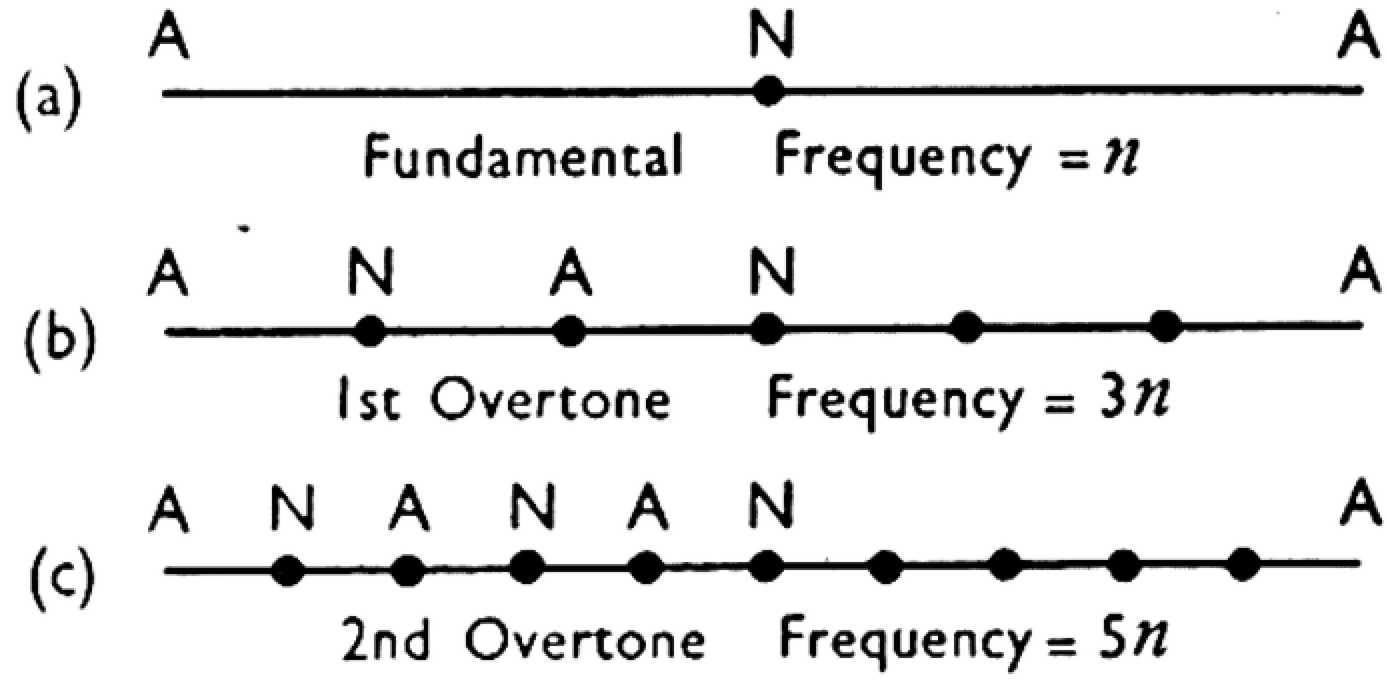
When a string which is vibrating longitudinally gives its fundamental note, there will be a node at each end and an antinode between. Hence $\lambda=2l$ and $V=2nl$. In the case of a rod clamped in the middle, there must be a node at this position and antinodes at either end ; therefore, in this case also, $V=2nl$. It should be clear now that if we can measure the frequency of the note given by a string or a rod of length l when vibrating longitudinally, we have the means of measuring the velocity of sound in the medium of which the string or rod is composed.

The various overtones for a stretched string or a free bar require the condition $s\lambda=2l$, where s is the number of loops. Hence the expression for the frequency of these overtones is

$$n=\frac{s}{2l}\sqrt{\frac{T}{m}},$$

which is similar to the case when the string is vibrating trans versely.

If a bar vibrating longitudinally is clamped at its middle point, all the overtones which require an antinode at this point are suppressed, and only the *odd* overtones are present. This will be seen from the diagram (p. 112). In (*a*) the bar vibrates to give the fundamental note, with a node in the centre and antinodes at the ends ; hence the length of the bar is equal to $\lambda/2$. The next



mode of vibration to give the first overtone must have three nodes arranged as in (*b*), and $\lambda/2$ now is equal to one-third of the length. Similarly for the second overtone, there must be five nodes as in (*c*) and $\lambda/2 = l/5$. The frequencies of the fundamental and the successive overtones must therefore be n, $3n$, $5n$, etc. The frequency of longitudinal vibrations in rods is usually very high compared with that of the transverse modes, the ratio increasing rapidly as the diameter or thickness diminishes relative to length.

Methods of excitation. Rods of metal, wood or glass, clamped at the mid-point, are readily set into longitudinal vibration by the steady frictional drag of a resined cloth drawn along the rod towards an antinode, as in the Kundt's tube experiment. If the rod is a relatively stiff one, the vibrations may be set up by striking the end a sharp blow with a hammer. In this case, however, both transverse and longitudinal vibrations may be excited, though the one or the other can be rapidly damped out by clamping at a suitable point.

Electrical means may also be used for exciting longitudinal vibrations in bars of magnetic materials when an alternating current of resonant frequency is available. The A.C. is passed through an iron wire core which is mounted close to the end of a steel bar. The resonance is very sharp, necessitating careful tuning, and both fundamental and the overtones can readily be excited as almost pure tones.

Vibrations in non-magnetic rods have been obtained by electrostatic means, the end of the vibrating rod forming one plate of a condenser supplied with high-frequency A.C. from a valve oscillator.

When considering the longitudinal vibrations of a rod, it must not be forgotten that if a rod is stretched by a force parallel to its length, the extension is in general accompanied by lateral contraction in such a manner that the increase in volume is less than if the displacement of every particle were parallel to the

axis. If a rod is long compared with the diameter, the inertia of the lateral motion may be neglected. But if the rod is short, the lateral motion of a particle near the boundary would be comparable in magnitude with the longitudinal motion and could not be overlooked without risk of considerable error. Clearly in such cases, Poisson's ratio (μ) for the material of the rod would have to be brought into account, and Rayleigh calculated that the effect of the lateral motion is to increase the period in the ratio

$$1 : 1 + \frac{i^2\mu^2\pi^2}{4} \cdot \frac{r^2}{l^2},$$

where r is the radius of the rod, l is the length, and i is an integer.

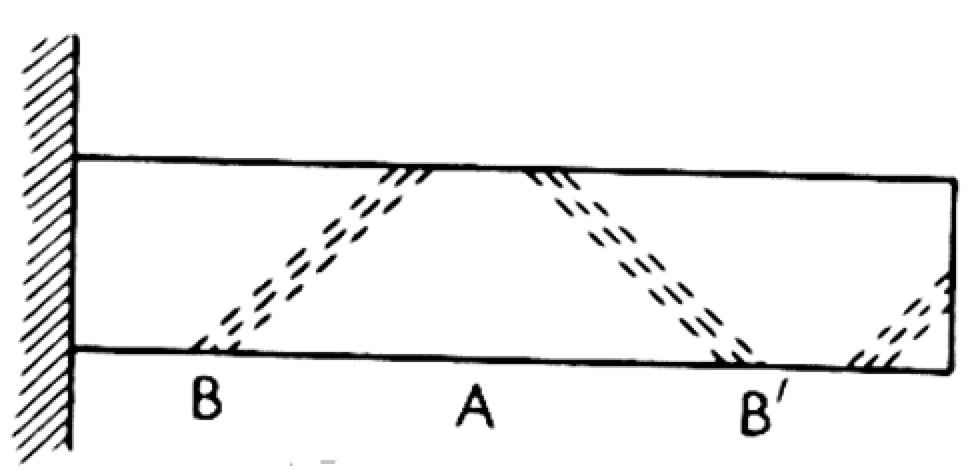
It is worth while pointing out here that the value of μ must lie between 0 and $\frac{1}{2}$. If μ were negative, a longitudinal tension would produce a lateral swelling, which can scarcely be possible in solids. If μ were greater than $\frac{1}{2}$, the lateral contraction would be great enough to overbalance the elongation and cause a diminution of volume on the whole; this would be inconsistent with stability. At one time, it was supposed that μ must of necessity be equal to $\frac{1}{4}$ for all solids, but it is now known that its value varies.

Torsional vibrations. If a rod or bar is clamped at one end, and the side is bowed transversely, a very high note is produced; the bar twists and untwists alternately and the vibrations are called **torsional vibrations.** If the bar is in the form of a rectangle and is held horizontal, the nature of the vibrations can be seen by sprinkling sand on the face. There is no doubt that transverse as well as torsional vibrations are set up. The position of the nodal lines is indicated in the diagram by the shaded portions. If the vibrations were transverse only, the lines would be at right angles to the edge, but owing to the presence of torsional vibrations the lines are inclined as shown when the bar is bowed at A and damped at B.

The force with which twisting is resisted depends upon the modulus of elasticity, called the rigidity. If we call this n, the relation between E (Young's modulus), μ and n may be written

$$n = \frac{E}{2(\mu + 1)},$$

showing that n lies between $E/2$ and $E/3$.



The velocity of wave propagation is $\sqrt{n/\rho}$, and it will be seen that the velocity of longitudinal vibrations is to that of torsional vibrations in the ratio $\sqrt{E} : \sqrt{n}$, or $\sqrt{(2+2\mu)} : 1$. For example, if $\mu = 1/3$, the ratio of frequencies would be

$$\sqrt{E} : \sqrt{n} = \sqrt{8} : \sqrt{3} = 1 \cdot 63,$$

corresponding to an interval rather greater than a fifth. In all cases, the ratio of frequencies must lie between $\sqrt{2}/1$ and $\sqrt{3}/1$.

It must be said in conclusion that torsional vibrations are of very small importance compared with longitudinal vibrations.

CHAPTER VI

CHARACTERISTICS OF MUSICAL SOUNDS

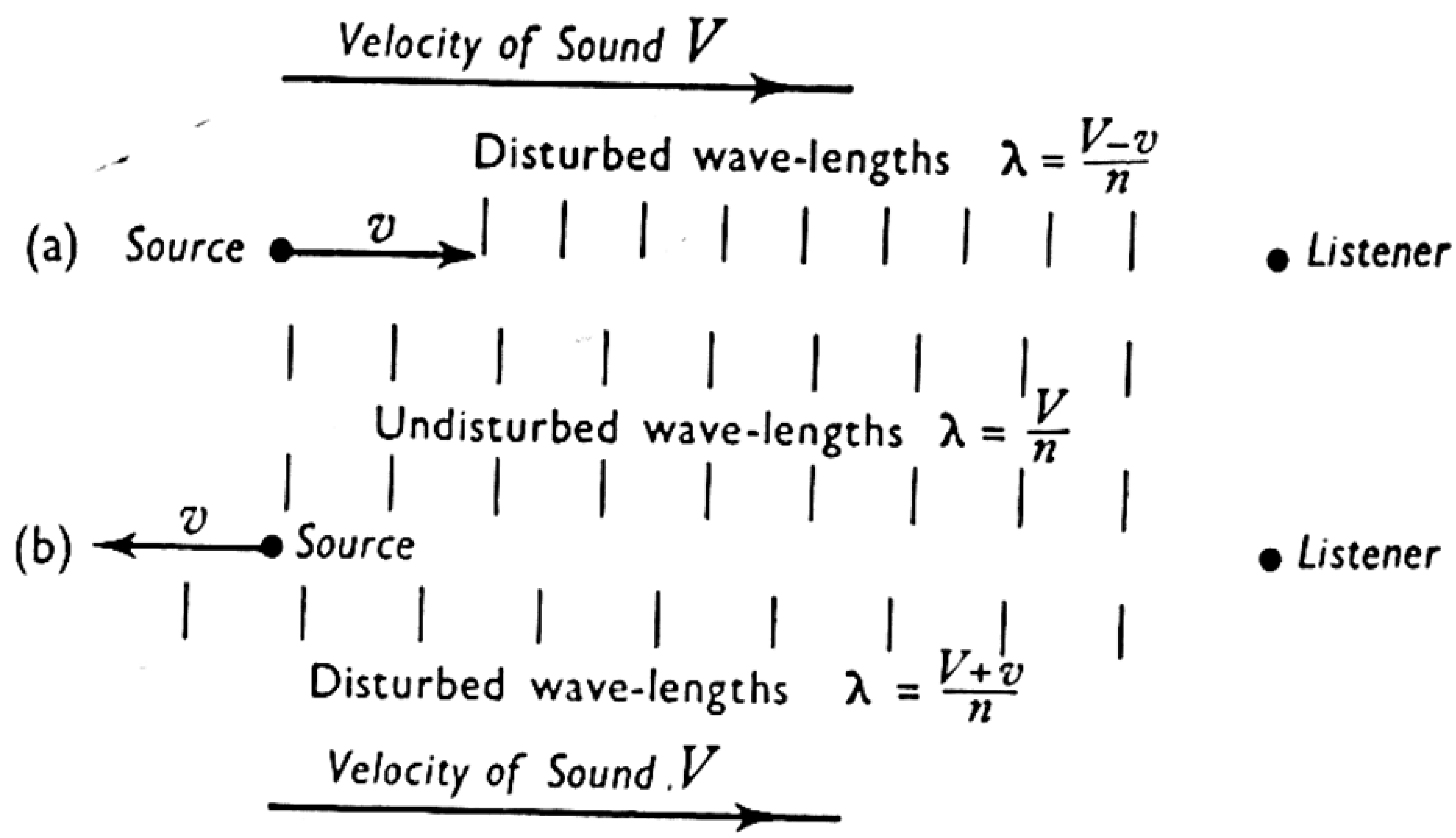
THE three characteristics of musical sounds are pitch, loudness and quality, and they will be considered in that order.

PITCH

The pitch of a note is that property of the sound which determines its position on the musical scale, which is a series of notes of definite and different frequencies. The sensation of pitch depends on the frequency with which the impulses succeed one another at the ear, and this principle can be easily tested by using the siren or the toothed wheel.

It must be remembered, however, that frequency is an *objective* rate of vibration, whereas pitch is a *subjective* sensation by which a listener classifies a note as high or low, and although the two terms are often used interchangeably it is possible for two *pure* notes of slightly different frequencies to have practically the same pitch. This is due to what is called the *differential frequency sensitivity* of the ear, which varies with sound pressure levels and with frequency. In 1931 Shower and Biddulph showed that the sensitivity is greatest for frequencies above about 1,000 c.p.s., and becomes relatively poor at lower frequencies. For example, at a sound pressure level of 10 decibels, a 30-cycle note must be changed in frequency to about 32·7 c.p.s. before any change in pitch can be detected. This applies to pure notes, but in the case of musical instruments where harmonics are present, a greater frequency discrimination is possible.

Musical notes of quite a large range in frequencies have been produced. The lowest note of very large organs has a frequency of about 16·5, whereas a note of frequency of more than 40,000 has been obtained by a special kind of tuning fork ; but these frequencies are outside the ordinary limits of audibility, which lie between about 20 and 20,000. The frequencies of the notes used in music lie between about 30 and 5,000, and the lowest and highest notes given by a piano have frequencies of about 27 and 3,500 respectively.



Doppler's principle. The interesting phenomenon, known as the **Doppler effect**, bearing on the relation between pitch and frequency, is the apparent alteration in the pitch of a note due to the relative motion of the source and the observer. As the source, for example, the whistle of a railway engine, approaches the listener, the waves which are emitted get crowded up into a smaller space. Between the instant when one compression is sent out and the instant when it is followed by the next, the source has moved forward so that the two compressions are separated by less than their normal distance apart. Hence the compressions arrive at the ear in *quicker succession* than would be the case if source and observer were relatively at rest. If the source is receding from the listener the converse happens, for in this case the compressions arrive *less rapidly*. Thus, in the first case, the apparent pitch is higher than the true pitch, while in the latter case it is lower.

Source moving, listener stationary. Suppose the source of sound to have a frequency n, so that it emits n waves per second with a velocity V; further suppose the source to move with a velocity v *towards* the listener. If λ is the undisturbed wave-length, we have $n\lambda = V$, and when the source moves, the n waves will occupy a distance $(V - v)$. The disturbed wave-length will therefore be given by $\lambda_1 = \dfrac{V-v}{n}$. Hence the apparent frequency of the note is:

$$\frac{V}{\dfrac{V-v}{n}} = \frac{nV}{V-v}.$$

If the source moves *away* from the listener the apparent frequency is $\dfrac{nV}{V+v}$.

Source stationary, listener moving. Suppose the listener moves with velocity v towards a stationary source. In one second he will receive n waves + the number of waves in a length v; hence he will receive:

$$n+\frac{v}{\lambda}=n+\frac{v}{V/n}=n+\frac{nv}{V}=n\left(\frac{V+v}{V}\right) \text{ waves per second};$$

this is the apparent frequency.

Now consider a numerical example in which $V=1{,}100$ ft. per sec., $v=100$ ft. per sec. and $n=800$.

In the first case, when the source is moving,

$$\text{apparent frequency}=\frac{nV}{V-v}=\frac{800\times 1100}{1000}=880.$$

In the second case, when the listener is moving,

$$\text{apparent frequency}=n\left(\frac{V+v}{V}\right)=\frac{800\times 1200}{1100}=872\tfrac{8}{11}.$$

Hence the pitch of the note is not the same in both cases, although the relative velocities of source and listener are equal. To account for this, however, we may note that the wave-length of the sound when the source is moving is altered, but in the second case the wave-length remains unchanged.

Effect of wind. Suppose the source moves towards a stationary listener with velocity v as before, and suppose a wind is blowing in the same direction with velocity w. The waves emitted in one second will occupy a distance $V+w-v$. Hence the disturbed wave-length λ_1 will be given by

$$\lambda_1=\frac{V+w-v}{n},$$

that is, the wave-length is changed in the ratio

$$\frac{V+w-v}{V}.$$

Also the apparent frequency of the note to the listener will be

$$\frac{V}{\dfrac{V+w-v}{n}}=\frac{nV}{V+w-v}.$$

Now consider the motion of the listener, the source remaining stationary. If there were no wind and if the listener were stationary, n waves occupying a distance V would reach him in 1 second. But when the wind is blowing, and the listener moving with a velocity v_l, the waves occupying a distance $V+w-v_l$ reach the listener in 1 second. Therefore the apparent frequency is

$$\frac{nV}{V+w-v_l},$$

that is, it changes in the ratio

$$\frac{V+w-v_l}{V}.$$

Hence when both source *and* listener are moving, the *total* change in pitch, measured by the ratio of the apparent frequencies, is given by

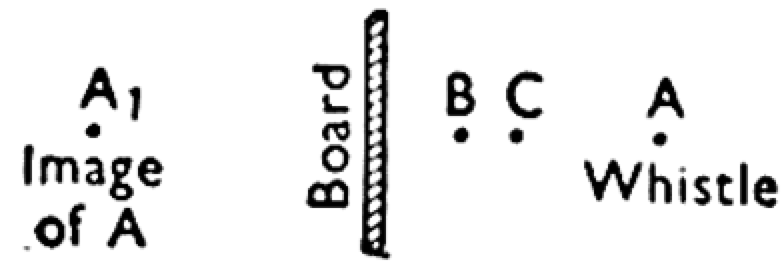
$$\frac{\dfrac{nV}{V+w-v}}{\dfrac{nV}{V+w-v_l}}=\frac{V+w-v_l}{V+w-v}.$$

If $v=v_l$, the above expression reduces to unity, in which case the wind does not affect the pitch of the note. Thus when both the source and the listener are stationary, or when they both have equal velocities in the same direction, the pitch is not affected by the wind.

Experimental determination of pitch by interference. The phenomenon of interference can be utilised to determine the frequency of a high-pitched sound. A Galton whistle, connected to a gas bag containing air so that it can produce a continuous high-pitched note, can be used as the source, and a sensitive flame as a detector. A drawing board is fixed in a vertical position on the bench and the sensitive flame is put near it. The Galton whistle is arranged in a suitable position A, say a yard or so from the board, and set in operation. The flame is moved slowly from the board until a position P is found when the flame ceases roaring and burns steadily. A second position Q is then found and the distance PQ measured ; this is the value of $\lambda/2$.

The sound from the source is reflected at the board, and the reflected and incident waves interfere to produce stationary waves, the nodes being the positions where the flame burns steadily.

Let B and C be two positions of the flame. If the distances AB and A_1B, where A_1 is the image of A, differ by a whole number of wave-lengths, the two waves assist one another and the flame will roar. But if the distances AC and A_1C differ by an odd number of half wave-lengths, the two waves destroy one another, and the flame burns steadily as it is at a node.



We have $A_1B - AB = n\lambda$, and $A_1C - AC = n\lambda + \lambda/2$;

$$\therefore\ (A_1C - A_1B) + (AB - AC) = \lambda/2;$$

whence
$$2BC = \lambda/2.$$

Hence in the experiment the distances BC, etc., are equal to $\lambda/4$. Instead of a Galton whistle, the valve oscillator mentioned on p. 54 can be used as the source.

Aeolian tones. The singing of telephone and telegraph wires and the whistling of the wind through trees are familiar sounds to everyone, and the sounds so produced are called **aeolian tones.** It has long been known that when a current of air strikes a stretched wire normal to its length, sounds are produced, the pitch of which is independent of the material, length and tension of the wire. In 1878 Strouhal investigated the effect by revolving a vertical, stretched wire about an axis parallel to its length, and he found that the frequency of the note is given approximately by the formula

$$n = \frac{0{\cdot}185v}{d},$$

when v is the relative velocity of the wire and air and d is the diameter of the wire. Lord Rayleigh afterwards showed that the vibrations of the wire are transverse to the direction of the wind.

Although no completely satisfactory theory has been given, the effect seems to be produced by the formation of unstable vortex sheets by the wind rushing past the wire, and the eddies form a mass vibrating from one side of the wire to the other. If the frequency of the sound so produced corresponds with the natural frequency of the wire the sound is greatly increased. In the **aeolian harp** a number of wires of the same low pitch, though of different thicknesses are stretched on a sounding board and exposed to the wind. The varying thicknesses of the strings result in a series of different notes.

Jet tones and Edge tones. Whenever a jet of air is forced through a slit, the jet becomes sinuous in character, and eddies are formed which appear alternately on opposite sides. The frequency of the note produced, the **jet tone**, is given by

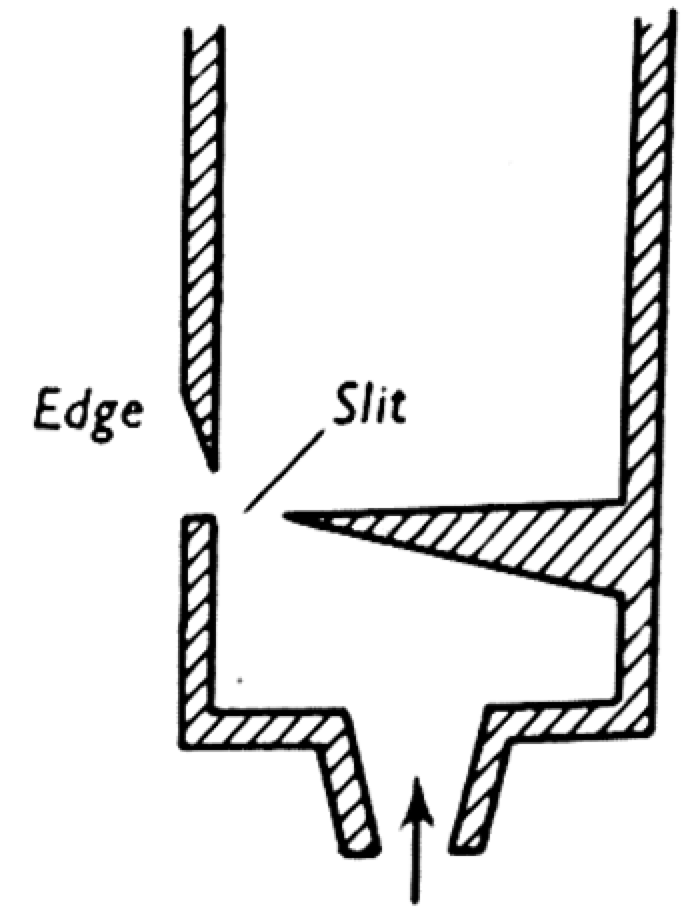
$$n = \frac{0{\cdot}055v}{d},$$

where v is the velocity of the air at its efflux and d is the width of the slit. If this is compared with Strouhal's formula for aeolian tones, it will be seen that the pitch of the jet tone is about two octaves below that of the aeolian tone due to a cylindrical object of the same diameter.

Jet tones are very feeble and unstable, but if a wedge of small angle is presented to the jet, the edge being parallel to the slit, stronger and more stable tones are produced. These tones, which are called **edge tones**, were first noticed by Sondhauss in 1854, and they certainly show the importance of vortex motion in the production of sounds.

A searching investigation into the nature of jet and edge tones has been carried out by G. Burniston Brown, and by photographing jets of air impregnated with tobacco smoke issuing from orifices of various types, he has obtained some instructive information concerning the growth of the vortices and their rate of formation, etc.; this, however, cannot be dealt with here.

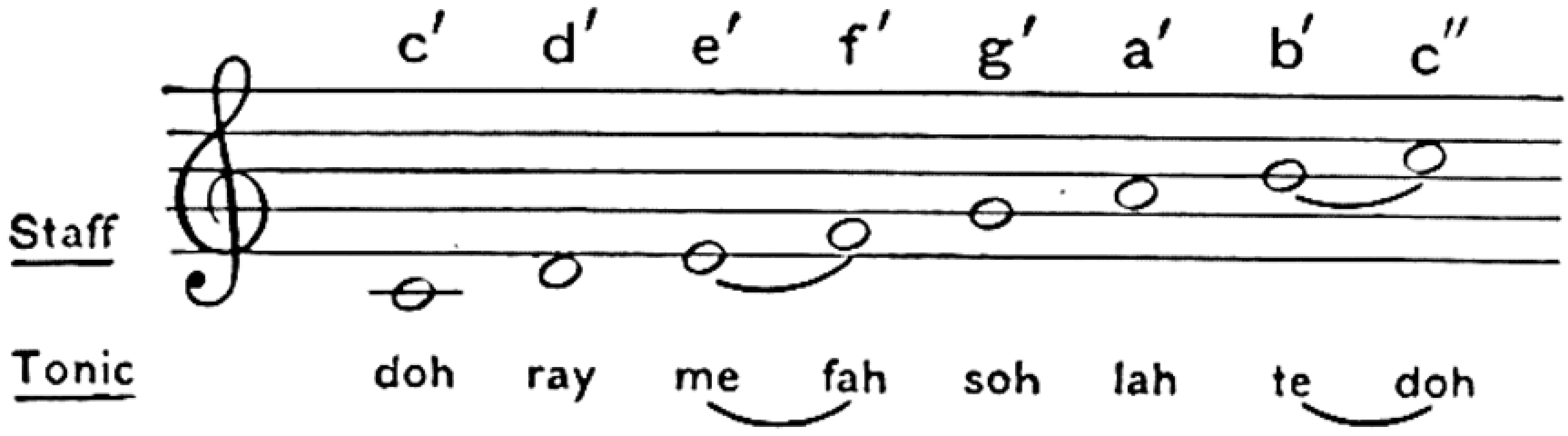
Edge tones play quite an important part in the production of sound in organ pipes. Air from the slit passes across the mouth and strikes the wedge-shaped edge, thus giving rise to an edge tone. In practice, the wind pressure is adjusted so that the frequency of this tone is the same as the fundamental of the pipe; this operation is called "voicing". If the edge tone and one of the free vibrations of the air column are in resonance, energy is rapidly absorbed by the column and the pipe "speaks" promptly. If, however, the two are out of tune, energy is



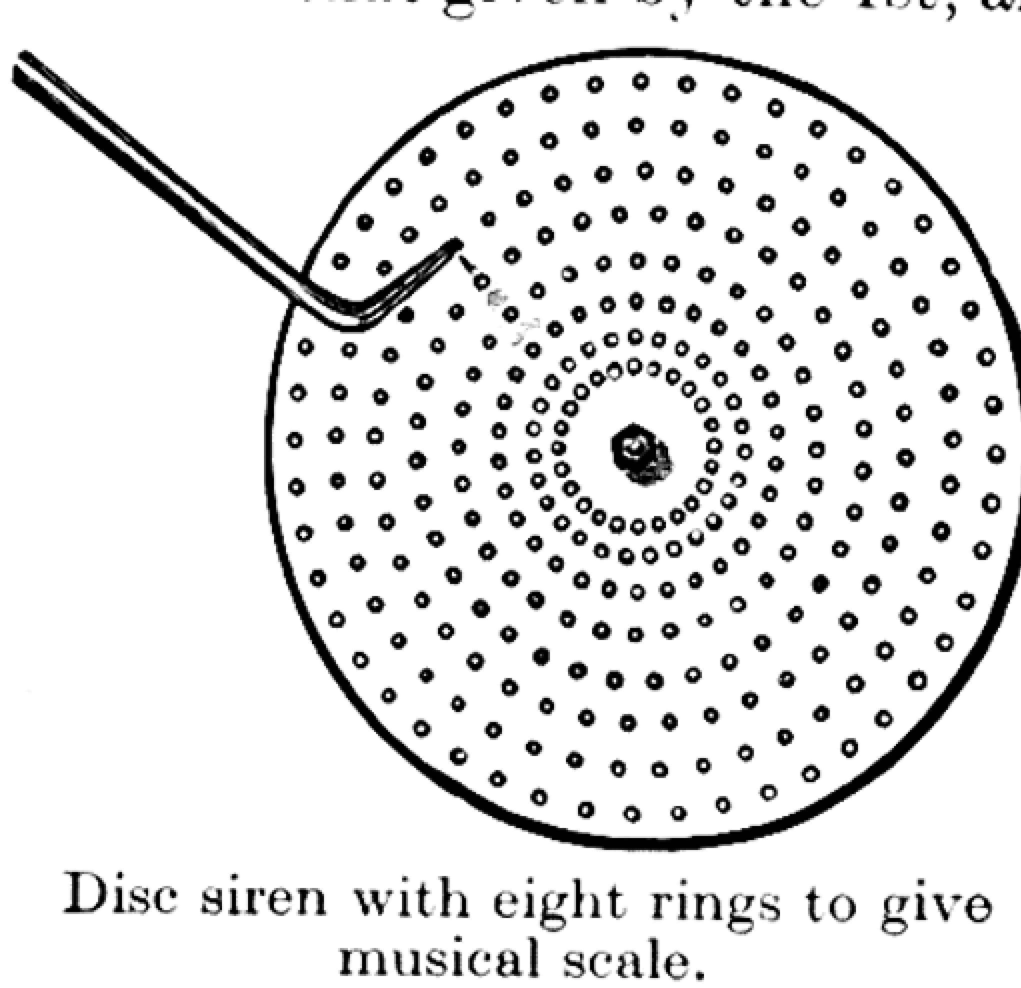
absorbed slowly and the pipe gives a slow response. The frequency of the edge tone can be adjusted for resonance by altering the distance of the edge from the jet.

The edge tone and the air in the pipe constitute what is known as a **coupled system**, formed of two components which can mutually interfere, and so prevent either from vibrating freely. When the inteference is slight, the system is *loosely* coupled. In the case of an organ pipe, there is a *closely* coupled system, for the dominant partner is the comparatively massive air column and the feeble partner is the edge tone.

Musical scale and intervals. The simplest musical scale is the **diatonic** scale which consists of two groups of four notes, each containing three tones and one semitone, and it can be written down in the staff notation or in the tonic notation as indicated below.



The *ratio* of the frequencies of any two notes in the musical scale is called a **musical interval**, and to investigate the relation between two notes constituting a simple musical interval a modified form of siren, in which there is a rotating disc with concentric rings of small holes, can be used. A cardboard disc mounted on an electric motor has rings in which the number of holes may be in the ratio 4, 5, 6, 8 from the innermost ring ; a jet of air playing on the disc can be directed on to any ring of holes at will. The note given by the 4th ring is an octave above that given by the 1st, an interval which can be easily recognized by most people, and the ratio of the two frequencies is 2 : 1. This relationship remains true for all speeds of rotation, for although the respective frequencies will alter and the *difference* between the two frequencies will either decrease or increase, yet the interval always remains an octave. Hence a musical interval is determined by the *ratio* of the



Disc siren with eight rings to give musical scale.

frequencies of the two notes and not by their difference. Further confirmation of this may be obtained by using different rings of holes.

The various musical intervals are named—or numbered—by counting up the scale from one note to another and including both notes. Thus *c* to *d* is a second, *c* to *e* a third, and so on. The interval from *e* to *g* is also a third, but when we compare the frequencies we find that *e* to *g* is a smaller interval than *c* to *e*. It is therefore called a minor third, while the interval *c* to *e* is called a major third.

The chief intervals with their frequency ratios are given in the accompanying table.

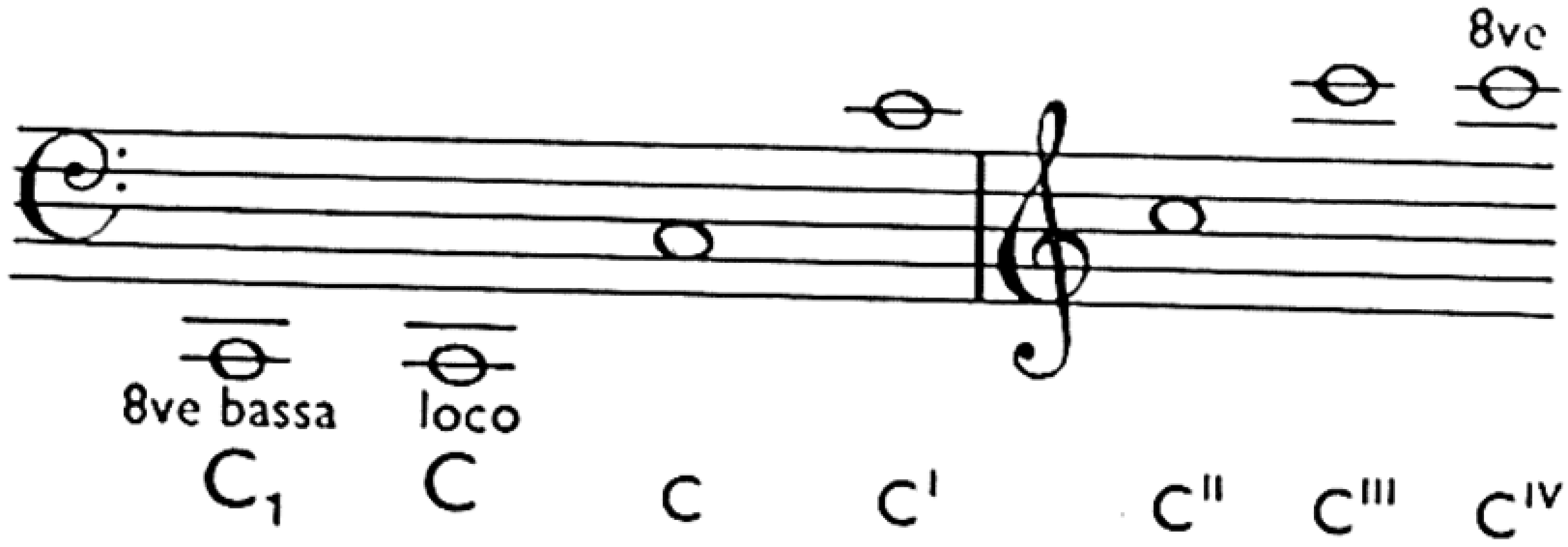
Interval	Ratio of Frequencies	Notes within a single octave giving the interval
Octave	2 : 1	*c* to *c′*
Major Sixth	5 : 3	*c* to *a* *d* to *b*
Minor Sixth	8 : 5	*e* to *c′*
Fifth	3 : 2	*c* to *g* *e* to *b* *f* to *c′*
Fourth	4 : 3	*c* to *f* *d* to *g* *e* to *a* *g* to *c′*
Major Third	5 : 4	*c* to *e* *f* to *a* *g* to *b*
Minor Third	6 : 5	*e* to *g* *a* to *c′*

Frequency ratios. The relative frequencies of the notes comprising an octave in the diatonic scale are as follows :

$$\begin{array}{cccccccc} c & d & e & f & g & a & b & c', \\ 1 & \frac{9}{8} & \frac{5}{4} & \frac{4}{3} & \frac{3}{2} & \frac{5}{3} & \frac{15}{8} & 2 \end{array}$$

or, clearing of fractions, we have

$$24 \quad 27 \quad 30 \quad 32 \quad 36 \quad 40 \quad 45 \quad 48.$$



(We are adopting the Helmholtz notation as above for the names of the notes, where middle *C* on the piano is represented by *c'*.)

To obtain the intervals between successive notes of the scale we divide the number representing each note by the number representing the one immediately below. Thus the interval from *c* to *d* is given by $\frac{27}{24}$ or $\frac{9}{8}$, and so on. Hence we obtain the following ratios for the intervals between successive notes :

$$c \underbrace{\qquad}_{\frac{9}{8}} d \underbrace{\qquad}_{\frac{10}{9}} e \underbrace{\qquad}_{\frac{16}{15}} f \underbrace{\qquad}_{\frac{9}{8}} g \underbrace{\qquad}_{\frac{10}{9}} a \underbrace{\qquad}_{\frac{9}{8}} b \underbrace{\qquad}_{\frac{16}{15}} c'$$

Notice that these intervals are of three different sizes. The largest ratio $\frac{9}{8}$ is called a large or **major tone**, the next ratio $\frac{10}{9}$ is a small or **minor tone**, while the ratio $\frac{16}{15}$ is a **semitone.**

It will be seen from the table opposite that the interval *c* to *f* is a fourth and the interval *f* to *c'* is a fifth ; if these two are added together they give the octave *c* to *c'*. But to obtain this result in terms of frequency-ratios we have to *multiply* the corresponding ratios. Thus, to obtain the interval when we add a fifth to a fourth, we multiply $\frac{3}{2}$ by $\frac{4}{3}$, which gives 2, representing the octave. To simplify the method of measuring intervals the whole octave is divided into 1200 units called **cents**, the cent being used because 100 cents make the semitone of those instruments in which 12 equal semitones are the intervals occurring in one octave. In terms of cents, we get the following values for the various intervals :

Octave	1200 cents	Major Third	386 cents
Major Sixth	884 ,,	Minor Third	316 ,,
Minor Sixth	814 ,,	Major Tone	204 ,,
Fifth	702 ,,	Minor Tone	182 ,,
Fourth	498 ,,	Semitone	112 ,,

Hence we can compare the different intervals by simply adding and subtracting. For example,

a fifth plus a fourth $= 702 + 498 = 1200$ cents $=$ one octave.

It must be observed that the state for which the above calculations hold is an *ideal* one ; this will be referred to again when we discuss temperament.

Standards of pitch. Widely different pitches have been adopted for the musical scales at various times and places and for distinct musical uses. For scientific purposes the standard of pitch generally accepted is a frequency of 256 for c' and 512 for c'', thus giving a frequency of 426·7 for a' ; but for musical purposes the Philharmonic Society fixed, in 1896, a frequency of 439 at a temperature 68° F. for a', and this is now an international standard and is known as the " low pitch ". Previous to 1896 the frequency adopted for a' was 452·4, giving a frequency of 526·8 for c'' ; this was the " high pitch ".

Since the pitch of a note generally alters with temperature, it is of course necessary to define a temperature when adopting a standard.

LOUDNESS

It has already been stated that loudness of a sound corresponds to a degree of sensation produced in the ear, whereas intensity refers to a definite physical quantity which is determined by the rate of supply of vibrational energy. Loudness must, of course, depend on intensity, but it also depends on the sensitiveness of the ear under particular conditions ; for example, near the limits of audibility the loudness of a sound may be very feeble but the intensity quite large. The relation between sensation (loudness) and stimulus (intensity) is given by **Weber's law : The increase of stimulus necessary to produce a just perceptible increase of sensation is proportional to the pre-existing stimulus.**

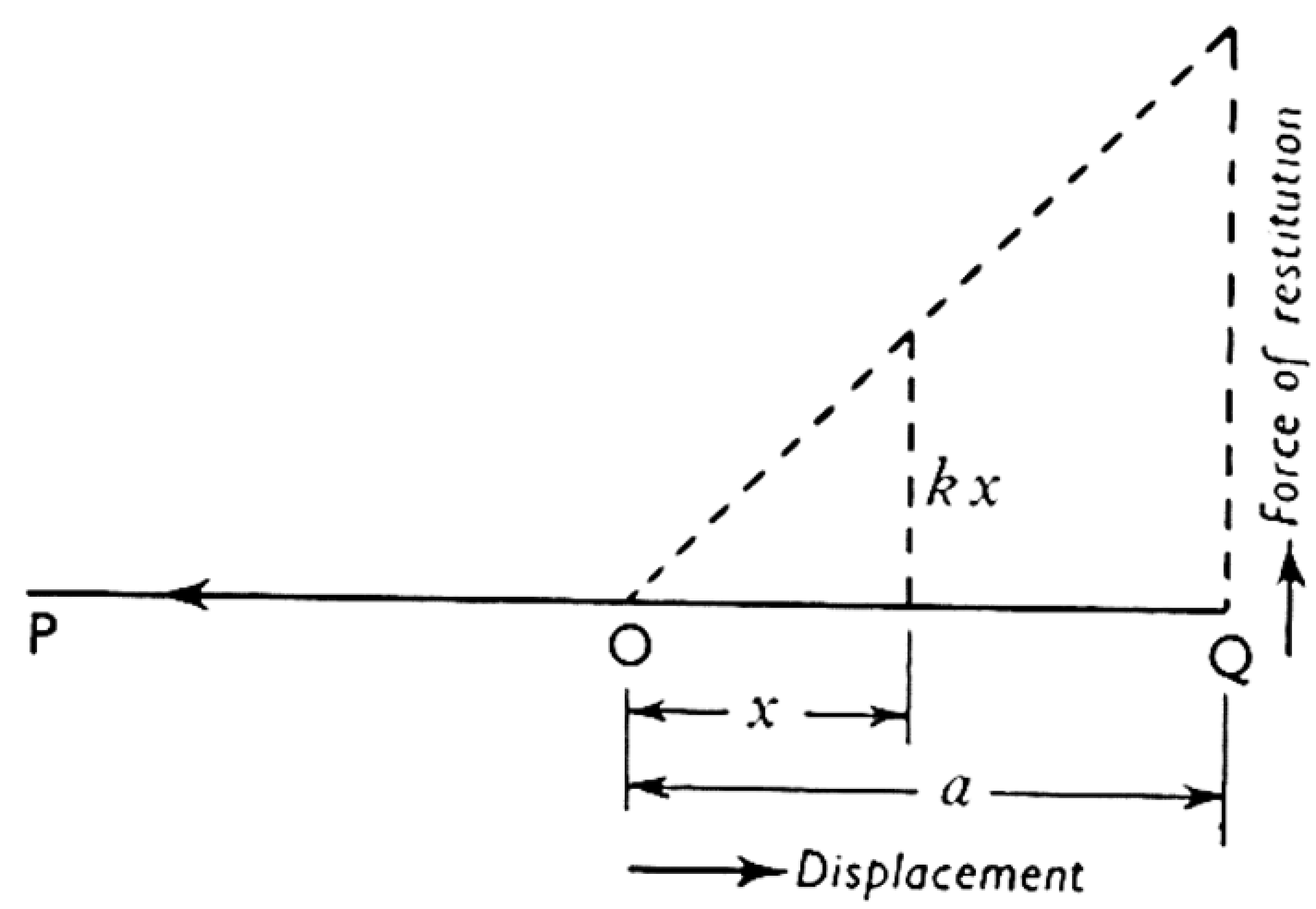
Thus, if the intensity of a sound is progressively doubled, the loudness is not increased in the same ratio ; it increases by equal additions. In fact, the loudness is proportional to the logarithm of the intensity.

The measurement of the loudness of a noise has been referred to in Chapter I and it will be further discussed in Chapter XIV. What we are chiefly concerned with here is the intensity of a sound, and to consider the factors on which this depends.

There is no doubt that energy in one form or another must be supplied before an object emits a sound. The energy supplied is usually in the form of mechanical energy, but vibrations may be caused by other forms. For example, heat can produce vibrations, as is evidenced by the apparatus known as Trevelyan's rocker, and also by " singing " flames, while, as we have seen, ultrasonic

vibrations may be caused by electromagnetic energy. It is to be expected that the greater the amount of energy initially used the greater will be the intensity of the sound produced ; also, we know that the more energy we put into, say, the plucking of a string, the greater is the amplitude of the vibration. Hence we shall expect that the intensity will depend on the amplitude.

Energy of a vibrating object. Suppose an object of mass m is vibrating with simple harmonic motion between the two extreme positions P and Q. At these points the velocity of the mass will be zero, but it will have a maximum velocity, say v, at O the middle point, and the kinetic energy at this point is $\frac{1}{2}mv^2$. Now,

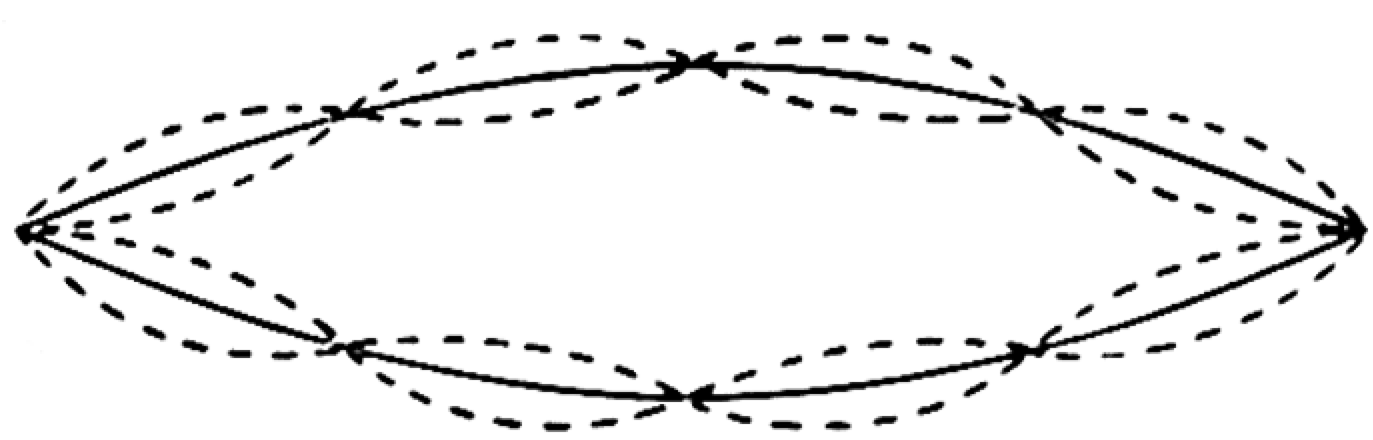


when the mass is displaced from O, it is acted upon by the force of restitution which, for any given displacement x, is proportional to x, and the work done by this force is $kx^2/2$ (see force-displacement diagram), where k is a constant. When the mass is at P or Q, the displacement is a maximum and is equal to the amplitude a of the vibration. Hence we have $\frac{1}{2}mv^2 = \frac{1}{2}ka^2$, and as k is a constant for any given value of displacement, the energy of the vibrating system is directly proportional to the square of the amplitude. As intensity has been defined as a rate of flow of energy, it follows that **the intensity of the sound emitted by a source is directly proportional to the square of the amplitude.** Also, since the time T of one complete vibration is $\frac{2\pi}{\omega} = \frac{1}{n}$, and since $\omega = v/a$, we have $v = 2\pi an$.

Hence the kinetic energy $= \frac{1}{2}mv^2 = 2\pi^2 ma^2 n^2$. Therefore, **the intensity of the sound is also proportional to the square of the frequency.**

QUALITY

The quality of a musical note is the property which enables us to distinguish between two notes of the same pitch and the same loudness when produced by different kinds of instruments or even by two different voices, and we shall find that this depends on the nature of the vibration giving rise to the note. We know from the study of light that the resultant colour of, say, an opaque object, is due not only to the number of the constituent colours but also to the relative proportions in which these constituents exist. We have also seen that most musical notes are not pure notes as in the tuning fork, but consist of the fundamental accompanied by various overtones, and it is the resultant sound of this mixture which determines the quality. It is on account of this similarity with light that we might use the term **sound-colour** as an alternative to quality.



Representation of simultaneous vibrations in a string.

There is strong evidence for the belief that sounding objects such as stretched strings are not only capable of being made to vibrate successively in various modes giving a series of different notes, but also that they give *simultaneously* a number of different notes which are simple and unanalysable and which we shall call *tones*.

Let a sonometer wire be plucked somewhere near one end, and while it is sounding let its middle point be lightly damped with a feather. This will suppress the fundamental node of vibration, which of course requires an antinode at this point, and the prime tone will disappear. But we shall now hear the first overtone or harmonic, a tone an octave above the fundamental. Now merely damping the centre of the wire could not bring into existence a mode of vibration not previously present, and we are forced to conclude that this mode of vibration was already in existence together with the fundamental mode. The overtone could not be heard as a distinct tone while the fundamental was sounding because of the prominence of the latter tone ; though,

now we know that it does coexist with the fundamental, if we again pluck the string we shall probably be able to pick it out of the resulting note. In a similar way, it can be demonstrated that the fundamental tone is also accompanied by other and higher overtones. Hence, it is concluded that the resultant note obtained by plucking the string is a complex one made up of the fundamental, which gives the pitch of the note, and various overtones all sounding together; it is this complexity which gives rise to the distinct quality of the note. If the string is plucked at the centre instead of near one end, there is a marked difference in quality, for whereas previously the quality was harsh and brilliant owing to the richness in overtones, now the quality is dull and nasal. In this case all the even overtones must be absent, for these modes of vibration need a *node* at the centre.

In a piano the quality depends mainly on the point on the strings which the hammers are made to strike, on the hardness of the hammers, on their time of contact with the strings, and on the sounding board. The point at which the strings are struck is usually selected so as to make the seventh and higher overtones rather weak, but to allow the lower overtones to be present. If the hammers are sharp and hard, the displacement of the strings is more abrupt and the upper overtones become more prominent; thus the quality is brilliant and perhaps harsh. Soft flat hammers tend to dulness of quality; hence hammers are usually faced with compressed felt, so that if the quality is too brilliant it can be softened down to any extent by teasing out the felt.

In a violin the bow is usually applied to a point of the string about one-twelfth of the length from the bridge, though the skilled violinist has great control over the quality, since he can determine the overtones which shall accompany the fundamental by altering the point on the string at which the bow is applied. The quality of the note given by a violin also seems to depend upon the wood chosen for the body, its shape and even the varnish used.

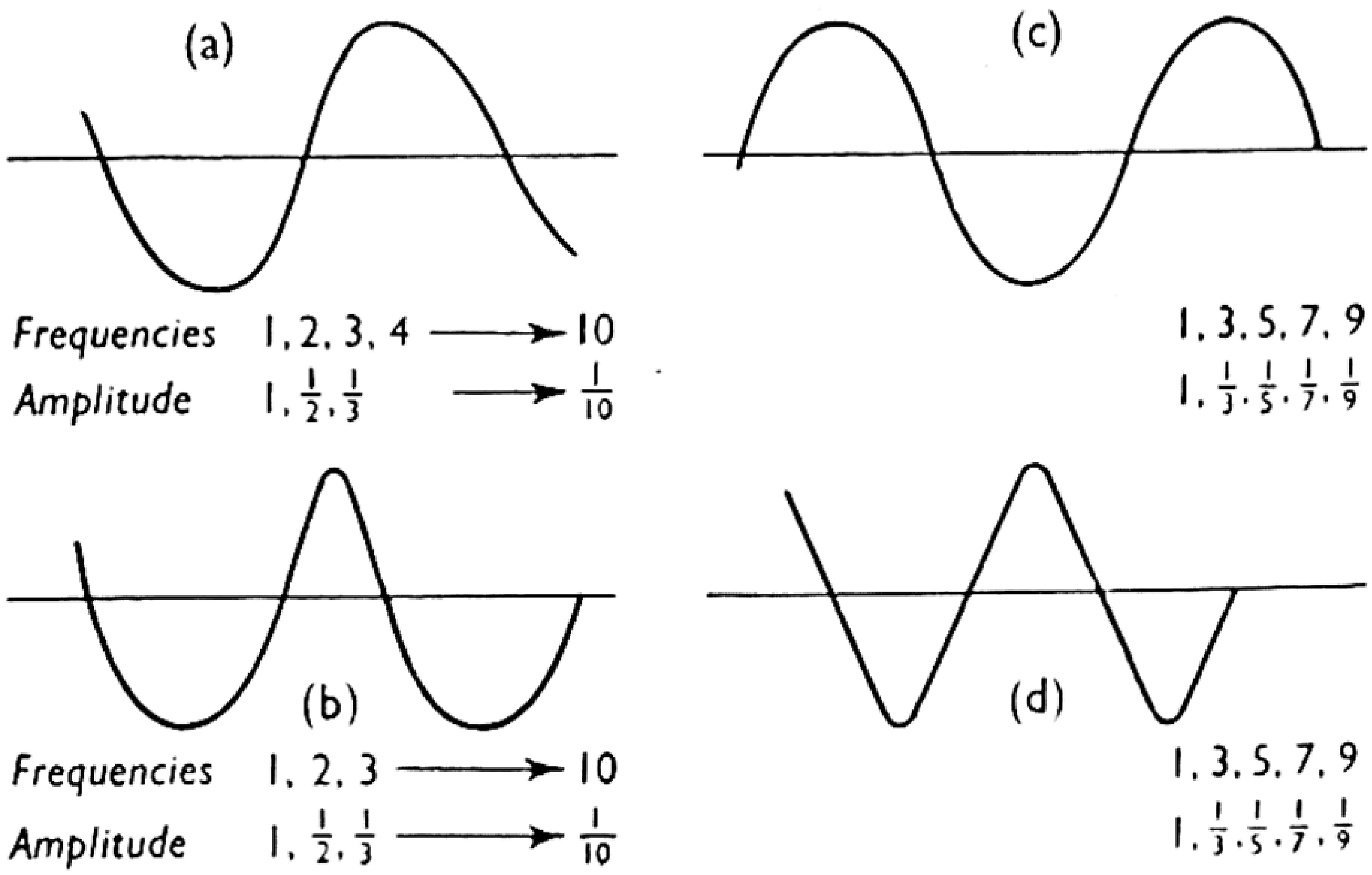
So far as instruments in which the string is excited by plucking are concerned, if a broad, soft plucking device such as the finger is used, the note is sweet and soft as in the harp; on the other hand, the ivory plectrum as used in a mandoline gives a harsh metallic quality.

In the case of brass instruments, such as trumpets, trombones, etc., the sound acquires great penetrating force due to the strength of the higher overtones.

Summing up the discussion so far on the quality of a musical

note, it may be said that the quality depends on the number of overtones present with the fundamental, and on the relative intensities of the overtones; also, as we have seen, in certain musical instruments the quality is influenced by other factors. In addition to the above, however, there is another factor which must be considered, namely, the possible change in phase relations between the various tones which make up the complex note. There can be no doubt that the character of the displacement diagram really determines the quality of a musical note; and if the resultant displacement curve is different for two sounds, then

No Difference in Phase



Difference of Phase 90°

the quality is different (see oscillograms on p. 228). Koenig performed the synthesis of musical notes experimentally by compounding the sine curves corresponding to the different overtones with their proper relative amplitudes and altering the relative phases between the overtones. The curves obtained are shown above. In (*a*) and (*c*) the phases are all the same, but the number and the amplitudes of the overtones are different. In (*b*) and (*d*), however, although the curves are similar to (*a*) and (*c*) respectively, so far as number and amplitude are concerned, the phase of the first tone is 90° behind that of the fundamental, the second overtone is 90° behind the first and so on. It will be noticed that the curves are very different from (*a*) and (*c*). In this way, Koenig concluded that the relative phases of the component tones has an influence on the quality of the note.

Effect of transients on quality. One other factor which helps to determine the quality of a note must also be mentioned, for studies by Fletcher and others have revealed the importance of the *transient period of vibration*, that short time during which the sound is being built up and is dying down (see also p. 236). It has long been noticed that when a *sustained* note is played on a violin or a 'cello, the ear tends to confuse the two instruments; but no such confusion arises when the instruments are played normally. This is due to the fact that when the notes are sustained, the initial transient notes have had time to die out.

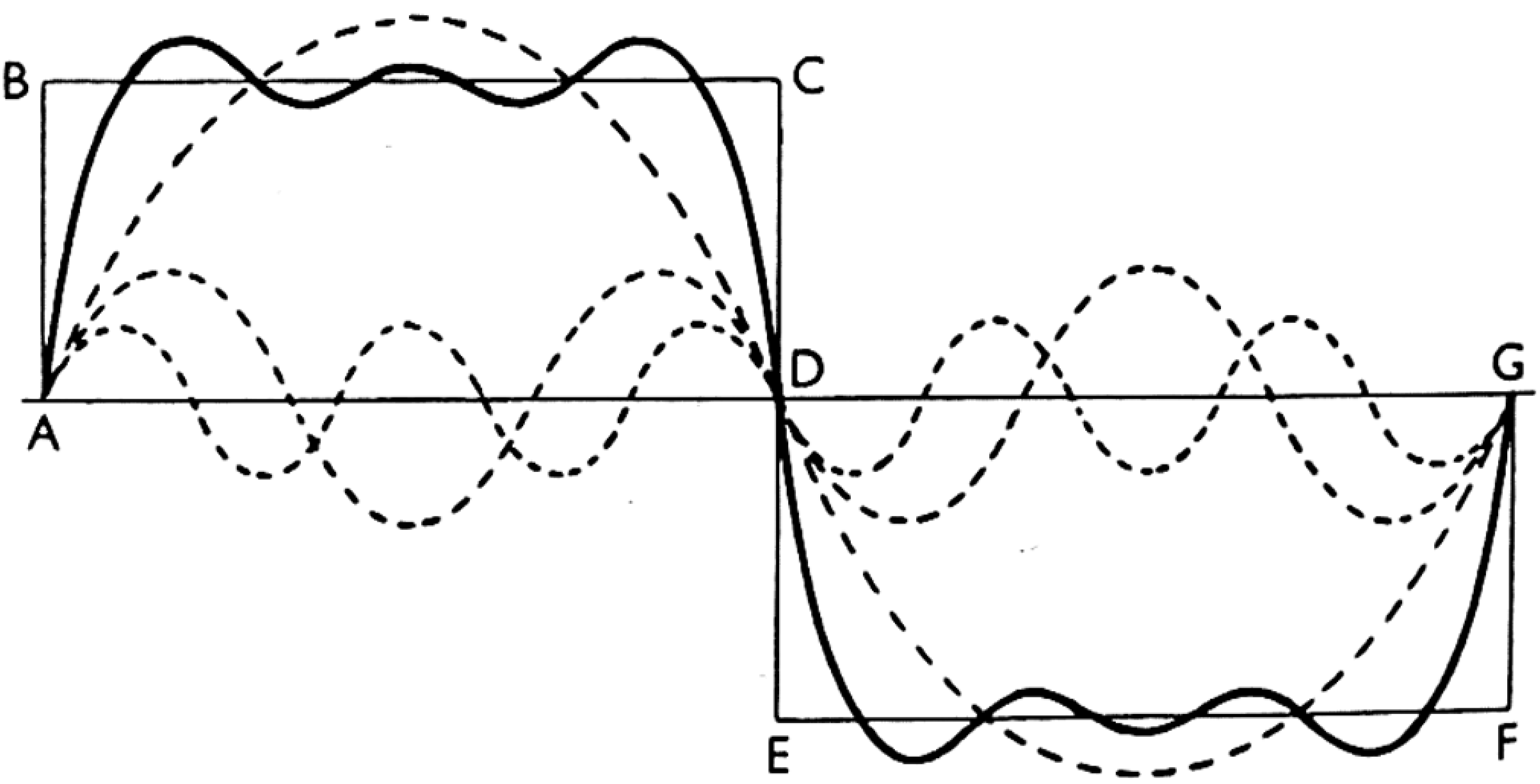
The characteristic sound of a drum is due entirely to a transient, for the motion of the drumhead is highly damped and the applied force is of short duration.

ANALYSIS AND SYNTHESIS OF MUSICAL SOUNDS

Fourier's theorem. Any form of single-valued periodic vibration can be analysed or synthesised, and we have already considered how two simple periodic curves can be compounded to form a resultant curve (beats, p. 86). The same method can be applied to compound any number of harmonic curves, and very often it will be found that the resultant periodic curve is of a type much different from the simple sine curves of the individual components. Such resultant curves, representing the various sounds, abound in music and speech. In 1822 Fourier, a celebrated French mathematician, showed mathematically that "any single-valued periodic function can be expressed as a summation of a simple harmonic series having frequencies which are multiples of that of the given function". This statement is known as Fourier's theorem, and by use of it a complex sound can be analysed into its various components. Conversely, a periodic curve can be built up by compounding together a finite number of harmonic curves, the periods of which are commensurate; and if T is the period of the complex periodic motion, the periods of the component simple harmonic motions will be included in the numbers T, $T/2$, $T/3$, etc.

The mathematical analysis or synthesis of complex wave-forms may become very laborious, and machines called *harmonic analysers* have been devised to simplify the process; in these the necessary mathematical calculations are performed by a direct mechanical process.

As an example, suppose the required resultant curve to be re-



presented by the lines *ABC–FG* and consider the compounding of three simple harmonic curves of which the frequencies are in the ratio $1:3:5$, while the amplitudes are as $1:\frac{1}{3}:\frac{1}{5}$. The resultant of these three simple curves is represented in the diagram by the thick line, which is approaching the form of the required curve. If more simple curves are compounded, the more nearly will the required curve be reproduced.

Analysing power of the ear. When a musical note represented by a complex periodic motion is sounded, the ear is able to pick out the fundamental tone and assign a definite pitch to the note, and it is also able to hear separately each of the components. It is thus a practical harmonic analyser, for it analyses the complex vibration into a series of simple harmonic vibrations (the members of the Fourier series) each corresponding to a simple tone. Again, when listening to a vocal or instrumental quartette, the ear can concentrate on any one part and follow that, in addition to appreciating the general effect. Its power of analysing complex sounds thus far surpasses any power of analysis possessed by the eye.

To account for the action of the ear in this and other respects, Helmholtz propounded a theory of audition. According to this theory, the different threads (Corti's fibres) of the basilar membrane, which is found in the cochlea, act as vibrators, and these are tuned to frequencies within the limits of hearing. Each vibration is able to excite its appropriate nerve fibre or fibres, so that a nerve impulse, corresponding to the frequency of the vibrator, is transmitted to the brain. The mass of each vibrator is such that it will be easily set in motion, and after the stimulus has ceased, it will readily come to rest on account of damping.

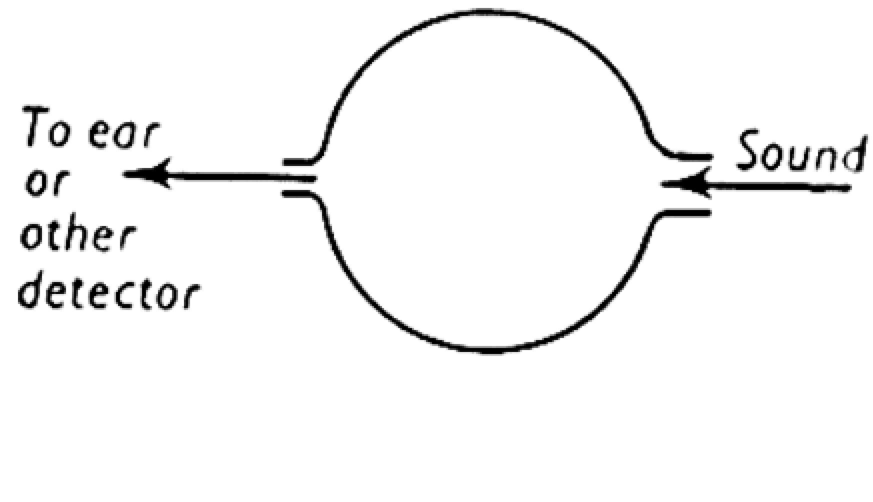
If a simple tone falls on the ear, then only one, or at any rate a very small proportion, of the vibrators is affected, and the brain is not able to resolve the sound into any simple sensation; this will account for discrimination in pitch. If the sound is a complex one, it will be resolved into its constituents by the vibrators corresponding to the tones existing in it, each picking out its own portion of the wave and thus stimulating the corresponding nerve fibres. The nervous impulses are transmitted to the brain, where they are fused to give rise to a sensation of a particular quality, but fused so imperfectly, that each constituent may be separately recognised.

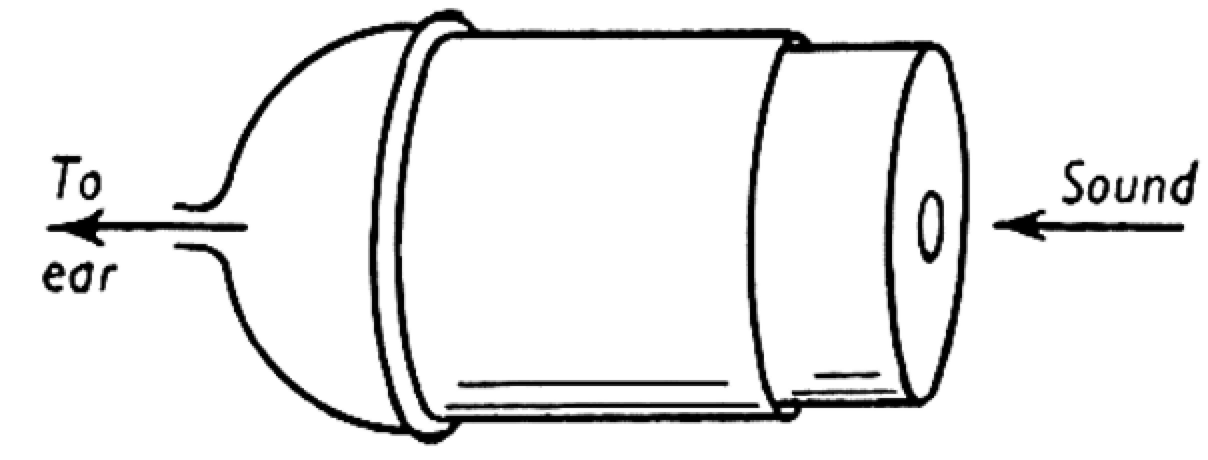
When two tones of nearly the same frequency are sounded simultaneously, some of the vibrators will respond to both tones, but the vibrations will be intermittent in character on account of the production of beats. If the beats are slow, the vibrators have sufficient time to come to rest in between the successive maxima of sound. If, however, the beats are too rapid, there may not be sufficient time for the nerve fibres to recover completely between the stimuli, and the effect will be noticed as a roughness or discord (see Chapter VII).

The resonance theory of Helmholtz as described above has not been universally accepted as the true explanation, and a rival theory known as the *telephonic* theory has been postulated. In this it has been suggested that the basilar membrane vibrates as a whole, and that the brain distinguishes between notes of different pitch because the actual nerve currents have the same frequency as the sound.

Other theories have also been advanced and it appears that a satisfactory interpretation of audition has not yet been found. Probably a thorough biophysical and biochemical study of the auditory nerve process will be necessary before a satisfying explanation can be found.

Helmholtz resonator. In order to show the existence of overtones in notes produced by stringed and other musical instruments, Helmholtz used a series of resonators. These are small hollow air chambers of the shape shown in the diagram, with two apertures, one of which can be connected to the ear or other detector, while the other is directed towards the source of sound. The shape and size of the resonators can be adjusted so that the contained air will resound to a tone of definite pitch. If the note



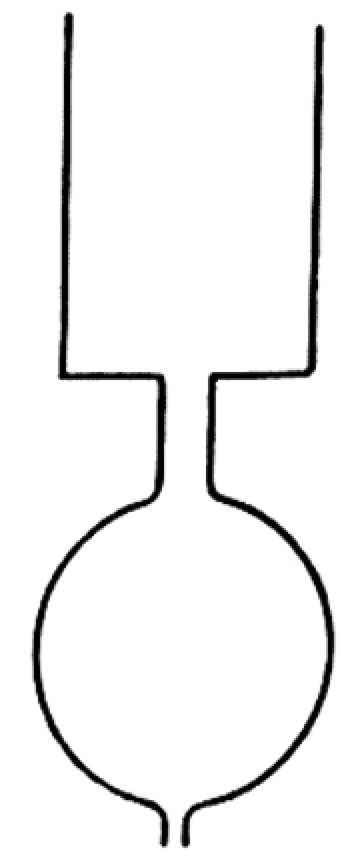


Koenig resonator.

to be analysed contains an overtone which corresponds in pitch to the natural pitch of the resonator, then this overtone will cause the resonator to "speak". Such a resonator does not radiate much energy and it does not resound very loudly; consequently, the vibration of the air in it persists for a considerable time. Thus it is lightly damped and is highly selective.

For some purposes, two Helmholtz resonators are coupled together to form a compound resonator, and it is interesting to note that the natural frequencies of the coupled resonators is *not* the same as those of the two regarded separately. A special type of compound resonator is the Boys type. In this a long open cylindrical tube is coupled at one end with a Helmholtz resonator through a short neck. E. T. Paris has shown that the natural frequencies (n) of the combination are determined by the following equation:

$$\tan\frac{\pi n}{2n_1} = -\frac{2\pi a n}{V c_0}\left(1 - \frac{n_2^2}{n^2}\right),$$

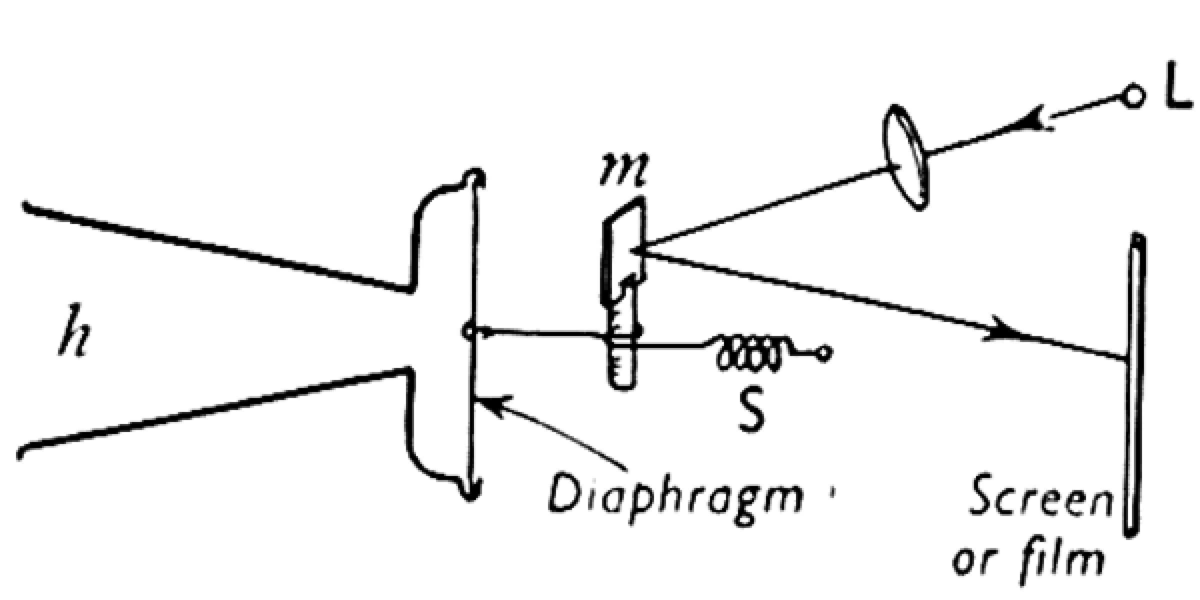


Boys resonator.

where n_1 and n_2 are the fundamental frequencies of the tube and the Helmholtz resonator respectively, a is the cross-sectional area of the tube, V is the velocity of sound in air and c_0 is the conductivity of the neck.

Koenig also devised a form of resonator in which the volume of the enclosed air could be altered, and so the pitch of the note.

Miller's phonodeik. Many attempts have been made to record wave-forms of sound by means of diaphragms and stretched membranes, and the movements of the vibrating diaphragm have been recorded in various ways, mechanically, optically and electrically.



An example of the optical method is Miller's phonodeik. In this apparatus the sound is collected by the horn h, at the

narrow end of which is a diaphragm of glass about 3/1,000 in. thick held lightly between soft rubber rings. To the middle of the diaphragm a few silk fibres are attached, which, after passing once round a tiny pulley, finish at the spring *s*. The pulley is on a spindle carrying a mirror, *m*, 1 mm. square, the whole mass being less than 1/500 gm. Light from a lamp *L* passes to the mirror and is brought to a focus by means of a lens on to a photographic film. Thus a displacement-time graph is obtained.

The apparatus can be arranged to give a visual picture of the sound by using a rotating mirror and screen instead of the film.

Mechanical recording is exemplified in the gramophone, while electrical methods involve the use of a microphone or similar device which responds to the vibrations of the diaphragm. The corresponding electrical oscillations are recorded by using some form of oscillograph, for example, a cathode ray tube.

It must be noted that the majority of diaphragm receivers are subject to resonance at certain frequencies, and this gives rise to a distorted record. Hence great care is necessary in the choice of a receiver and recorder for a particular type of sound, otherwise a number of factors may contribute to distortion. As was mentioned in Chapter I, one of the best non-resonant receivers is the condenser microphone of Wente, while the best form of recording system is a combination of a piezo-electric crystal receiver with a cathode ray oscillograph.

Synthesis of a musical note. In addition to analysing compound notes into the component tones, Helmholtz also performed the inverse operation, namely, building up a note of a given quality by the combination of a number of simple tones. We have also seen (p. 128) that Koenig performed a similar piece of work. The apparatus used by Helmholtz consisted of ten tuning forks which were tuned to give a fundamental of frequency 256 and the first nine harmonics. Each of the forks is arranged in front of a resonator tuned to unison with it. An eleventh fork is kept vibrating electrically, and is so arranged that it makes and breaks an electric circuit once in each vibration. Each of the other forks is provided with an electromagnet, through which the intermittent current produced by the extra fork is sent. The result is that the first fork is acted upon by a periodic force which recurs regularly after 1, 2, 3, etc. complete vibrations, and thus keeps the fork vibrating. Fork number 2, in the same way, receives an impulse every other vibration, and so on.

Each of the resonators is fitted with a clapper worked by a string attached to a key-board, by means of which the mouth of

the resonator can be closed, and when the resonator is closed the sound of its corresponding fork is negligible. The resultant note heard corresponds to the note produced by the coexistence of the tones given out by the forks whose resonators are uncovered. The intensity of each tone can be regulated by varying the opening of the resonator.

A more modern method of building up a note from the various components is quoted by Dr. P. C. Buck in his *Acoustics for Musicians*. He states: " an ingenious instrument has been built by Mr. Rothwell, the well-known organ builder, on which some experiments can be made with partial tones. A prime note (low *G*) is produced by a soft bourdon pipe, and its partials, up to the twenty-fifth, can be sounded in any combination desired (by wedging down the notes) on soft dulciana pipes. When the five lowest partials are sounded the result is simply a soft and pleasant chord of *G* major; but as other partials are added the sound of the chord gradually vanishes, whilst the prime note advances into the foreground with ever increasing volume. When all twenty-five are sounding together—each, be it remembered, quite soft by itself—the result is one enormous low *G* of the unmistakable quality of a trombone ".

GENERAL DISCUSSION OF COMBINATION OF VIBRATIONS

When a single particle is acted upon by a number of distinct forces, each of which would cause it to perform simple harmonic motion, the question arises as to the resultant motion, and there are several important cases which should be considered.

If two vibrations of a particle take place in the same direction, the resultant displacement is the sum of the separate displacements; but it must be borne in mind that the resultant displacement curve will be influenced by the frequencies, the phase difference and the amplitudes of the individual vibrations. A system of vibrations of the same frequency and in the same straight line, but where the phases and amplitudes are different, can always be reduced to a single resultant by means of a vector polygon, the lines representing the amplitudes, the angles and the relative phases, in exactly the same way as a system of forces acting at a point.

If two vibrations of *nearly* the same frequency, and the same amplitude are acting in the same direction, we obtain the phenomenon of beats which was discussed earlier.

Vibrations at right angles. Consider the case when two vibrations of the same frequency but different amplitudes and phases act on a particle. Let one of the vibrations be in the line Ox and let it be represented by the equation $x = a_1 \sin \omega t$. The other vibration will be in the direction Oy, and its equation is

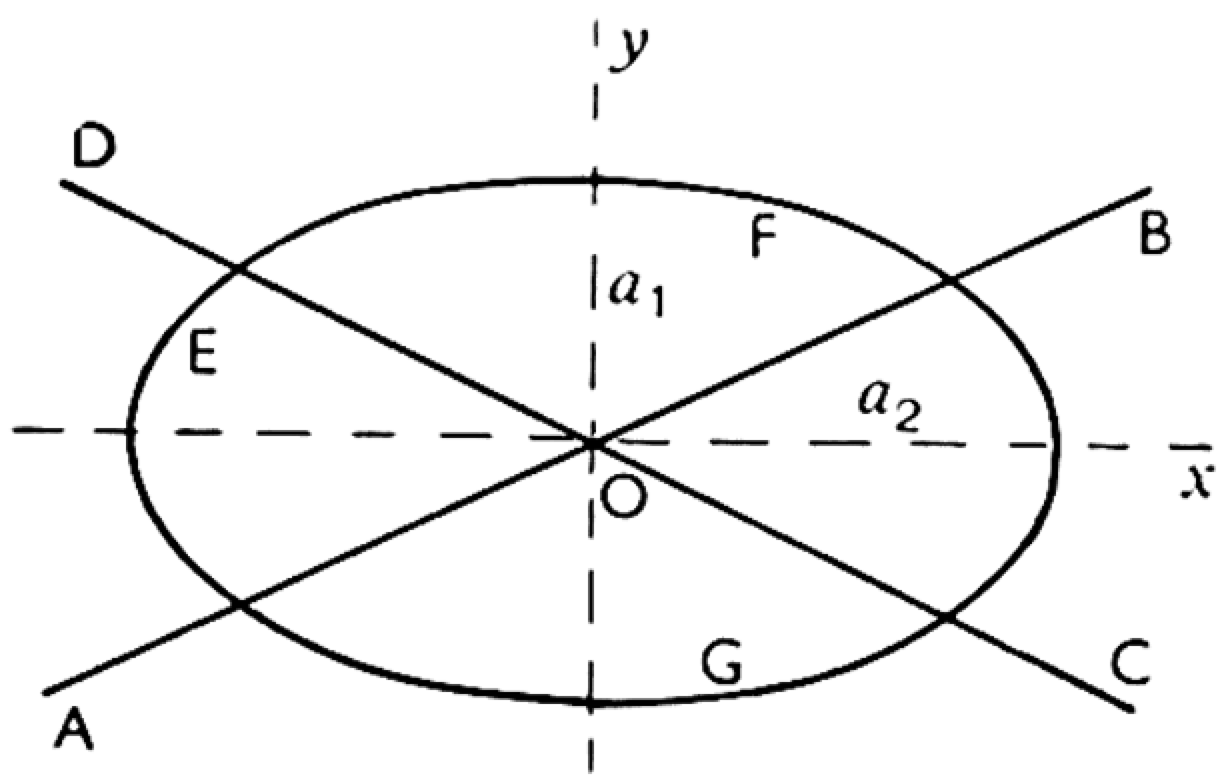
$$y = a_2 \sin(\omega t + \theta),$$

where θ is the phase difference.

(*a*) If $\theta = 0$, the two vibrations are in phase, and the displacement for both vibrations is zero at the same instant. The equations now become

$$x = a_1 \sin \omega t \quad \text{and} \quad y = a_2 \sin \omega t.$$

$$\therefore \quad \frac{x}{y} = \frac{a_1}{a_2}.$$



This is the equation to a straight line, which in this case is represented by the line AB passing through O, and is the resultant path of the particle.

(*b*) If $\theta = \pi$, the equations become

$$x = a_1 \sin \omega t \quad \text{and} \quad y = -a_2 \sin \omega t.$$

$$\therefore \quad \frac{x}{y} = -\frac{a_1}{a_2},$$

and the path of the particle is the line CD.

(*c*) If $\theta = \pi/2$, we have

$$x = a_1 \sin \omega t,$$

and

$$y = a_2 \sin\left(\omega t + \frac{\pi}{2}\right) = a_2 \cos \omega t.$$

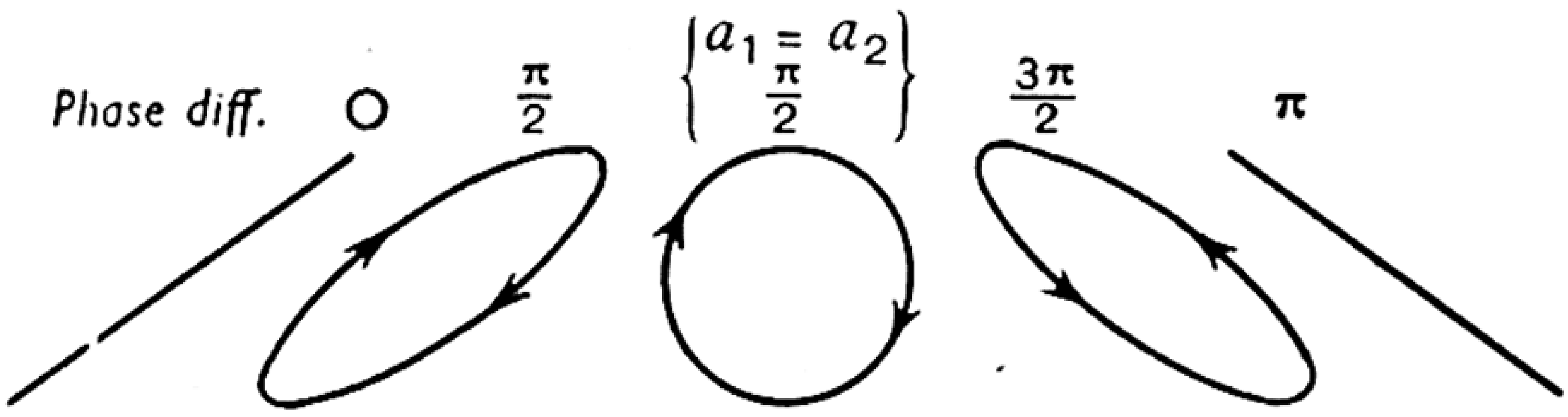
$$\therefore \quad \frac{x^2}{a_1^2} + \frac{y^2}{a_2^2} = 1.$$

This is the equation to an ellipse. Thus the path of the particle is the ellipse EFG in a clockwise direction.

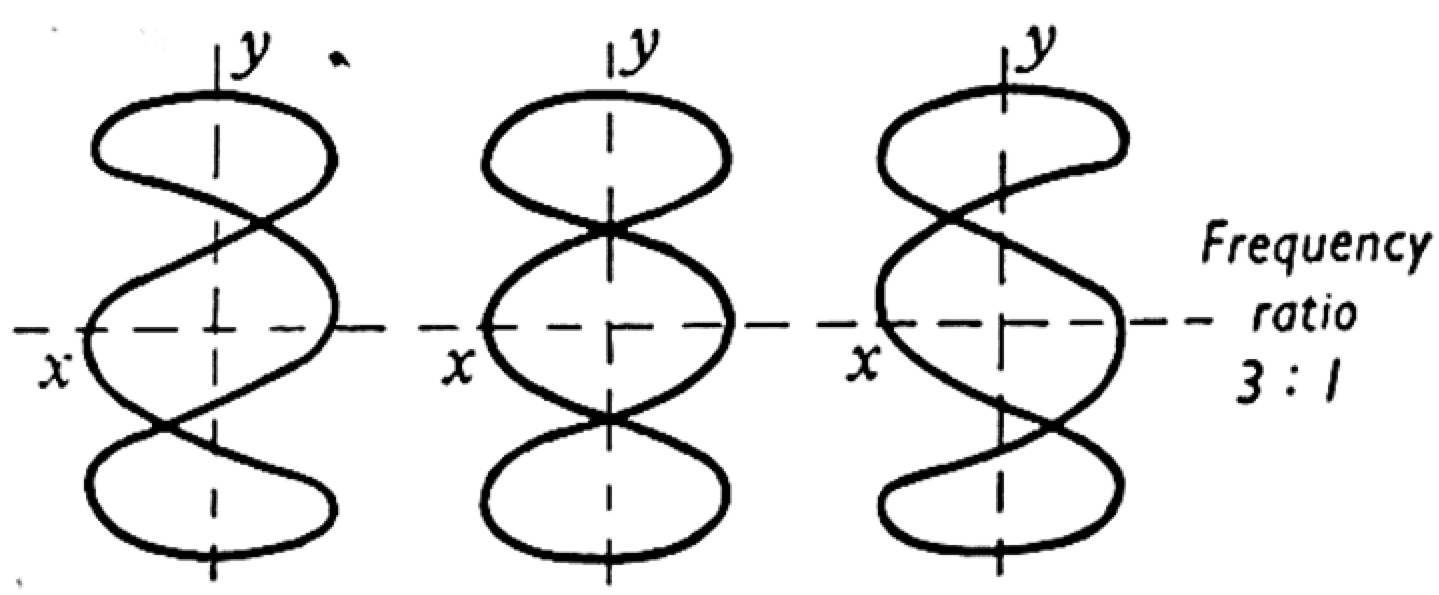
Also, if $\theta = -\pi/2$, the figure is still an ellipse, but the direction is anti-clockwise.

(d) If, in addition to the condition imposed in (c), $a_1 = a_2$, the equation of the figure reduces to $x^2 + y^2 = a_1{}^2$, and the figure is a circle.

Thus, when the frequencies are equal, the particle traces an elliptical path which may vary from a straight line when the phase difference is π, to a circle when the phase difference is $\pi/2$ and the amplitudes equal.



Frequencies of Vibrations Equal



Curves cut y-axis in 6 points and x-axis in 2

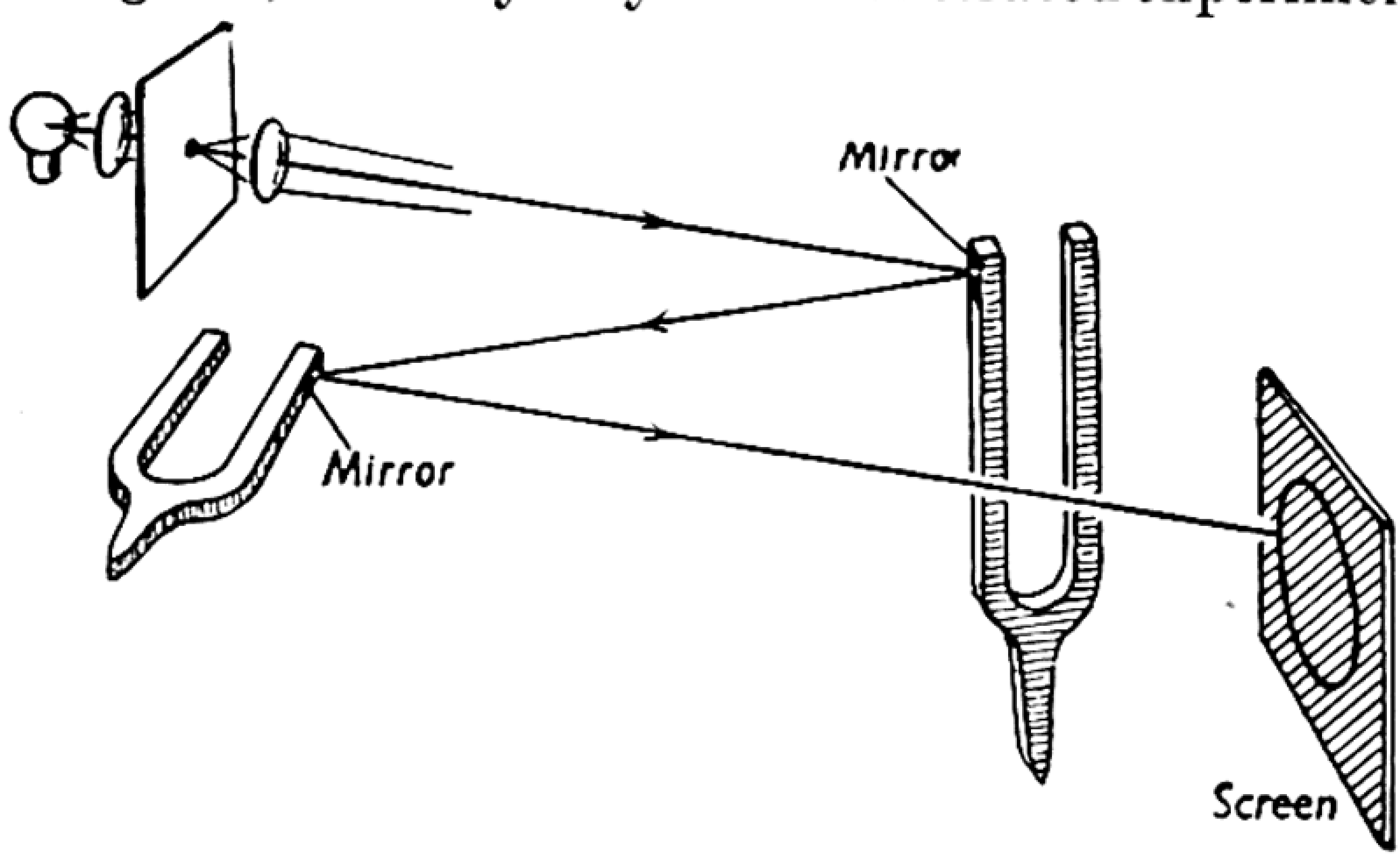
For any other difference in phase, the path is still an ellipse, but the major and minor axes are inclined to the directions of the component vibrations. The equation may be found by resolving the first motion into two components, one in phase with, and the other component 90° in phase from the second simple harmonic motion. By combining the two in phase, a linear motion results, and when this is combined with the component 90° out of phase, an elliptical motion results. The axes of the ellipse are no longer the same as the original axes.

If the frequencies of the two vibrations are not quite, but *nearly* equal, the particle follows a path which slowly changes through the various forms, straight line, ellipse and circle, due to the slowly changing phase difference. The frequency of the performance of a complete cycle of figures will be the difference between the nearly equal frequencies.

If the frequencies are commensurable, that is, in a definite ratio such as 2 : 1, 3 : 1, etc., the particle traces out a curve having a certain number of loops, this number being equal to the ratio of the frequencies. For *nearly* commensurate frequencies, the curve slowly changes as the phase difference varies.

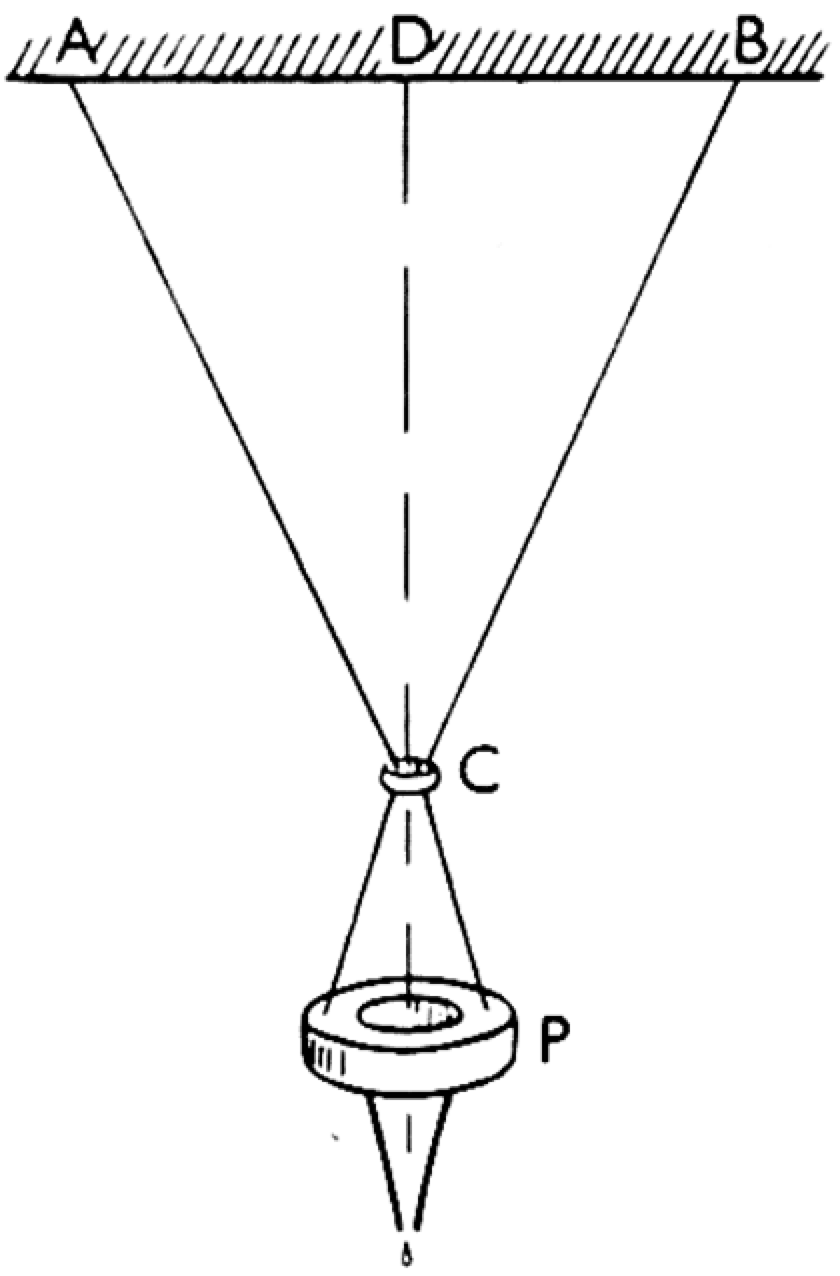
When the shape of the curves corresponding to the frequency relations is known, the approximate ratio of the frequencies can be recognised. For example, the curves shown in the diagram cut the y-axis in six points and the x-axis in two, so that the frequencies are in the ratio 6 : 2 or 3 : 1, for the vibrating point makes three vibrations in one direction in the same time that it makes one vibration in a direction at right angles.

Lissajous' figures. The various curves obtained by compounding two simple harmonic vibrations at right angles are known as Lissajous' figures, and they may be demonstrated experimentally in



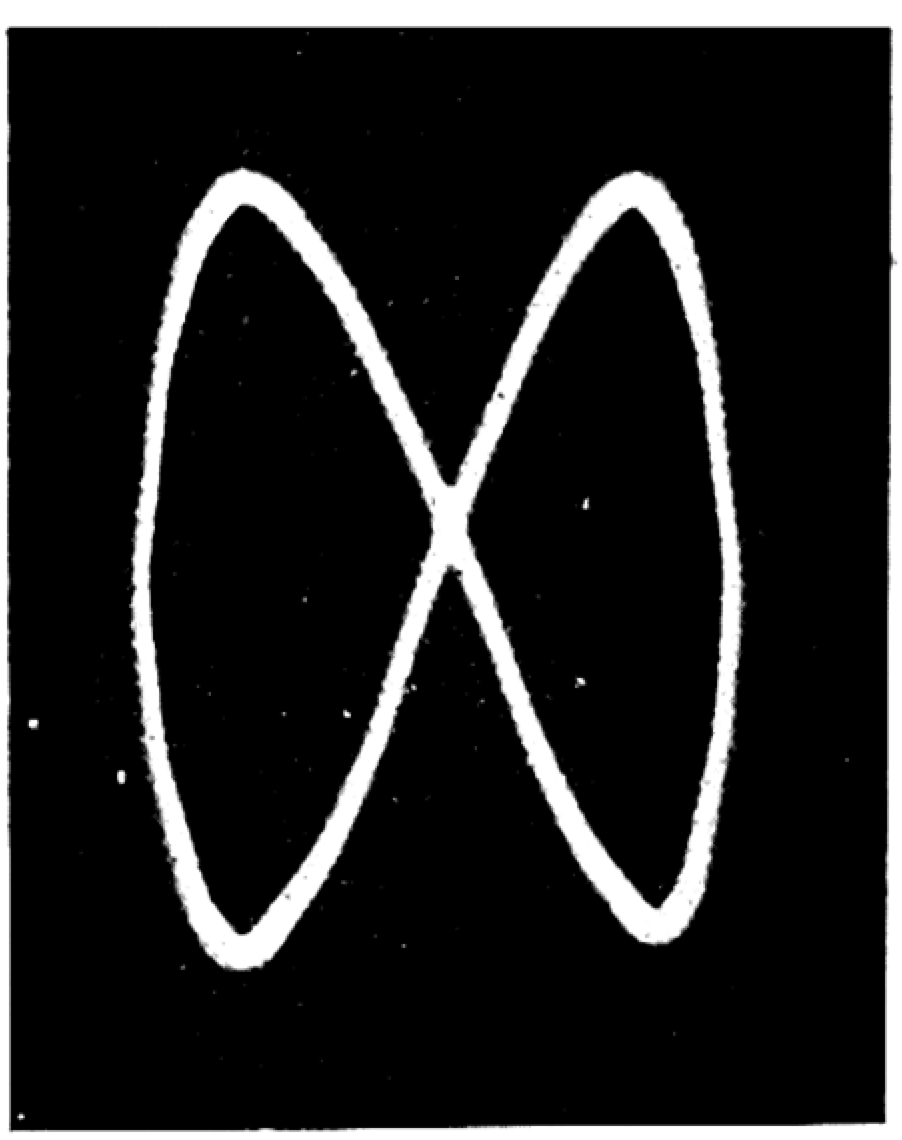
a variety of ways. In one optical method two tuning forks can be used as follows. A mirror is attached to one prong of a fork, and a narrow beam of light is focused on it. After reflection the light falls on a mirror attached to a second fork, this fork being placed so that the motion of its prongs takes place in a direction at right angles to that of the prongs of the first fork; after reflection here the light falls on a screen. If neither fork is vibrating, there will be a steady spot of light on the screen; if one fork only vibrates, the spot will trace out a straight line in either a horizontal or vertical direction. But when both forks vibrate, the spot will trace out a characteristic curve depending on the relative frequencies of the forks.

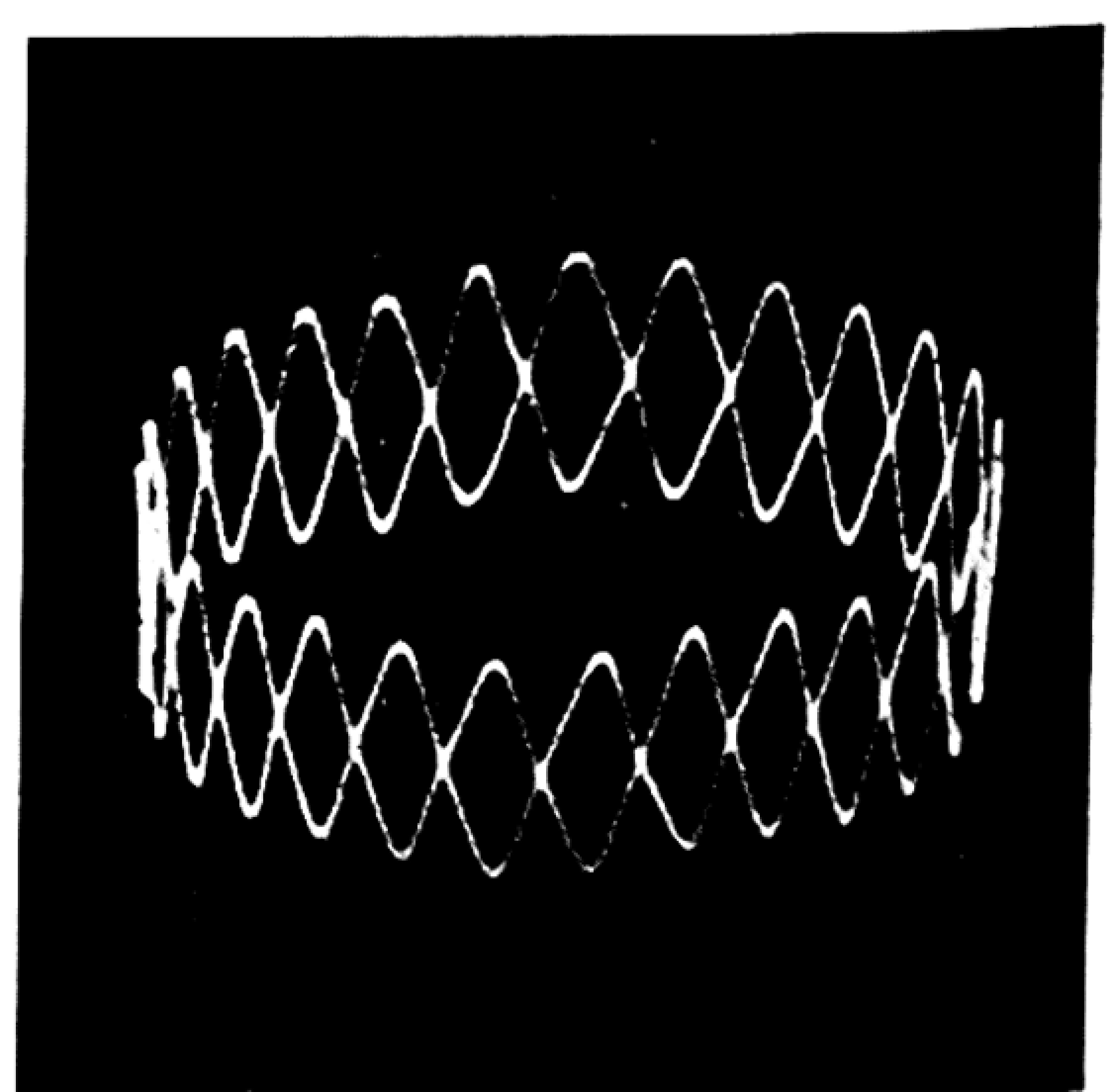
Perhaps the simplest method of arranging a vibrating system so that the motion of a point shall consist of two simple harmonic motions at right angles, with their periods in any desired ratio, is



to use a device known as **Blackburn's pendulum.** This consists of a thin wire or piece of string with its two ends fixed to a horizontal rod at A and B. The string is cut in the middle and the two ends are attached to a heavy lead ring P carrying a glass funnel with a narrow exit tube. A clip C enables the string to be caught up and so allows the length CP to be varied. The whole arrangement forms a pendulum DP for vibrations perpendicular to the plane of the figure and one of length CP when the vibrations are in the plane of the figure. When the bob is displaced outwards in a slanting position and then released, the two motions operate, and if some dry sand is put in the funnel and allowed to escape on to a piece of paper immediately below a record is traced on the paper. The periods of vibration are in the ratio of the square roots of CP and DP; hence if $DP = 4CP$ the frequencies are in the ratio 1 : 2.

A very convenient way of obtaining the figures if the necessary apparatus is available is by using a cathode ray oscillograph.





By Courtesy of the General Electric Co. Ltd.

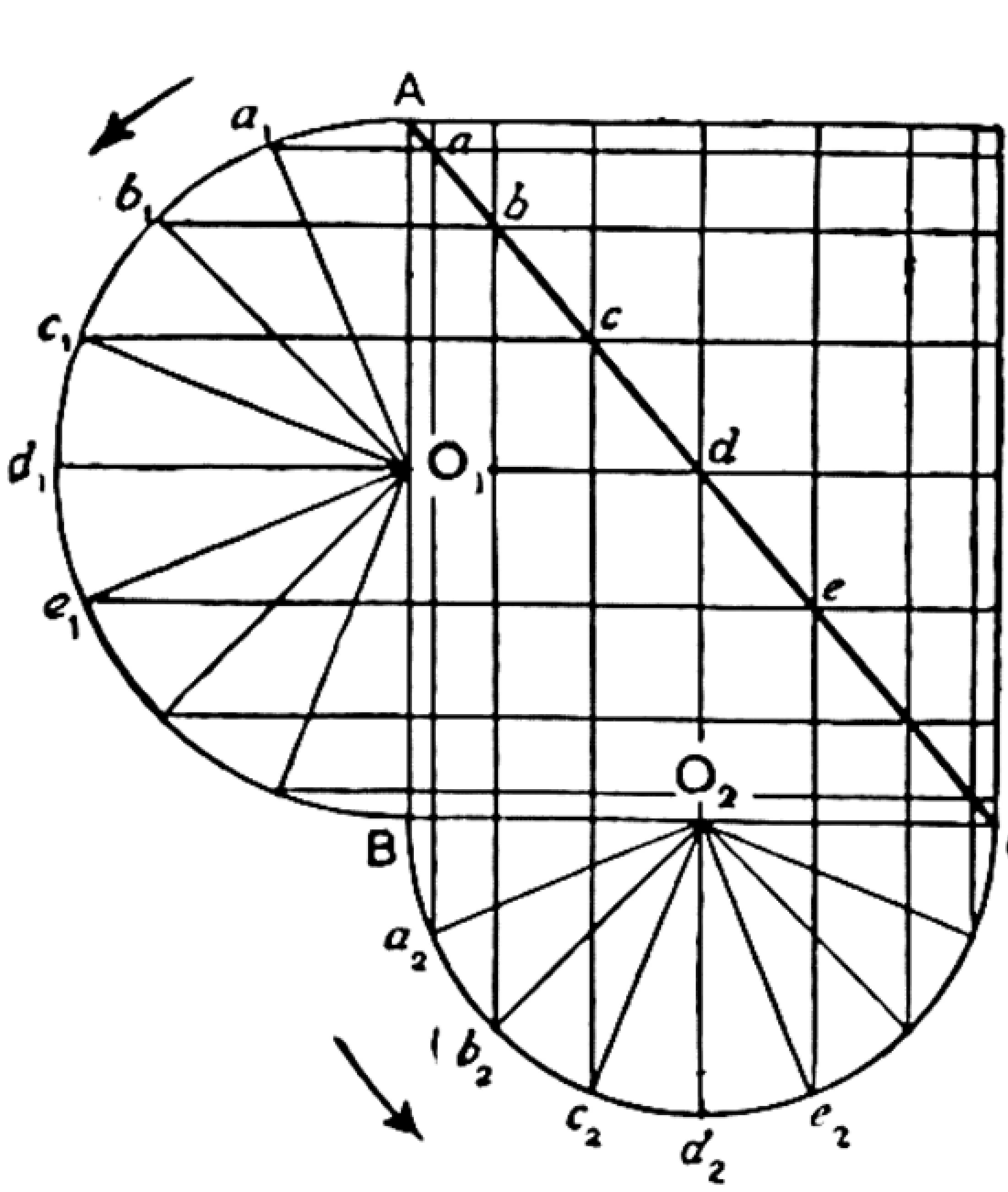
(a) (b)

Lissajous' figures recorded by a cathode ray oscillograph.
Ratio of frequencies (a) 2 : 1, (b) 29 : 2.

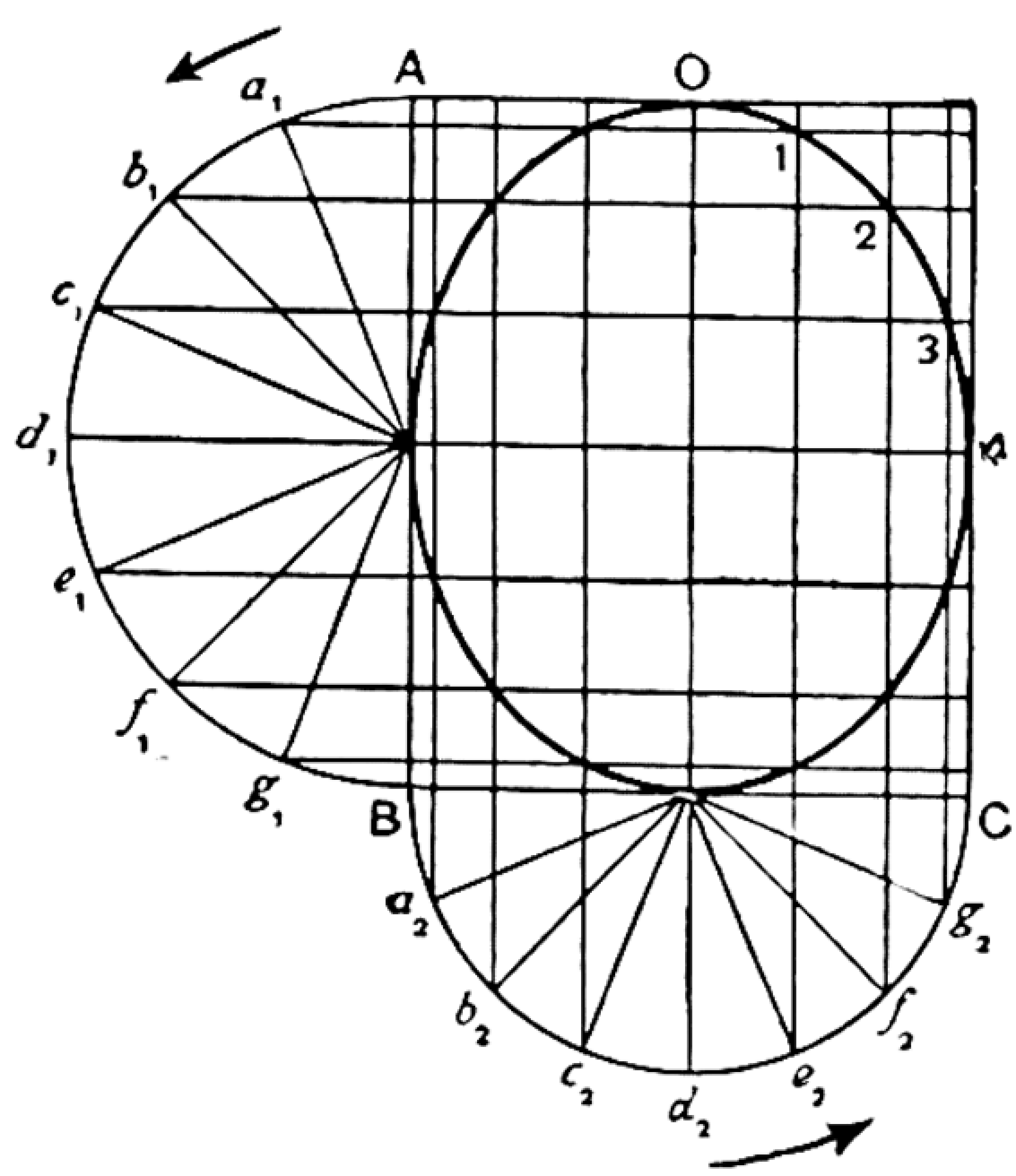
Lissajous' figures afford a good method for testing the accuracy of tuning of some simple interval between two forks (see p. 215), while the principle may be employed to investigate the way the period of vibration of a rod varies with the length of the rod.

Graphical treatment. The figures for the various combinations of two simple harmonic motions at right angles may be obtained by a graphical method as follows. Suppose that the two motions are of the same frequency, in the same phase but of different ampltidues, represented by AB and BC with O_1 and O_2 the mean positions (diagram a). With centres O_1 and O_2 describe semi-circles on AB and BC and divide them into equal parts Aa_1, a_1b_1, etc. and Ba_2, a_2b_2, etc. If the particle is regarded as starting at A, its position on account of the first vibration is given by the projection of O_1a_1, and on account of the second vibration its position is given by the projection of O_2a_2. Thus, its actual position due to both motions is represented by a. Similarly, the positions b, c, d, etc., are obtained as far as C, and the second half of the vibration will be the return from C to A; the resultant motion therefore is represented by the straight line AC.

If the two components differ in phase, the resultant motion is represented by an ellipse as shown in (b). In this diagram there is



(a)



(b)

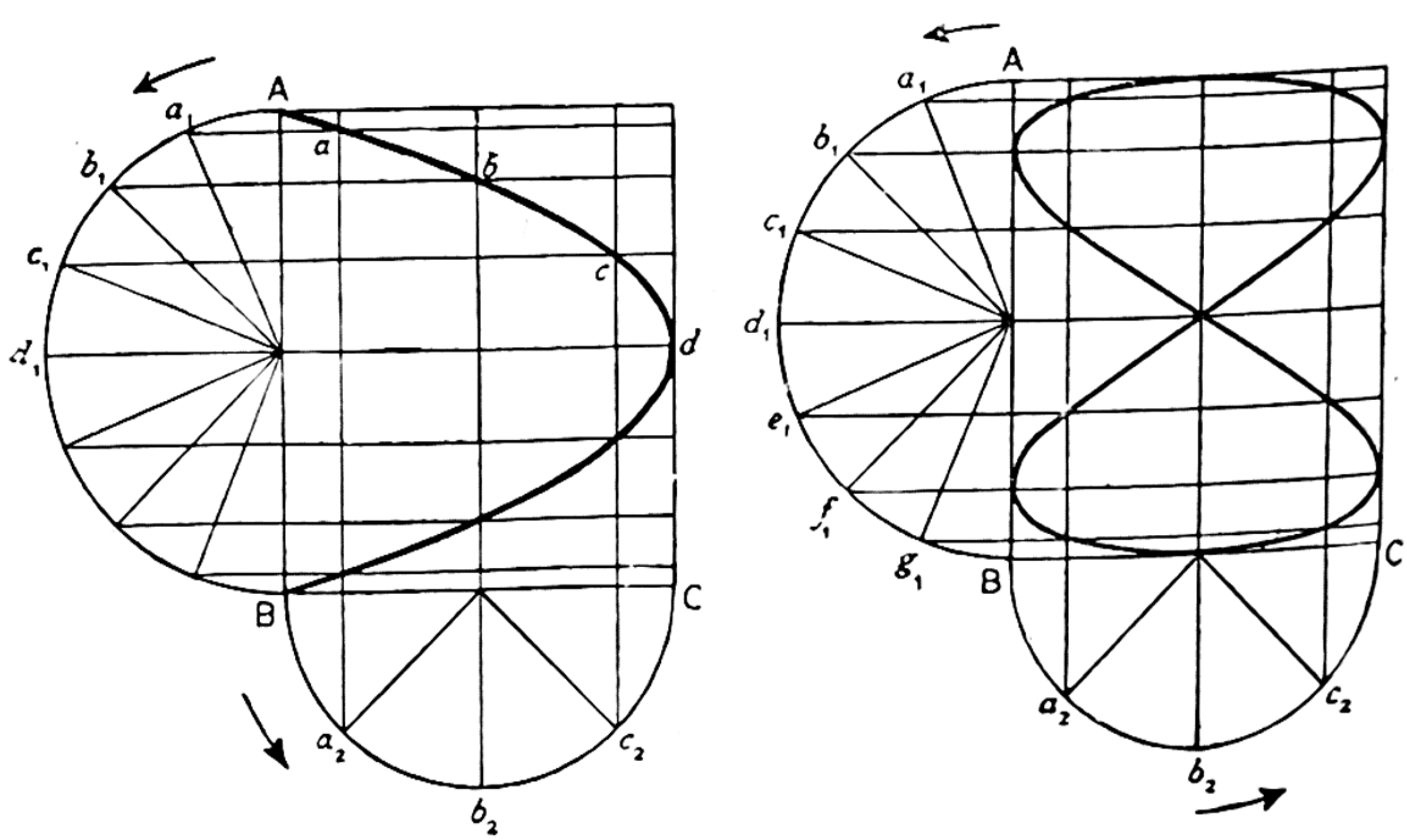
a phase difference of $\pi/2$, one motion starting at A and the other midway between B and C, and the resultant positions are O, 1, 2, 3, etc. It will be clear that if the amplitudes of the component vibrations are equal, AB and BC are equal, and the resultant motion will be represented by a circle. In the case of the ellipse, the equations representing the two simple harmonic motions may be regarded as

$$x = a \sin \omega \quad \text{and} \quad y = b \sin (\omega + \theta)$$

where θ represents the phase difference. The ratio of the amplitudes, b/a, is given by $\tan \alpha$, where α is the inclination of the axis of the ellipse shown in the figure (p. 141) to the x-axis. The value of θ can also be found, for OB gives the maximum value of y, which is b; and OC, the value of y when $x = 0$, is $b \sin \theta$; hence

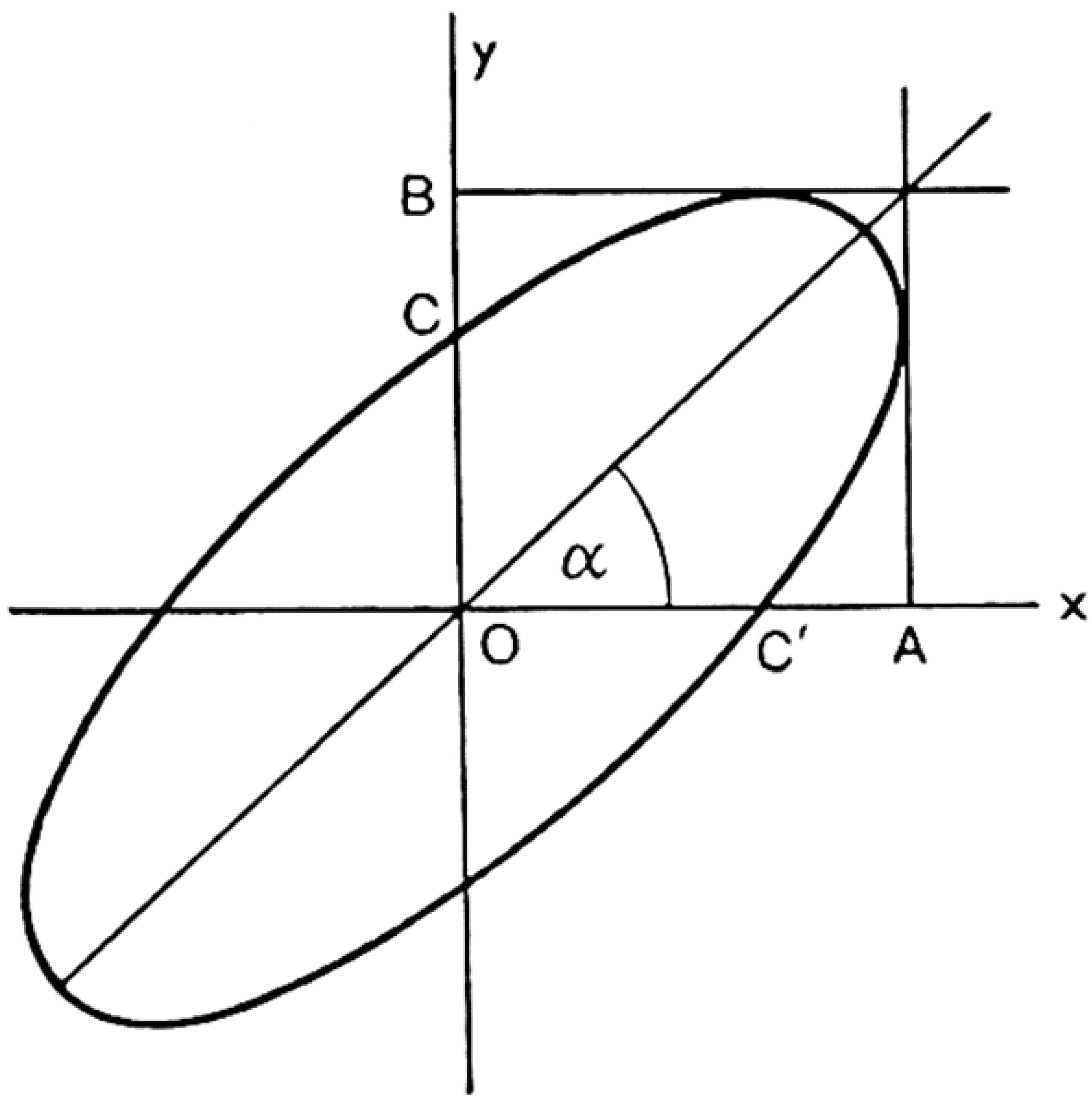
$$\sin \theta = \frac{OC}{OB}.$$

If the periods of the two vibrations are different, the same method of finding the resultant motion is used as in the above cases, but now the arcs of the circles denoting equal intervals of



(c) (d)

time do not subtend equal angles at the respective centres. In (c) the ratio of the frequencies is 2 : 1, and the two components are in the same phase, while in (d) the frequencies are in the same ratio, but there is a phase difference of $\pi/2$.



It will be obvious to the student that the Lissajous' figures described above are comparatively simple. But any combination of two simple harmonic motions at right angles can be dealt with in a similar way, and the student should try a few more difficult exercises in which the two motions are of different amplitudes and periodic times and also have a phase difference.

The method of procedure is the same in all cases. If the amplitudes of the two motions are different, the diameters of the semicircles must be made proportional to the amplitudes. If the periodic times are different, the circumferences of the semi-circles must be divided into a number of equal arcs proportional to the two times, and when the two motions are not in phase, due allowance must be made when fixing the starting points on the circle.

As is indicated on p. 137, the frequency ratio of the two motions can be read off from any particular pattern that is produced.

CHAPTER VII

DISSONANCE AND CONSONANCE: COMBINATION TONES

DISSONANCE AND CONSONANCE

Simple tones. When two *tones* of the same frequency are sounded together, no beats are produced; but if the frequency of one is gradually increased, the number of beats increases. Very slow beats are not unpleasant, but as the number increases so does the unpleasantness, and a stage is reached when this becomes a maximum. The phenomenon is known as **dissonance** or **discord,** and it may be compared with the irritating effect on the eye caused by a flickering light. Thus so far, it appears that discord is due to beats. But the effect does not depend merely on the *number* of beats, for it varies with the absolute frequencies of the tones which give the beats. In the neighbourhood of c'', frequency 512, the harshness seems to be a maximum when the number of beats is about 32 per second. But if we sound C, frequency 64, together with G, frequency 96, thus producing the interval the *fifth*, we also have a frequency difference of 32 and yet there is no trace of harshness. The accompanying table indicates a few musical intervals with a frequency difference of 32 between the two tones, producing a varying degree of dissonance.

Interval	Tones	Frequencies	No. of beats per second
Semitone	b', c''	480, 512	32
Tone	d', e'	288, 320	32
Minor Third	e, g	160, 192	32
Major Third	c, e	128, 160	32
Fourth	G, c	96, 128	32
Fifth	C, G	64, 96	32

The following table, due to Mayer, who investigated this subject, gives the series of values for the frequencies of the beats

when discord is a maximum and when the sensation of harshness disappears.

Frequency of lower tone	No. of beats per second	
	Maximum discord	Harshness disappears
64	6·4	16
128	10·4	26
256	18·8	47
384	24·0	60
512	31·2	78
640	36·0	90
768	43·6	109
1,024	54·0	135

It will be noticed that the ratio of the frequencies of the lower tone and the one which produces maximum discord gives an interval of approximately a semitone in every case, and this interval is usually regarded as a discord. Also the harshness disappears when the interval between the two tones is approximately 1·2 or 6/5, and it will be noticed that this interval is a minor third. So far, over a fairly wide range of frequencies, it seems that discord is only likely to occur when the two tones lie within the interval of a minor third. But some of the intervals greater than a minor third are extremely dissonant, the seventh being one of the most dissonant of all ; hence the problem must be examined further.

Complex musical sounds. It must be remembered that in the discussion above, the sounds have been simple tones. But musical sounds are complex, consisting of the fundamental tone and a number of overtones, and it is necessary to consider how far the overtones contribute towards dissonance and consonance.

If we assume that the source of the two sounds is such that the overtones are in a harmonic series, we find in the case of the *octave* that the frequencies of the overtones are as shown in the upper table, on page 144.

Here it will be noticed that all the overtones of the higher note are in unison with overtones of the lower note ; hence there cannot be any discord. Looking at the figures at another angle, we find that the smallest difference in frequency (except unison) between any two overtones of the notes is 256. Hence, since for

	1st Note	Octave
Fundamental	256	512
1st overtone	512	1024
2nd ,,	768	1536
3rd ,,	1024	2048
4th ,,	1280	2560
5th ,,	1536	↓
6th ,,	1792	
7th ,,	2048	
8th ,,	2304	

maximum discord this difference must be about 19 (see table, p. 143), we see that not only the fundamentals, but also the overtones are constant. Thus an octave is an interval of the highest consonance.

Now consider the upper note to be lowered a semitone in pitch, thus giving an interval of a *seventh*. The frequencies of the fundamental and several of the overtones are given in the table,

	1st Note	2nd Note
Fundamental	256	480
1st overtone	512	960
2nd ,,	768	1440
3rd ,,	1024	1920
4th ,,	1280	2400
5th ,,	1536	
6th ,,	1792	
7th ,,	2048	

and if the differences in frequency between various overtones are compared with the table on p. 143, it is clear that dissonance *must* occur. This can also be seen by noticing that each overtone of the upper note which in the octave coincided with an overtone of the lower note is now a semitone out of tune with it and so will give rapid beats and dissonance.

It is interesting to note that when the two sounds are simple tones, the dissonance of the *seventh* almost disappears, although it sounds a somewhat unusual interval. That it does not entirely disappear is partly due to combination tones (see p. 148).

Thus, in order to determine theoretically whether an interval

is consonant or not, we must consider not only whether beats may occur between the two fundamentals, but also whether beats may occur between the overtones.

Coincidence of two overtones. Now consider an interval, say the *fifth*, and assume that the frequencies of the notes forming the interval are 256 (c') and 384 (g'). If the frequencies of the fundamental and the various overtones are tabulated in the form shown, it will be seen that the third tone of the lower note coincides with the second tone of the higher note ; also the sixth tone of the lower coincides with the fourth of the higher.

Lower note.	256		512	$\boxed{768}$	1024		1280	$\boxed{1536}$
Higher note.		384		$\boxed{768}$		1164		$\boxed{1536}$

Hence the orders of the two coincident tones will determine the ratio of the frequencies of the two notes forming the interval. Thus the ratio in this case is 3/2, and this is true of all intervals.

Mistuned intervals. Now consider the effect on the pairs of coincident overtones of errors in the tuning of the notes, taking as our first example a mistuned *octave* in wihch the frequencies of the notes are 256 and 129. The frequency of the first overtone of the lower note is 258, and this will give 2 beats per second with the fundamental of the higher note ; a similar thing happens when the frequency of the lower note is 127. If we now consider the higher note to be mistuned so that the frequencies are 257 and 128, we find there is only 1 beat per second between the fundamental of the higher note and the first overtone of the lower. Thus, if the frequency of one of the notes is inaccurate by 1 vibration per second, the number of beats is 1 or 2 according as the inaccuracy is in the upper or lower note.

Now examine the mistuned *fifth*, in which the frequencies of the notes are 128 and 191 (instead of 192 for the accurate interval). The frequency of the second overtone of the lower note is 384 and that of the first overtone of the higher note is 382 ; thus there are 2 beats per second with these two tones. If the mistuning is such that the frequencies are 129 and 192, the number of beats given by the same corresponding overtones is 3. The relation between other overtones could of course be examined in a similar way. Hence in both cases, mistuning the lower note gives the larger number of beats per second.

Since we find, when an interval is not true, that those overtones of the two notes which ought to be in unison are in a condition for

producing beats, it follows that the greater the number of *common* overtones, and the stronger these overtones are, the greater will be the dissonance produced by mistuning the interval, and so the greater the accuracy with which the ear adjusts itself to such an interval. In the case of the *octave*, which has perfect consonance, all the overtones of the higher note are in unison with overtones of the lower ; hence this interval is rather easy to tune. In the *fifth*, the alternate overtones of the higher note are in unison with overtones of the lower ; this interval is used in string instruments and makes possible their accurate tuning. In the *fourth* every third overtone of the higher are in unison with overtones of the lower. The student should examine other intervals in a similar way ; it will be found that, as the consonance decreases, it is a higher and higher, and therefore less important, overtone that is in unison.

Thus, an interval is more consonant the greater the number and the lower the overtones which are common to the two notes.

Classification of intervals. Helmholtz classified consonant intervals as follows :

Absolute consonances : The *octave*, the *twelfth* and the double *octave*.

Perfect consonances : The *fourth* and the *fifth*. They are called perfect because they may be used in any part of the scale without important disturbance of harmoniousness.

Medial consonances : The major *third* and the major *sixth*. These intervals are distinctly dissonant if they occur in the lower part of the scale, but they are comparatively smooth in the higher part.

Imperfect consonances : The minor *third* and the minor *sixth*. Note that these intervals are the inversions (the defect of an *octave*) of the major *sixth* and the major *third* respectively.

It should be noticed in connection with the above classification that, if a given interval is increased by an *octave*, there is a very marked effect on the degree of consonance. For example, if the *fifth* is increased by an *octave*, the interval becomes a twelfth, and so it passes from the list of perfect consonances to that of absolute consonances. But if the *fourth* is increased it becomes more dissonant, as will be seen from the following arrangement of the two intervals :

Fourth			*f*		*f′*
Octave	*C*	*c*	*g*	*c′*	*e′* *g′*

there is marked beating between the fundamental of one note (f) and the second overtone (g) of the other, while there is also beating in the higher orders of overtones. The student should examine the effect of increasing other intervals in a similar way.

Attention has been limited so far to those sources of sound, such as a piano, where the overtones are in a harmonic series of diminishing importance, and only the first few need be considered. But there are several other interesting cases that can be mentioned. When a stopped organ pipe is blown *feebly*, very nearly simple tones are produced, and if a minor sixth is sounded with such pipes the interval loses much of its harshness. Then again a musical interval can be obtained by using two different instruments, the two notes then being of different quality. It is interesting to consider whether there is any difference in the degree of consonance according as to whether one particular instrument is selected for the high note or the low note. Suppose an oboe and a clarinet give a major *third* (say, C–E), and in the first instance let the lower note be assigned to the oboe. As an oboe gives the full series of overtones and the clarinet only the odd members of the series, the arrangement of the various tones may be written as follows :

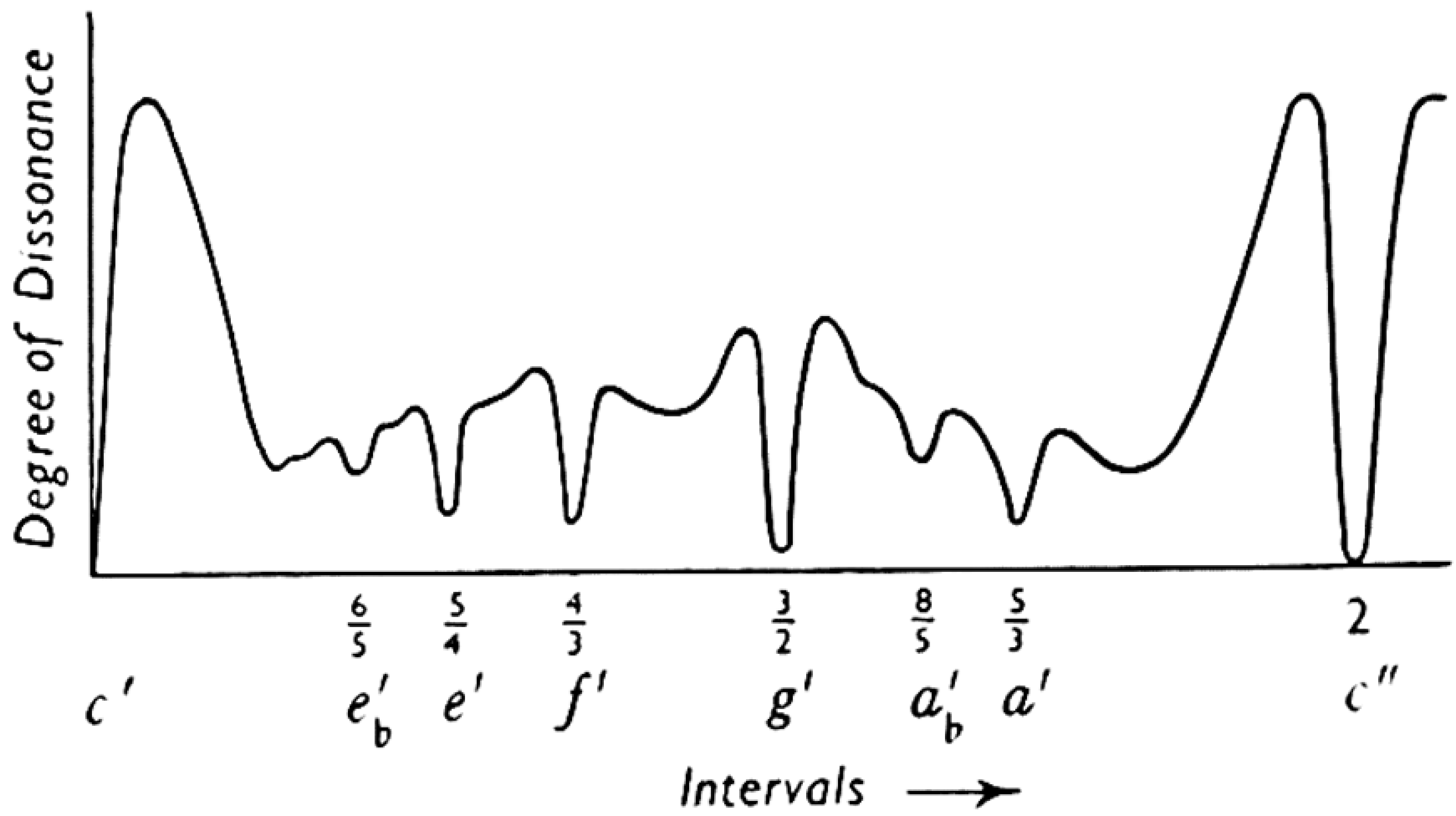
Oboe	C		c	g	c'	e'	g'
Clarinet		E			b		$g\sharp$

Here we have two semitones beating intervals as indicated, which will produce a certain amount of harshness ; but if the lower note is played on the clarinet we find that there are no intervals which beat and the interval is more harmonious :

Clarinet	C			g		e'
Oboe		E	e		b	e'

It will be seen therefore that the whole subject of dissonance and consonance of intervals produced by musical notes is quite a complex problem, since it involves in every case a consideration of both the number and the prominence of the overtones present. Later it will be shown that the problem is further complicated by the presence of combination tones.

A graphical representation of the changes in dissonance which take place when the interval between two notes is gradually altered is instructive. If both notes are originally tuned to c', and then, while one is kept at this pitch, the other is gradually



raised in pitch to c'', we shall get the conditions indicated in the diagram. The ordinates represent the relative amounts of dissonance and the abscissae the notes giving the well-known intervals. It will be seen in particular that when the two notes give a true *octave*, dissonance is a minimum at zero ; but if there is slight mistuning on either side, the curve rises very sharply to a maximum of dissonance. A similar state of affairs is noticeable when the *fifth* is sounded (c'–g'), but the effect is not so pronounced. In fact, all the well-known intervals are represented by dips in the curve and are more or less bounded by more or less strong dissonance.

Combination tones. If simple tones instead of compound notes are employed to obtain musical intervals, we still get dissonance when, say, a *fifth* or an *octave* is slightly mistuned ; and it is well known that the accuracy with which the ear is able to detect an untrue interval is very considerably less with pure tones than with compound ones.

The explanation of dissonance given above will not account for this. Helmholtz has explained the dissonance of simple tones as being due to the beats produced by what are called **combination tones.** These tones were first discovered by the organist Sorge about 1745, but afterwards became known through the Italian violinist Tartini, and were called Tartini's tones. The most important type of combination tone is known as the *difference tone* owing to the fact that its frequency is equal to the difference of frequencies of the two generators ; this is the type discovered by Sorge and Tartini. There is also a *summation* tone, the frequency of which is the sum of the frequencies of the generators ; this is much weaker than the difference tone and was discovered much later.

One important condition for the production of combination tones is that the generating tones shall be sounded *strongly*. Hitherto, in considering the result of the superposition of two systems of waves in air or in any other medium, we have assumed that the displacement of any particle due to the two systems is small, so that the restoring force is exactly proportional to the displacement. But in two systems where the amplitude is so large that this proportionality no longer exists, it has been shown by Helmholtz that, in addition to the two primary wave-systems of frequency m and n, there will be produced two secondary systems of which the frequencies will be $n-m$ and $n+m$. These will correspond to the **difference tone** and **summation tone** respectively.

Such tones have been produced by using two harmonium reeds as the source, and they have caused suitably tuned resonators to respond. In some cases the body in which these tones are produced may be the ear itself, for the bones and membranes which convey the sound from the outside drum to the nerve terminations form an arrangement such that, when violently disturbed, the restoring force would not be proportional to the displacement. The tones of frequencies $n-m$ and $n+m$ are known as *first-order combination tones* to distinguish them from those of higher orders. These latter are produced by combination of one of the generators with one of the combination tones ; for example, the *second-order tones* are given by one of the generators and one of the first-order tones, and so on.

Thus we shall have the following tones and frequencies :

First-order difference tone :	$n-m$.
First-order summation tone :	$n+m$.
Second order difference tones :	$n-2m$, m, n.
Second-order summation tones :	$2n-m$, $2n+n$, $n+2m$,

and so on.

It will be noticed that the first-order difference tone has a frequency exactly equal to the number of beats produced between the two generating tones. Koenig maintained that the production of these tones was due to the coalescence of the beats to give a musical tone, and he called such sounds beat-tones. This hypothesis, however, can not be regarded as entirely valid. In the first place, combination tones can be generated by pure tones separated by an *octave* ; but previously we have supposed that beats cannot occur between pure tones separated by an interval much greater than a minor *third*. Again beats can be heard when

two quite faint tones are sounded together, but combination tones only occur when the two generators are sounded strongly. Also, Koenig's hypothesis does not explain the existence of the summation tones, though it has been established quite definitely that such tones do exist. Koenig held to the view that, if such tones are heard, they were due to beat tones produced between some of the upper overtones of the generators. Combination tones can be readily heard on a harmonium if the generators are sufficiently strong, and particularly so if the upper of the two generators is kept the same while the lower one is made to descend down the scale, for here the difference of frequency between the two generators is increased and the pitch of the difference tone is raised.

To obtain the difference tone on a harmonium, sound the diad represented by c'' and a' as the generating tones. This interval is a minor *third*, and the frequencies can be taken as 512 and 427 respectively. The first-order difference tone will therefore have a frequency of 85, which is the tone F, an interval of a twelfth below c'. To get the summation tone, sound first the note c (128) and then add to it the note F (85). The frequency of the summation tone will be 213, which is the tone a, a major sixth above c.

A very good example of a difference tone is provided by the double whistle used by referees in football matches. In this there are two short pipes side by side giving tones of slightly different pitch. The first-order difference tone is therefore of low frequency and can easily be heard, and it is this tone which gives the characteristic quality of these whistles.

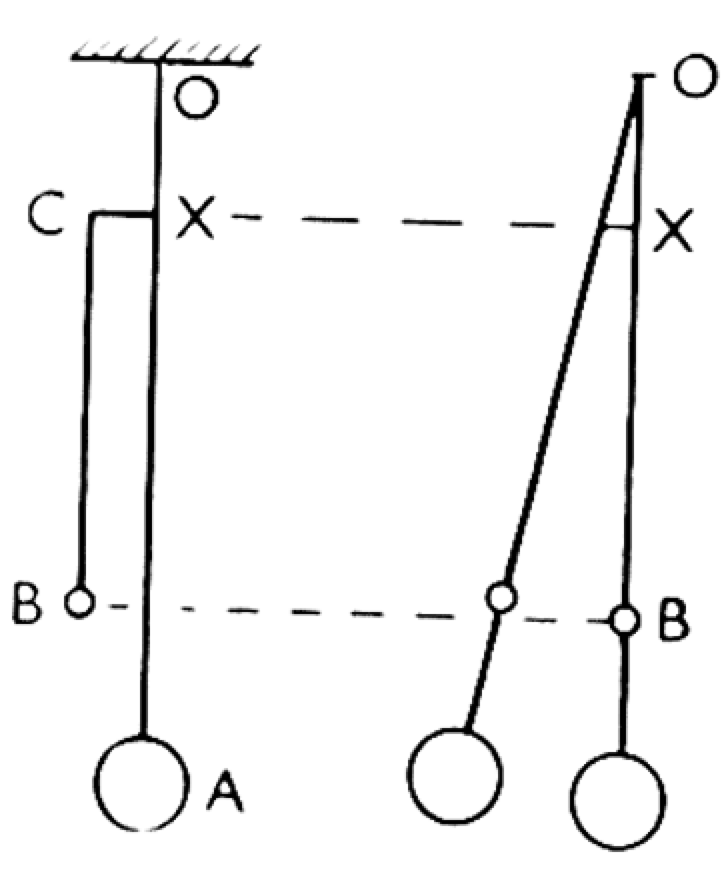
CHAPTER VIII

FORCED AND FREE VIBRATIONS: VIBRATIONS OF AIR COLUMNS

WHEN a simple harmonic force is applied to any object which is capable of vibrating, it produces a simple harmonic motion in that object, though the amplitude may be very small. Also, all objects capable of vibrating have a natural period of vibration which depends generally on the dimensions of the vibrating object. These two facts are used in the following discussion.

Consider a heavy pendulum A to be suspended from a point O and let a small pendulum B be attached to the suspension at X, below O. When A is set in motion B will be given a periodic force, the period being equal to that of A, and since A is large compared with B, the latter will not appreciably alter the motion of A.

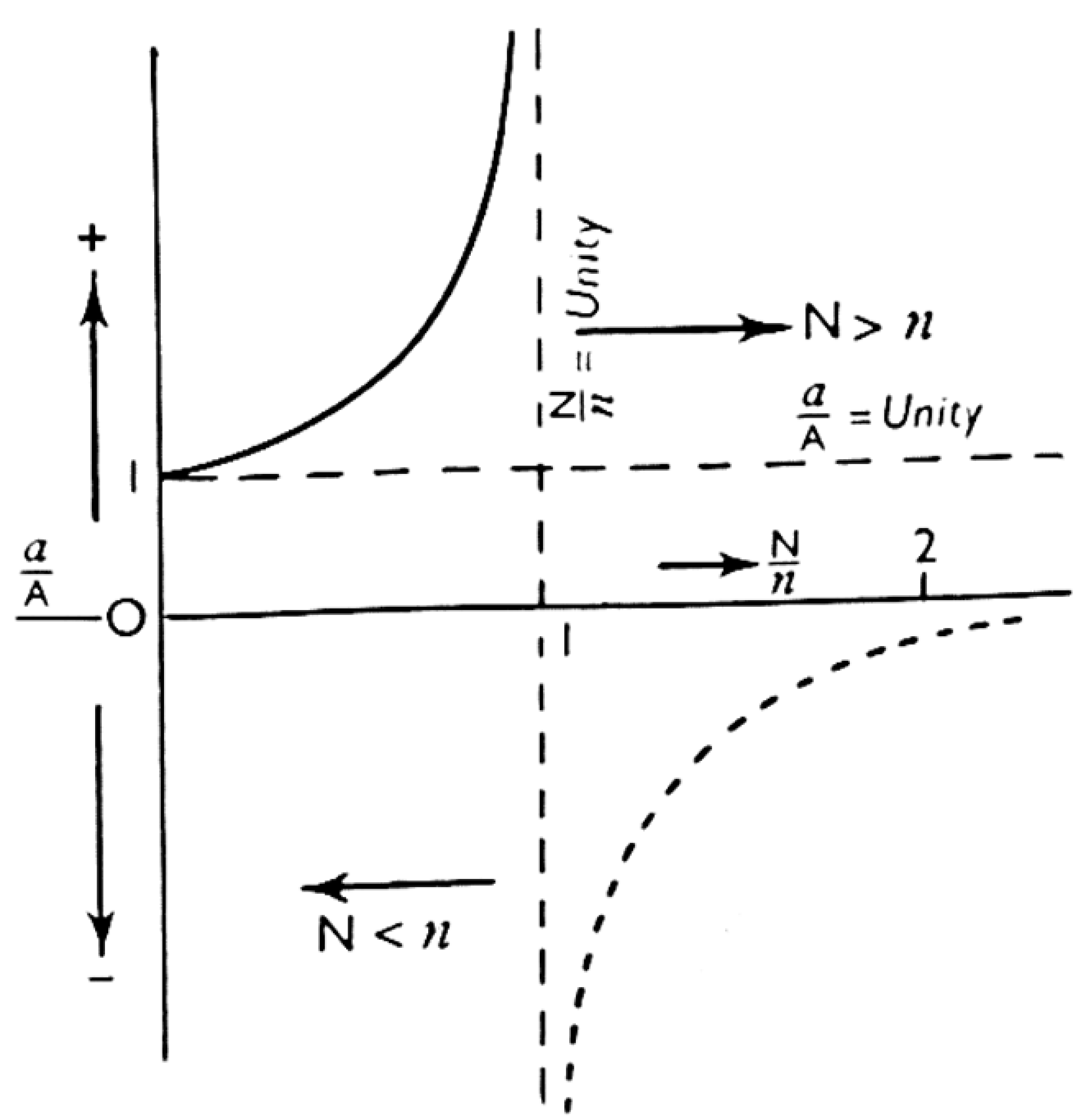
Let the period of A be T and the length of the equivalent simple pendulum be L. When A is started, B also will be set in motion, and the amplitude of the motion will first gradually increase, then decrease and then increase again. These graduations will soon become less marked, and finally the amplitude will remain at a constant value. It will be found then that the period of B is exactly the same as that of the large pendulum, and also both pendulums will be practically in the same phase, that is, when the bob of A moves to the right B does the same. Now, since the length of the small pendulum is less than that of the larger one its natural period of vibration is less; hence since it is vibrating with the same period as A, it is clear this is not its natural period, that is, it is forced to vibrate by the external agency with a period not its own. Vibrations produced in this way are called **forced vibrations**, and in such cases the period of the object forced to vibrate is equal to that of the applied force.



There are two other cases in connection with the above experiment which must be considered. In the first place, if CB is greater than L, so that the natural period is greater than the period of A, then at the start the same alternations in amplitude will occur. But when a steady condition has been reached, the period of B will again be equal to that of A. In this case, however, the phase of B will differ from that of A by nearly 180°. Secondly, if CD is exactly equal to L, there will be no alternations of amplitude when the motion starts, but the amplitude of B will steadily increase and finally reach a value much greater than in the preceding cases. Here both pendulums have the same period of vibration, but it is the natural period in each case. Thus, when a periodic force acts on a pendulum—or any other object capable of vibrating—and the period of the force agrees with the natural period, the amplitude of the resulting motion is much greater than when the two periods differ. In such cases the phenomenon is termed **resonance**.

If, when an object is executing forced vibrations, the applied force is stopped, the object will continue to vibrate, but the vibrations will be of the same period as the natural period of the object; such vibrations are said to be **free**.

The amplitude of any forced vibration can be calculated in terms of the amplitude of the applied force and the two frequencies concerned, and the results may be exhibited graphically as in the diagram. Here, the ordinates represent the ratio of the



amplitude (a) of the forced vibration to that (A) of the applied force, and the abscissae represent the ratio of the frequency (N) of the applied force to that (n) of the natural frequency of the object which is forced to vibrate. Looking at the smooth curve, it will be seen that a/A becomes infinite when N/n is equal to unity. This is the case for resonance, for $N=n$, and the amplitude of the vibrating object should be infinitely large. In practice, however, this is not attained, because there is always a damping force to be considered. As the ratio N/n becomes smaller, that is, as the frequency of the applied force becomes less than the natural frequency of the object, the value of a/A falls rapidly until it becomes equal to unity for small values of N/n, and in this case the amplitudes are equal.

Now consider the case when $N>n$, that is, the frequency of the applied force is greater than the natural frequency of the object, so that N/n is greater than unity. The results are represented in the diagram by the dotted curve. Here, the amplitude of the forced vibration again falls so that a/A decreases, and when N becomes very large the amplitude of the forced vibration is reduced to zero. In this case, the phases of the two vibrations are opposite and the ratio of the amplitudes a/A is regarded as negative.

Damping and sharpness of resonance. When a vibrating object produces a sound, the energy of the waves which travel outwards is derived from the energy of vibration of the object. But a certain amount of energy is always lost in the form of heat due to viscosity of the particles of the object and friction, and this loss causes a gradual decrease in the amplitude of the vibrations, which are now said to be **damped.**

If A_1, A_2, etc., are the amplitudes of successive vibrations, the ratio $A_1/A_2=A_2/A_3=\ldots$; this constant is known as the *decrement*, and this measures the damping. Hence we have

$$\log A_1 - \log A_2 = \log \text{(const.)}$$

and this difference between the logarithms of successive amplitudes is called the **logarithmic decrement** (cf. this with the logarithmic decrement of, say, a ballistic galvanometer).

The effect of damping becomes relatively more and more important as resonance is approached. In order to maintain vibrations under conditions of damping, it is necessary to supply correctly timed impulses from an external source, this rate of energy being called the power dissipation. At resonance, the amplitude and the power dissipation reach a maximum. By

considering how the power required to maintain vibrations against the losses varies near resonance, it can be shown that the energy dissipated at a frequency p very near resonance is half the resonance value when

$$\frac{p}{n} = 1 \pm \frac{k}{n} \quad \text{(approximately)},$$

where n is the natural frequency and k is the damping constant, which can be obtained from the logarithmic decrement of the vibrations (logarithm of decrement $= kT$, where T is the periodic time); that is, when the frequency of the applied force differs from that of the resonator by the fraction k/n. This ratio k/n constitutes a measure of the *sharpness of resonance*, and the reciprocal n/k is sometimes referred to as the *persistence* of the vibrations.

It will be seen that the smaller the damping and the higher the natural frequency, the sharper will be the tuning and the greater the persistence of the vibrations. For example, a tuning fork is a vibrating system in which the damping is very slight; hence resonance between two forks will not occur unless the tuning is very exact.

On the other hand, in cases where faithful reception or reproduction of sound over a range of frequencies is required, resonance is very undesirable. Hence in such cases the system should have a natural frequency as far removed as possible from the frequency of the applied force, or alternatively, the system should be heavily damped.

If a current of air is directed across the mouth of an empty bottle, a definite musical note will result, and the pitch of the note can be raised by partially filling the bottle with water, or lowered by partially covering the opening with a card. If now a fork of frequency, say 512, is set vibrating and is held over the mouth of a bottle which is tuned to this frequency, the sound of the fork will be reinforced, because the air in the bottle is forced to vibrate. If the fork be loaded with wax so as to reduce its frequency, and the experiment repeated, the bottle will still reinforce the sound, and almost as strongly as before. Thus the bottle which acts as a resonator is not very selective. The reason is that the rate of damping of the vibrations of the air is very great and the vibrations die out very quickly.

In the case of a stretched string, the damping is as a rule greater than for a tuning fork, but much less than for a volume

of air ; hence the sharpness of resonance is intermediate between that of the fork and of the air.

EXAMPLES OF FORCED VIBRATIONS

The phenomena of forced vibrations can occur with any vibrating system, and are not necessarily confined to those vibrations which produce sound. As an example of forced vibrations we might consider the behaviour of two clocks which *nearly* keep the same time when on separate stands. If they are put on the *same* stand, the vibrations of the pendulums cause the stand to vibrate slightly, and each pendulum exercises a periodic force on the other. The result is that, after a short time, both pendulums settle down to the same rate of vibration and keep exactly the same time. The vibrations in each case are forced vibrations, and the final frequency is slightly different from the natural frequencies of both.

Sounding boards. If a stretched string, or a tuning fork, is set in vibration, the sound from it is very feeble, unless it is attached to a **sounding board.** When this is done, as in the case of the sonometer and stringed instruments, quite a loud sound will be heard. The string alone is in contact with only a small quantity of air, so that when it vibrates it sets a very small amount of air in vibration. Also since compression occurs on one side of the string at the same time as rarefaction on the other, inteference further reduces the effective sound. But when the string is stretched upon a board and plucked, the periodic force is communicated to the board through the bridges, and the board is forced to vibrate, though probably not at its own natural frequency. As the board is in contact with a comparatively large amount of air, the energy of the air set in motion in a given time is much greater than if there were no board. Of course, the vibrations of the string die away more rapidly when the sounding board is used, as the energy is radiated away more quickly.

It may be noted that a sounding board is a very important factor concerning the quality of the note. If the vibrations of the string itself are exactly all that are desired, then the bridges and the sounding board must reinforce the sound without change in character and convey them to the air. But if the vibrations of the string are defective in any way, then it is the duty of the bridge and board not only to reinforce but also to improve the vibrations so that the desired quality is obtained. It is not possible to specify what kind of sounding board is necessary for

any particular purpose, but it is probably true to say that the nature of the wood and its seasoning, also its dimensions, are very important.

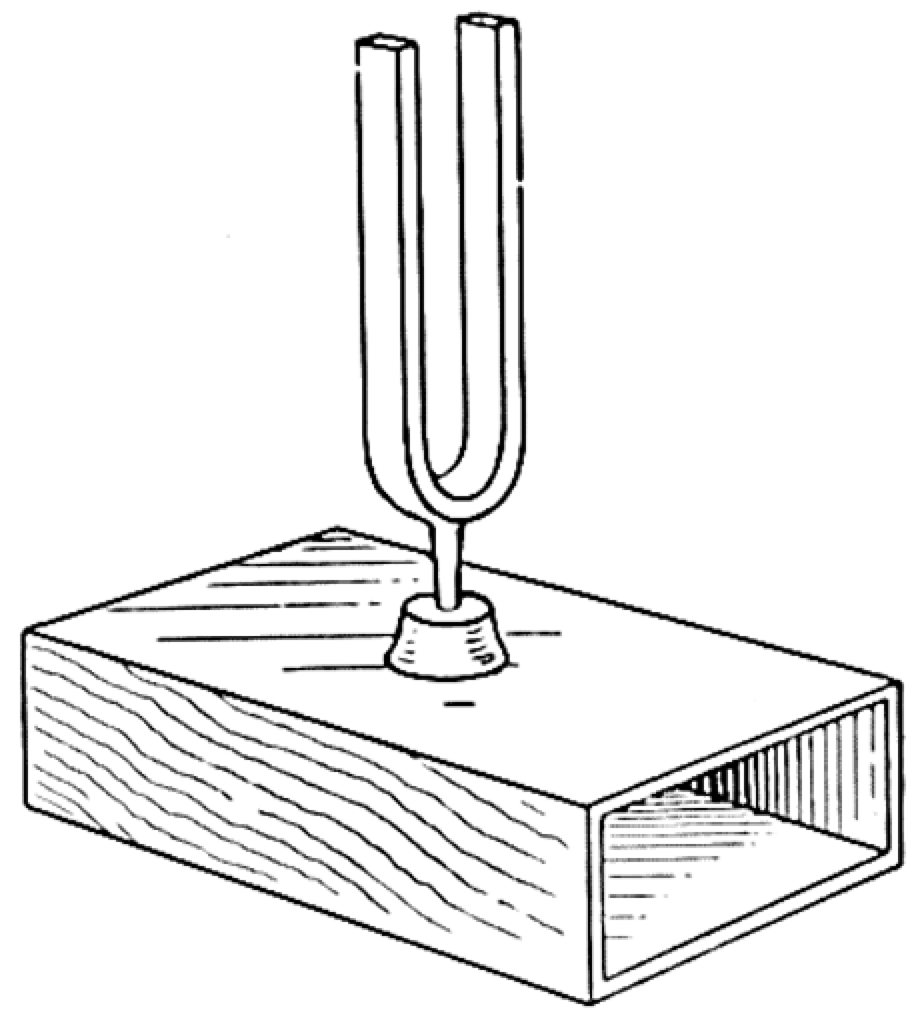
The case of forced vibrations produced by a tuning fork is similar to that of the string. If held in the hand, the sound of the fork is feeble, but if the stem is pressed upon a table, this is set into vibration, and so a greater quantity of air is also set vibrating. That this is not a case of resonance is seen from the fact that the same table will reinforce the sound when tuning forks of different frequency are used.

EXAMPLES OF RESONANCE

There are many examples of resonance. If two forks have exactly the same frequency and one is vigorously bowed and then held near the other, the second will be found to be vibrating; similarly, if two strings stretched upon the same board have the same frequency, then on bowing one of them, the other will vibrate. Soldiers marching over a suspension bridge always break step in case the period of vibration of their marching should coincide with that of the bridge and cause the latter to start vibrating dangerously. Again, when the "loud" pedal of a piano is depressed and a note is sung loudly, some of the strings will be found to be "resounding". This is because the natural frequencies of the strings are exactly equal to the frequencies of the vibrations of which the sung note is comprised. The "tuning" of a wireless receiver is an example of resonance between electrical circuits. When we "tune" the receiver, we merely adjust the natural frequency of the oscillations in the receiving circuit to that of the incoming waves. This type of resonance will be more fully dealt with later.

The phenomenon of resonance is of great practical importance, particularly to engineers, for if any quite small periodic force is brought to bear on some structure or machine having the same natural period, vibrations of very great magnitude may be caused. These vibrations are generally accompanied by stresses, and if the amplitude becomes great the stresses may exceed the elastic limit and the structure be damaged. Hence the period of the applied force should not agree with the natural period of the structure. An example where the effect of resonance is very marked occurs in ships fitted with reciprocating engines. Owing to the inertia of the reciprocating parts, a periodic force is applied to the hull, and if the period agrees with the natural period of the hull,

marked vibration is set up. If, however, the two periods are different, resonance no longer occurs and the vibration caused is very much reduced.

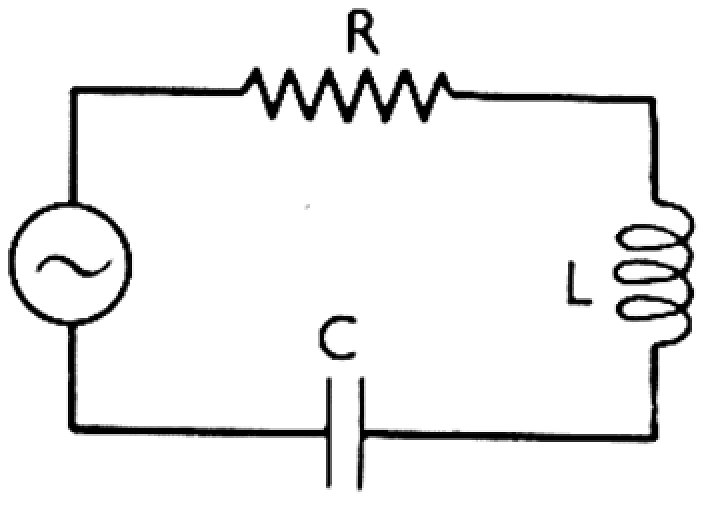


The intensity of the sound emitted by a tuning fork can be considerably strengthened by the aid of resonance. The prongs of a fork have not a very great area, and they are not capable of setting any great quantity of the surrounding air in violent vibration, for the air on the side to which the prong is moving can slip round the edge of the prong, and so partly fill up the rarefaction produced on the other side. In addition, the interference which takes place between the waves emitted from the two prongs reduces the intensity of the motion produced in the surrounding air. But if the fork is mounted on a hollow box of such a size and shape that the air inside the box has a natural frequency equal to that of the fork, when the fork is sounded resonance occurs and the intensity of the sound is greatly increased.

Electrical resonance. It has already been indicated that the phenomenon of resonance can occur with electrical vibrations, and various experimenters such as Sir Oliver Lodge and Hertz have demonstrated how such resonance effects can be obtained.

Theoretically, it can be shown that in a circuit containing an alternator, a resistance (R), a capacitance (C) and an inductance (L) connected as indicated in the diagram, the maximum current flowing in the circuit is given by :

$$I_{\text{max.}} = \frac{V_{\text{max}}}{\sqrt{R^2 + \left(2\pi nL - \dfrac{1}{2\pi nC}\right)^2}}.$$



The denominator here is called the **impedance**, and the quantity

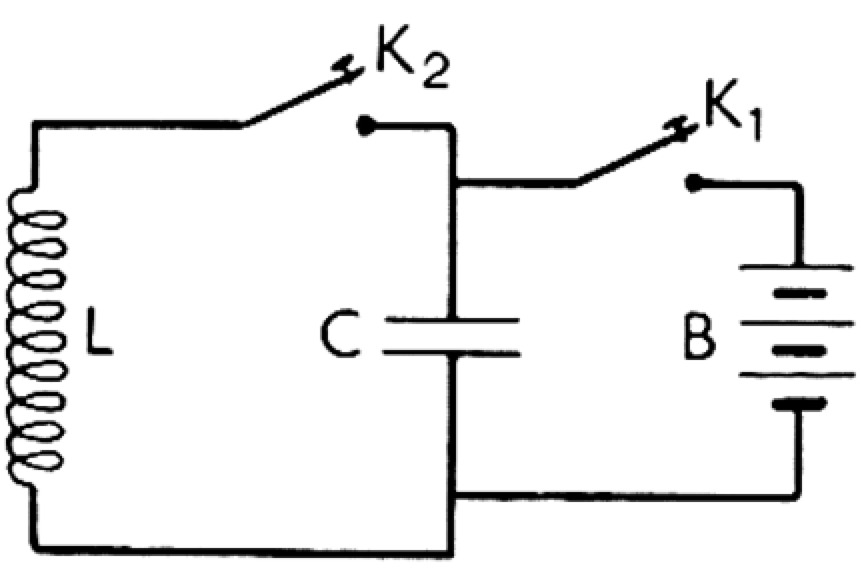
$$2\pi nL - \frac{1}{2\pi nC}$$

the **reactance**, n being the frequency of the oscillations. If

$$2\pi nL = 1/2\pi nC$$

the inductance effect neutralises the capacitance effect and the current has the same value as in a circuit of resistance R only. When this state of affairs occurs, the frequency n is $1/2\pi\sqrt{LC}$ and the period T is $2\pi\sqrt{LC}$. If the frequency n of the alternating current is fixed, we can arrange for this state to occur by altering L and C and the circuit is then said to be in resonance with the alternating current.

In the above case the oscillations are due to the applied alternations of the generator. But oscillations may occur as the result of a single disturbance in the electrical state of a circuit containing inductance and capacity, and such a circuit will have its own natural period depending on the values of L and C. Consider the circuit represented in the diagram. When K_1 is closed, the condenser is charged by the battery and electrical energy is stored in it. On opening K_1 and closing K_2, current flows through the LC circuit and continues until the condenser is discharged. But the inductance L prolongs the current, keeping up the flow in the same direction, with the result that C becomes charged in the opposite direction. Discharge again occurs with the direction of flow reversed, and the process is repeated until all the energy is dissipated. Thus a high-frequency oscillation is started in the LC circuit, and this would persist indefinitely if there were no resistance at all in the circuit, a state of affairs which, however, does not obtain in practice. This oscillating circuit has a natural frequency of its own which can easily be found. At the instant when maximum current is flowing, the P.D. across L is $2\pi fLI_{\text{max.}}$, and across C it is $I_{\text{max.}}/2\pi fC$. But these two P.D.'s are equal.



Hence $$2\pi fLI_{\text{max.}} = I_{\text{max.}}/2\pi fC\,;$$

and the frequency $$f = 1/2\pi\sqrt{LC}.$$

When an alternating E.M.F. is applied to a circuit, the impedance is $\sqrt{R^2 + (2\pi fL - 1/2\pi fC)^2}$, and this will be a minimum when

$$2\pi fL = 1/2\pi fC,$$

that is, when the frequency of the applied E.M.F. is equal to $1/2\pi\sqrt{LC}$. But this same expression $1/2\pi\sqrt{LC}$ gives the natural

frequency of a circuit of inductance L and capacitance C when the resistance is low. Thus the alternating current in any circuit due to an applied E.M.F. is greatest when the frequency of the applied E.M.F. is the same as the natural frequency of the circuit, that is, when the two are in resonance. Such circuits in practice are known as *series-resonance circuits*, and they are used in tuning wireless receivers.

Other cases of resonance. Since, according to the electromagnetic theory, light and heat are assumed to be identical with the electro-magnetic disturbances which are radiated from objects in which electrical oscillations are taking place, and since the electrons in the medium through which the energy passes are capable of vibrating with a definite period, resonance effects are found in both light and heat. For a full discussion of these the reader should consult appropriate books on the subject.

VIBRATIONS OF COLUMNS OF AIR

Closed and open pipes. When a tuning fork or some other source of definite frequency is sounded over the top of a column of air in a pipe, the air is forced to vibrate, and if the air column is of such a length that its natural period of vibration is identical with that of the source, resonance occurs and the sound is thereby strengthened.

It has been stated previously (p. 82) that stationary waves can be set up in both a closed pipe and an open one, and the vibrations of such a column of air are analogous to the longitudinal vibrations of a solid rod. Since a pulse of compression is reflected as a rarefaction at an open end and as a compression at a closed end, it is evident that a wave must travel twice the length of a closed pipe before the wave repeats itself. The *simple* theory indicates that in such a pipe, there must be a node at the closed end and an anti-node at the open end, and the length (l) of the pipe must be an odd multiple of $\lambda/4$, that is, $l = s\lambda/4$, where $s = 1, 3, 5$, etc. The overtones therefore form an *odd* harmonic series, the frequencies being given by

$$n = \frac{s}{4l}\sqrt{\frac{E}{\rho}},$$

where s is an odd integer. For a pipe *open* at both ends, each end must be an antinode and the length of the pipe a multiple of $\lambda/2$, that is, $l = s\lambda/2$, where $s = 1, 2, 3, 4$, etc. In this case, therefore, we have a *complete* harmonic series. It will further be noticed

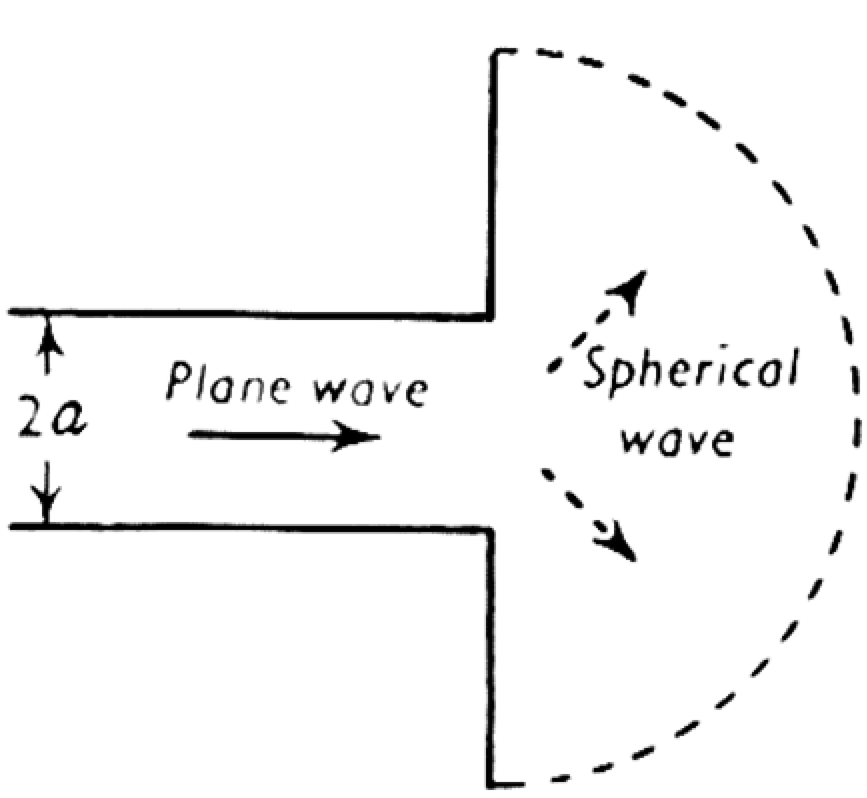
that, according to the simple theory, the frequency of the fundamental note of an open pipe should be twice that of a closed pipe of the same length (but see below).

Extension of simple theory. In the above discussion, it has been assumed that there is an antinode at the open end of the pipe; but this is never quite true. A *true* antinode is a point of zero pressure variation and maximum displacement amplitude. Now at the open end of a tube the stationary plane waves inside are changing to spherical progressive waves outside; consequently sound energy is being radiated in all directions from the end of the tube. The presence of this radiation involves the existence of excess pressure, which therefore means that there can be no antinode just at the end; it is a short distance beyond. Hence the resonating column of air is longer than the length of the tube by an amount, known as the **end correction**, equal to the distance between the end of the tube and the position of the true antinode. Therefore, when a closed pipe is sounding its fundamental, the true relationship between length of pipe and wave-length is given by $(l+e)=\lambda/4$, where e is the end correction. In the case of a pipe open at both ends, the relationship is given by $(l+2e)=\lambda/2$, since there is an end correction at both ends. Thus the interval between the notes given by an open and a closed pipe of the same length will be

$$\frac{4(l+e)}{2(l+2e)},$$

and this is less than 2. Hence, the open pipe, instead of giving the octave of the note given by the closed pipe, gives a somewhat lower note.

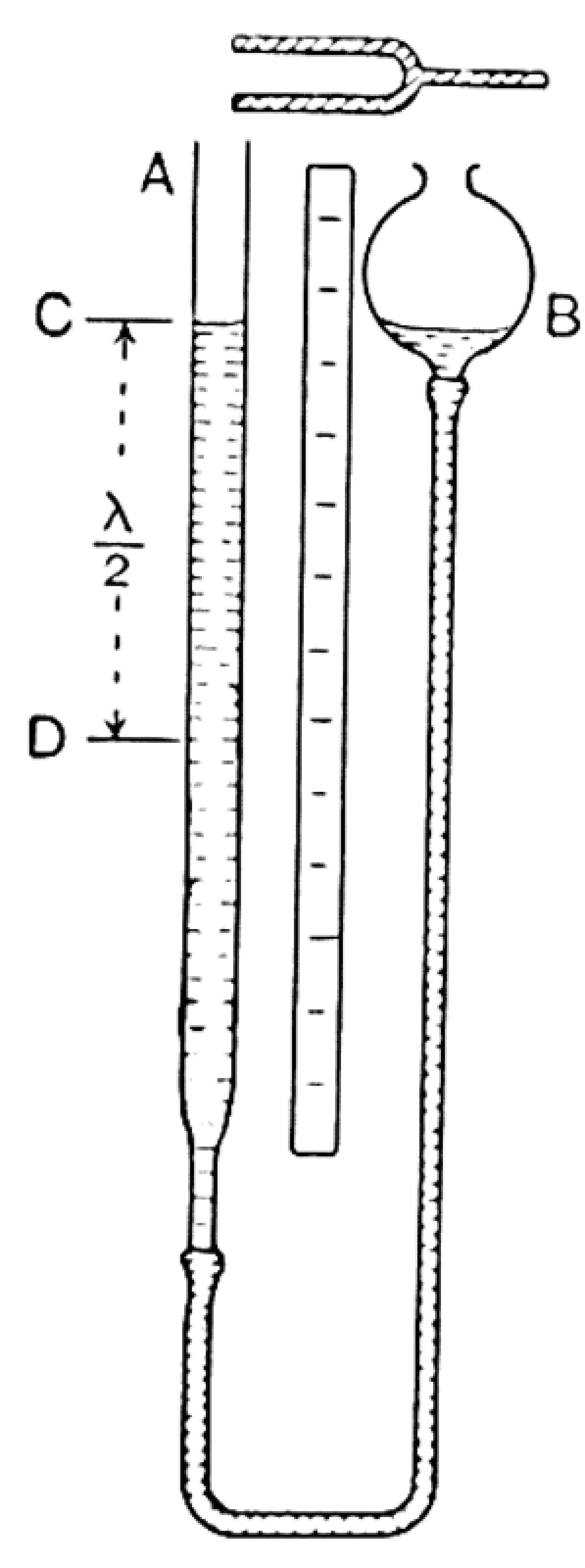
At this stage we might consider briefly the effect of having a flange at the end of the pipe. In the diagram is shown a pipe of diameter $2a$, small compared with the wave-length, and with the open end fitted with a theoretically infinite flange so that the spherical waves radiate only in the hemisphere to the right. It can be shown that, under the conditions stated, the ratio of the radiated intensity to the incident intensity at the end of the tube is approximately $8\pi^2a^2/\lambda^2$, while if the flange be removed, the corresponding ratio is approximately $4\pi^2a^2/\lambda^2$. Thus,



if the flange is removed, we get *less* radiation—a strange result at first sight, for we should expect that the opening to the free air would allow the energy to escape more easily than when a flange is present. But it must be remembered that as the area is increased so is the amount of reflection, and the amount radiated out becomes less.

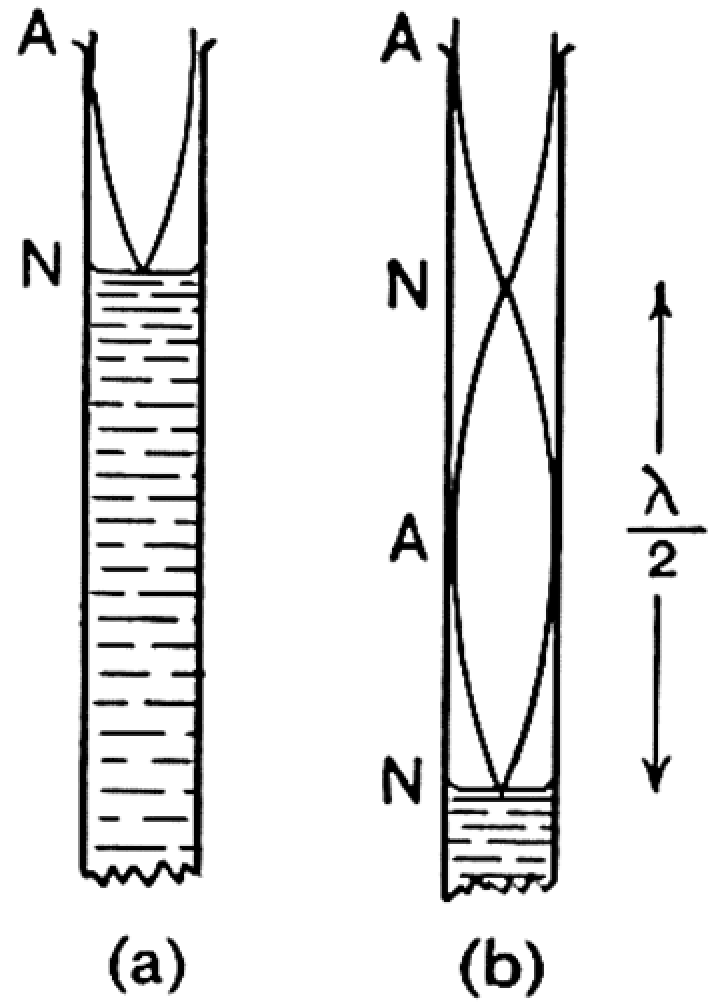
By giving values to a and λ in the above expression, we can obtain an indication of the amount of radiation at the end of the pipe. For example, if $a=1$ in. and $\lambda=50$ in., we get for the ratio of the radiated intensity to the incident intensity a value of 3·2 per cent., which indicates how small is the amount of sound energy dissipated from the open end of a pipe when the radius is small compared with λ, a necessary assumption in the above discussion. It follows then that a small opening is rather inefficient for the egress of sound, and incidentally also for the entrance of sound. It would appear that if one wanted to get more sound out of a pipe the thing to do is to increase the diameter of the opening; but if this is done suddenly, reflection is encouraged. If, however, the end is built up bell-like, the transmission from the inside to the outside and vice versa is increased, and in this connection it may be noted that a funnel-shaped end acts not so much to collect the sound as to supply easier ingress.

So far as the value of the end correction is concerned, it has been calculated that without a flange it is $0{\cdot}57r$, where r is the radius of the pipe; but if the tube has a flange at the end, the correction to be applied is $0{\cdot}82r$. The correction to the end of a pipe can also be expressed in the form A/c_0, where A is area of cross-section of the pipe and c_0 is the *conductivity* of the opening of the pipe (see p. 174).



EXPERIMENTAL WORK

The principle of resonance can be used to determine the velocity of sound in air and other gases, and for the purpose the apparatus shown in the diagram can be set up. Raise the reservoir B until the water in the tube stands near the top of the resonance tube A. Set the fork vibrating and hold it over the mouth of A, at the same time adjusting the length of the air column in A by moving the



reservoir up or down until the first position of resonance is obtained; mark the position (C) on the tube corresponding to this length. Now lower the reservoir and consequently the water level in A until the air column again resonates with the fork, and let the water-level now be at D. CD is equal to $\lambda/2$, from which the value of λ can be obtained and the velocity calculated from the relationship $V = n\lambda$. By obtaining the two positions of resonance there is no need to worry about any end correction. For, if l_1 and l_2 are the lengths of air columns corresponding to the positions of resonance, we have

$$(l_1 + e) = \lambda/4 \quad \text{and} \quad (l_2 + e) = 3\lambda/4$$

from which

$$(l_2 - l_1) = \lambda/2.$$

Since the velocity of sound is influenced by temperature, it is of course necessary to read the temperature at the time of the experiment.

The above apparatus can also be used to determine the velocity of sound in a gas such as carbon dioxide. The tube A should be filled by the method of downward displacement, but since some of the gas will dissolve in the water, care should be taken to ensure that the tube is full of the gas and not a mixture of gas and air. The ratio of the experimental results for air and carbon dioxide can be checked by calculation. Since, in general,

$$V = \sqrt{\frac{\gamma \rho}{\delta}},$$

we have

$$V_{\text{air}} = \sqrt{\frac{1{\cdot}41\rho}{\delta_{\text{air}}}} \quad \text{and} \quad V_{CO_2} = \sqrt{\frac{1{\cdot}29\rho}{\delta_{CO_2}}},$$

and if we consider ρ to have the same value in both cases, we have:

$$\frac{V^2_{\text{air}}}{V^2_{CO_2}} = \frac{1{\cdot}41 \times \delta_{CO_2}}{1{\cdot}29 \times \delta_{\text{air}}}.$$

It is left as an exercise for the student to consider how the experiment would have to be modified to find the velocity in a gas less dense than air. It is important to realise however, that in any

calculation using the relationship $V = \sqrt{\gamma\rho/\delta}$, the correct value of γ should be taken. In the case of the rare gases, helium, argon, neon, krypton and xenon, which are regarded as monatomic, the value of γ is about 1·66. For gases like hydrogen, oxygen and nitrogen, regarded as diatomic, and also the mixture constituting air, the value is 1·41, while for triatomic gases like carbon dioxide it is 1·29.

In the experimental work above, it is instructive to use a graphical method to find both the velocity of sound and the end correction. A series of forks should be chosen and the corresponding lengths of air column in resonance found. If a graph of l (ordinates) against $1/n$ (abscissae) be plotted, the result should be a straight line, the slope of which is $V/4$ for a closed tube. The negative intercept gives the value of e, for since

$$l + e = \lambda/4 = V/4n,$$

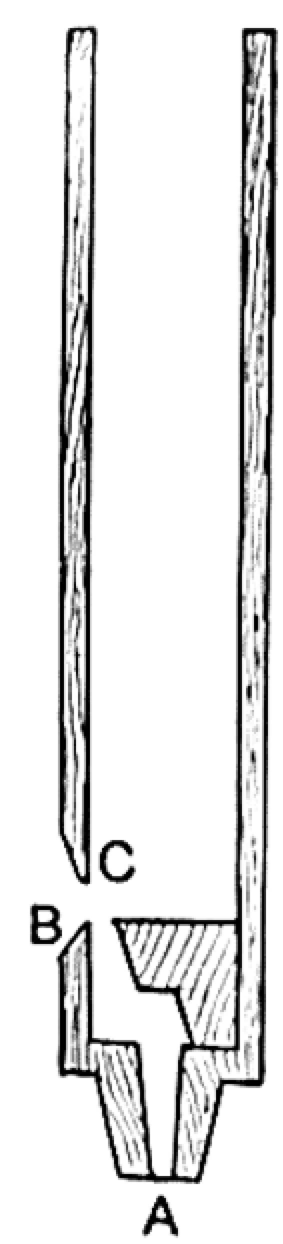
we have :

$$l = \frac{1}{n} \cdot \frac{V}{4} - e.$$

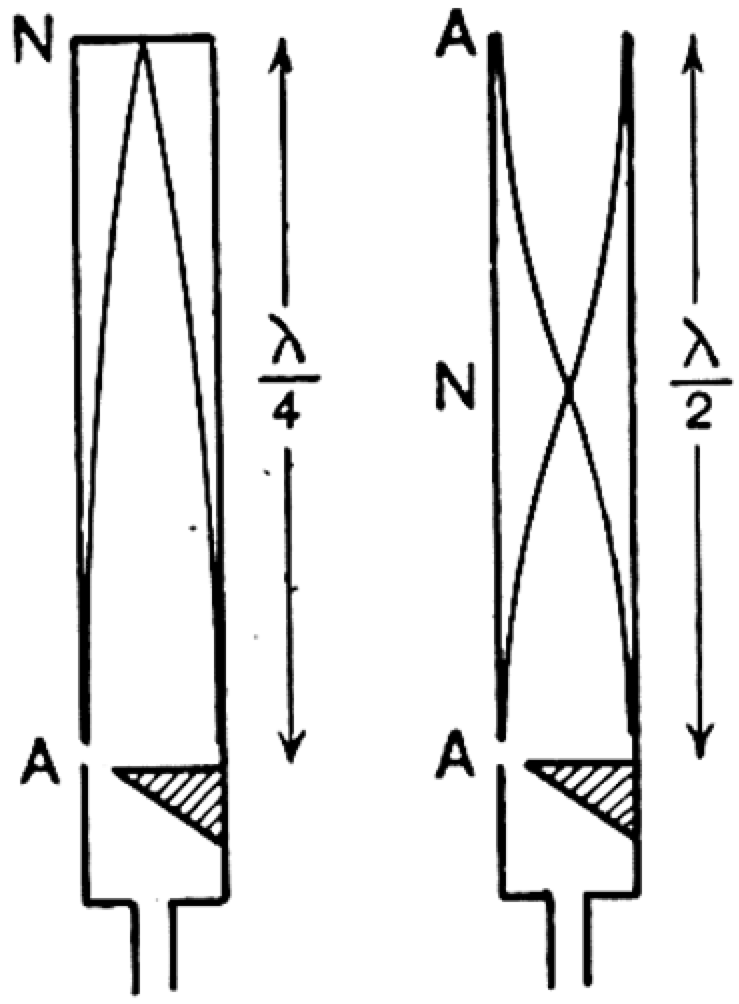
ORGAN PIPES

The most familiar example of the vibration of columns of air occurs in the case of organ pipes, though of course the organ is only one of many belonging to the class of *wind* instrument. A further reference to such instruments will be found in Chapter XI.

The organ comprises a series of pipes, closed and open, in which air is caused to vibrate in a manner similar to that described earlier. There are two main types of organ pipes: (1) **flue pipes**, (2) **reed pipes**.



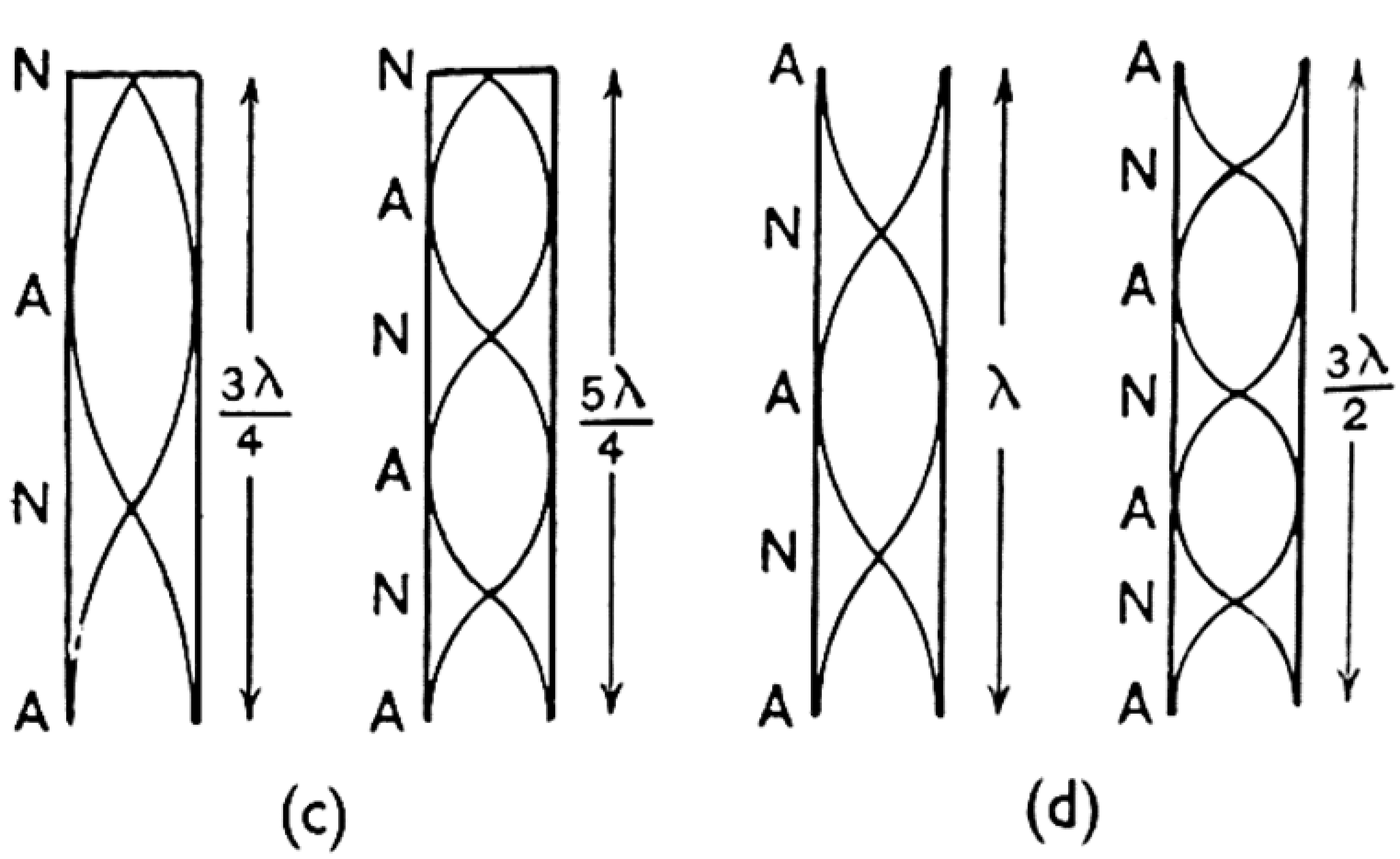
(1) **Flue pipes.** These may be made of metal, in which case they are usually cylindrical in shape, or of wood, in which case they are square in section. Air enters the pipe at A and issues from the slot B, known as the *mouth*, in a fine jet which impinges upon the "lip" C. Here a feeble compression or rarefaction is started which will travel up the pipe and be reflected at the other end. When once the vibration is started, it is increased and maintained by the energy of the jet, which is derived from the air forced through A under pressure. Consequently, a steady vibration of a definite frequency is set up in the pipe and a sound of definite pitch results.



(a) (b)

The simplest mode of vibration of the air in a closed pipe is indicated in diagram (*a*). According to the simple theory, there must be a node at the closed end and an antinode at the "lip"; hence the length of the pipe is *one-quarter* of the wave-length of the note sounded. The corresponding case for the open pipe is indicated in diagram (*b*) and it will be seen that the length of the pipe is *one-half* of the wave-length. In each case the note is the fundamental, and as we have seen previously, the fundamental note given by the open pipe is the octave above that given by the closed pipe.

An air column, like a stretched string, can vibrate in more than one way at the same time. The different possible modes of vibration are all subject to the condition that there must be a node where the pipe is closed and antinodes where the air in the pipe is in contact with the outside air. The next two simple modes of vibration for closed and open pipes are represented in diagrams (*c*) and (*d*). These more complicated modes of vibration give rise to the overtones. It will be seen that in the closed pipe the overtones are $3n$, $5n$, $7n$, etc., where n is the frequency of the fundamental; thus the even-number frequencies are missing. In the case of the open pipe, the frequencies of the overtones are $2n$, $3n$, $4n$, etc., that is, we have a complete harmonic series. Hence the quality of the note given by an open pipe is different from that given by a closed pipe. Closed pipes are usually stopped by a plunger at the upper end, which can be slightly raised or



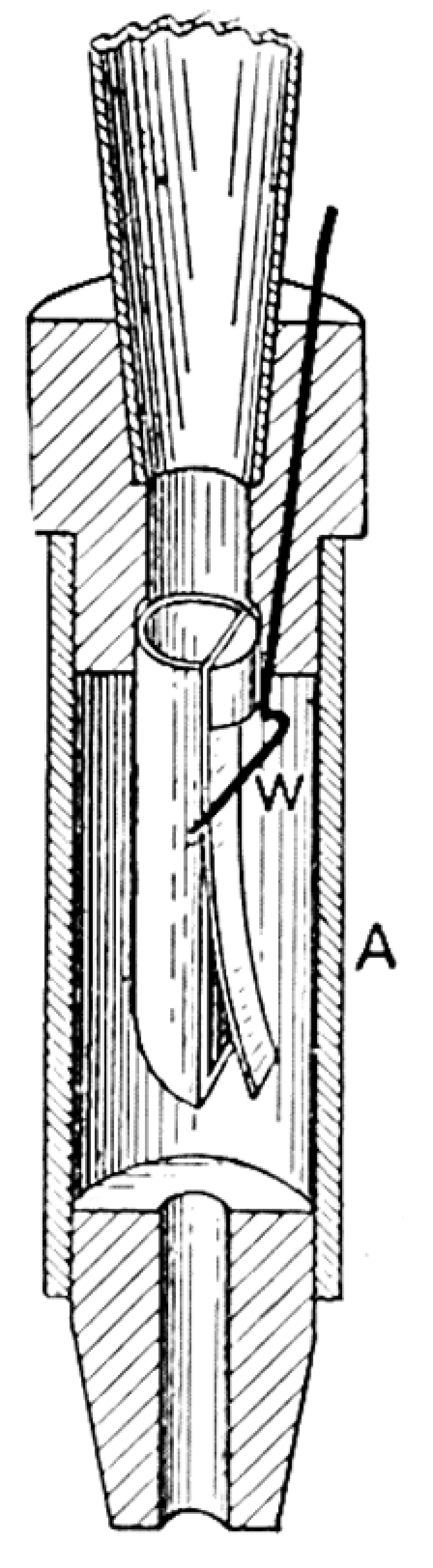
(c) (d)

lowered to alter the effective length of the air column. This affords a convenient method of tuning such pipes.

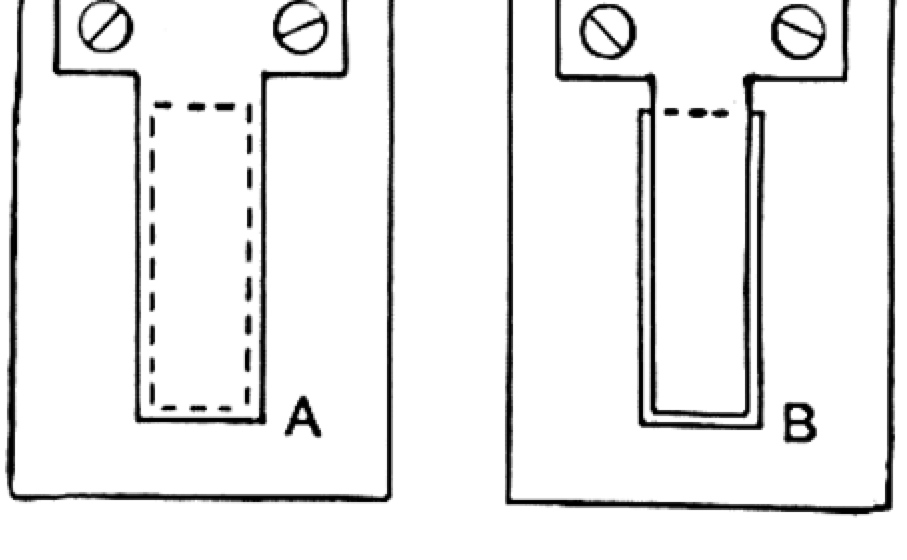
Open pipes are sometimes provided with a short sleeve at the upper end which can be moved up or down to alter the effective length for tuning purposes.

Wooden open pipes are generally tuned by bending a metal piece which shades the top ; thus the opening at the top can be varied in size as desired. The pitch of the note is flattened by lowering the flap. This is equivalent to *lengthening* the pipe, for the virtual open end is always beyond the actual end, and the *less* open the end is, the greater the discrepancy between the two. In this connection it should be noted that whereas the end correction at the open end of a pipe without a flange is $0{\cdot}6r$, that at the mouth which is a much smaller aperture may be as much as $1{\cdot}36$ times the diameter. To sharpen the pitch of the note of the pipes under consideration, the flap is raised and the pipe virtually shortened.

If an organ pipe is not very narrow, the note when blown gently is very nearly a pure tone. If the pipe is narrow or the wind pressure is great, the pipe will give a note in which the first overtone is very marked, while if the pipe is blown *very* strongly the first and second overtones are so strong as to drown the fundamental.



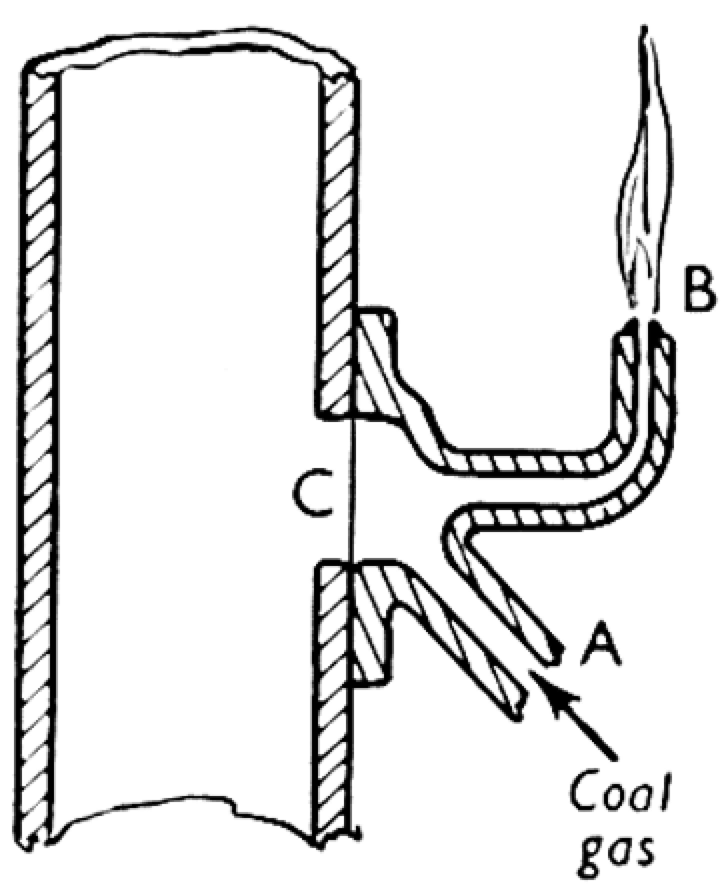
(2) **Reed pipes.** A reed consists of a flexible strip of metal which wholly or nearly covers the aperture through which the air passes to the pipe. If the reed completely covers the aperture, it is called a **beating reed** as in (A) ; but if it nearly, but not quite, closes it, it is called a **free reed** as in (B). The beating reed is always curved outwards, so that as the air pressure closes it, the closing does not take place suddenly, but gradually. This is necessary, for otherwise the note produced would be very harsh.



Beating reeds are used almost exclusively in organ pipes, and the free reeds are used in mouth-organs and harmoniums. The pitch of the note emitted is governed mainly by the frequency of the reed, but in the case of the organ the vibration of the air column and the reed react on each other, the resulting note being determined by them both.

A reed is tuned by means of a wire *W* which can be pushed further down to restrict the motion of the reed. A free reed is usually tuned by carefully scraping away metal. If the scraping is done at the tip of the reed, the note is sharpened ; if at the base of the reed, the note is flattened.

Experimental work. The positions of maximum pressure disturbance of the air in an organ pipe may be conveniently shown by means of manometric flames let into the side of the pipe. A manometric capsule or flame is shown in the diagram. Coal gas from the supply enters a small chamber by the pipe *A* and leaves by the jet *B* where it burns with a tall thin flame. This flame attachment is connected with a hole in the organ pipe which is covered by a thin india-rubber membrane *C*. When the air in the pipe is in a state o steady vibration, the variation in pressure drives the membrane in and out.



This communicates a varying pressure to the gas, and the jet jumps up and down correspondingly. The variations occur so rapidly that, owing to the persistence of vision, the eye cannot follow the movements of the flame without the aid of a rotating mirror driven at a constant speed. But when this is used, the image will appear serrated when the jet is in the position of a node, though when it is at one antinode the flame will be steady. Also, the difference in the effect produced when the pipe is blown gently and when it is blown hard can be demonstrated with the apparatus above. In the second case there will be twice the number of serrations observed in the first case, showing that the frequency of the note is doubled.

Another method of finding the positions of the nodes and antinodes is to lower into the pipe a small paper tray containing some fine sand. When the tray is in the position of an antinode the sand grains move about on the paper, but at a node there is no movement. To observe the effects, it will be necessary to have one of the sides of the pipe transparent.

Electronic organ. A short reference must now be made to the modern electronic organ which is rapidly coming into use especially in places where space is limited. One such instrument, known as the Compton Electrone, has been invented by Mr. L. E. A. Bourn and developed and built by the John Compton Organ Co. Ltd.

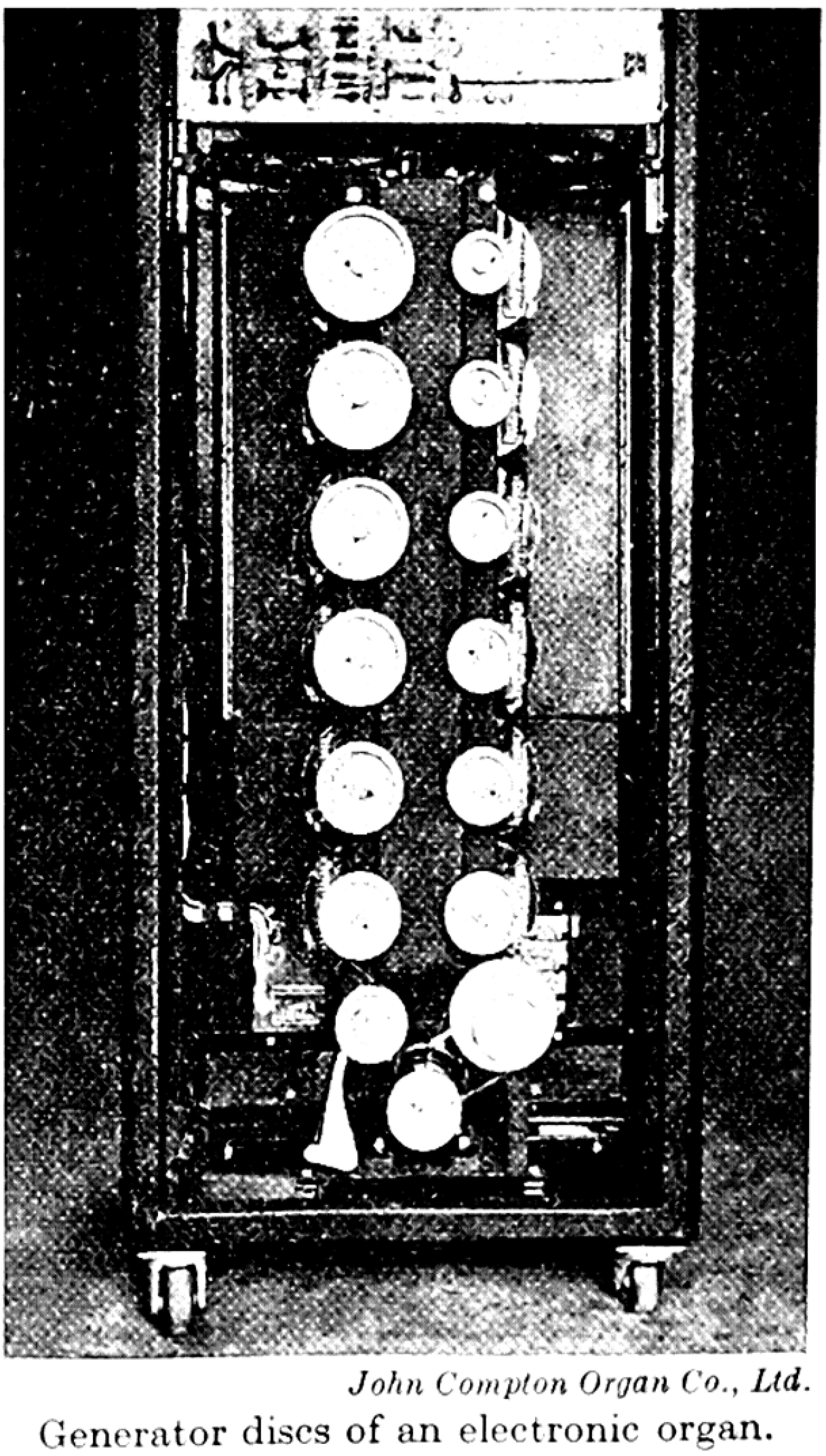
The whole instrument consists of three components, the console, the generating cabinet and the sound cabinet which houses the loudspeakers, and all three are easily transportable.

The console contains all key, stop, piston and coupler actions, and complies in every respect with modern organ-building technique. These actions are operated electrically; a rectifier unit converts the A. C. supply to 16-volt direct current which supplies all the current necessary. The generating cabinet contains the generator unit, manual relays and amplifiers, and is supplied by a separate mains plug. The method of obtaining the electrical oscillations, which are amplified and eventually fed into the loudspeakers, is by electrostatic induction. Each frequency is generated by the variation of electrical capacitance caused by the rotation of one member (rotor) in close proximity to a stationary member (stator), these two, together with a pick-up plate, constituting the generating unit. The stator plate is engraved with a complicated pattern of wave-forms representing the fundamental of the particular note together with its harmonics, and the black lines (see Plate 5, facing p. 262) are insulating barriers between one element and the next which are conducting surfaces. The rotor is in two parts, each having projecting radial lines on the face to suit the wave-forms, and the complete rotor runs between the stator and the pick-up plate with a very small air gap between each part. The pick-up plate as well as each half of the rotor is divided into two annular sections corresponding to the bass and treble notes. Thus two wires emerge from the pick-up plate and connect with the bass and treble sections of the amplifier.

Due to the relative motion of the stator and rotor, the electrical capacitance between the two varies in a manner corresponding to the wave-form. The mere variation of capacitance alone does not generate an alternating voltage, but it will do so if a D.C. potential difference is applied between the two members. Thus, although the generators are rotating all the time, no sound is generated except when a key is depressed, which then applies the necessary polarising potentials to the members of the particular note. All the wave-forms are connected to the polarising circuit

and the rotor connected to the amplifier, and when a stop is drawn, say clarinet, all the wave-forms necessary to build up the clarinet note are brought into action ; also matters are arranged so that unless stops are drawn the keys are inoperative.

Twelve such disc units comprise the whole generator assembly, each corresponding to the twelve semitones of the musical octave.



John Compton Organ Co., Ltd.

Generator discs of an electronic organ.

Each is identical in construction and each has a driving pulley of a diameter which causes it to run at its semi-tonal speed. All are driven by a single belt, and a spring-loaded jockey pulley keeps the belt at constant tension. The belt also passes over a second jockey pulley which is mounted eccentrically on a separately mounted wheel. The wheel is made to rotate by the action of the tremulant stop on the console, and when this is drawn the normally uniform velocity of the main driving belt is modulated,

and a varying velocity of rotation is imparted to all the disc units; this causes the tremulant effect.

All types of sound cabinets have two separate loudspeaker units, one to handle the bass frequencies, and the other the treble frequencies. Tonally, the electronic organ provides a substantial basis of diapason tone, a generous supply of solo orchestral effects (clarinet, flute, etc.) and a carillon. Certainly this type of organ has come to stay, though it probably will not replace what is generally regarded as the " king of instruments ", the pipe organ.

Vibrations in conical pipes, etc. No account of the vibration of air in pipes would be complete without some reference to pipes other than cylindrical in shape, since such differently shaped pipes and horns are largely used in musical instruments, loudspeakers, etc.

The simplest type of these pipes is the straight conical pipe. In such a pipe closed at its vertex there is an antinode at the open end, and other antinodes occur equidistantly just as if the whole pipe were cylindrical. The nodes on the other hand are slightly displaced towards the apex from the positions they would occupy in a cylindrical pipe. It is noteworthy that in the conical pipe the overtones form the *complete harmonic series*, having frequencies n, $2n$, $3n$, etc., thus differing from a cylindrical pipe closed at one end. Therefore, in the case of a conical pipe excited by means of a reed in the mouthpiece, which is equivalent to a closed end, the whole harmonic series of overtones may be present. This is the case in instruments such as the oboe, which is a conical tube with one end open and the other closed by a reed. With a clarinet, although one end is open and the other closed by a reed, only the odd overtones are present as a rule; this is because the tube of the instrument is cylindrical. A flute and a piccolo are also cylindrical, but they are open at both ends; hence the full series of overtones may be present.

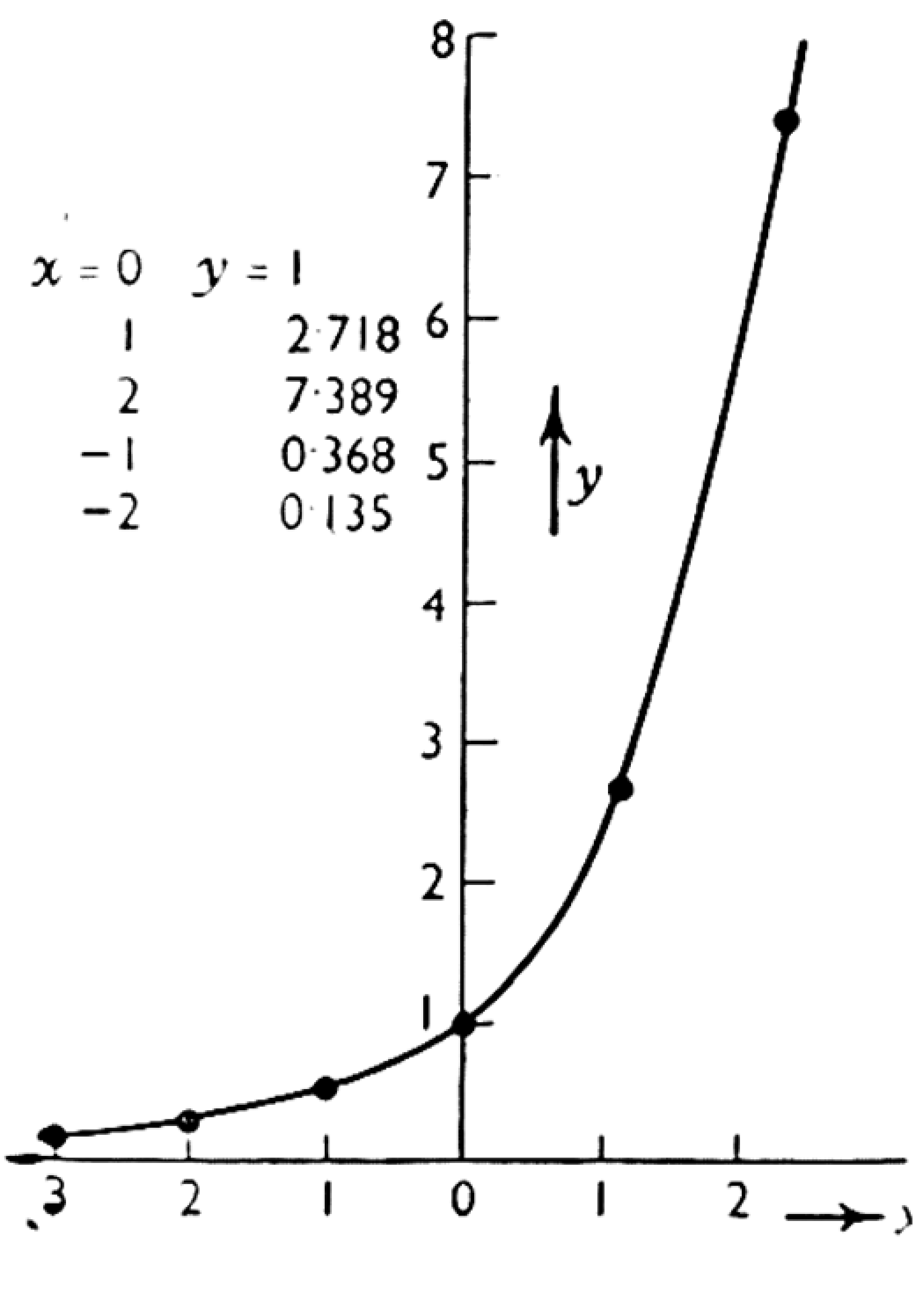
If, as is done in some musical instruments, the open end is made bell-shaped, this, together with the cupped mouthpiece, introduces slight departures from the ideal simple cone. In order to counteract the disturbing effects of the bell and mouthpiece, the end of the pipe is curved in the shape of a hyperbola. All brass instruments with cupped mouthpieces fall under this class, for here it is necessary to have the overtones as nearly in the harmonic series as possible. Without the curvature it is difficult to model a pipe so as to retain all the overtones in their strict harmonic relation.

Helmholtz showed that if the bell mouth of a cylindrical pipe is shaped as a hyperbola such that the radius of the mouth is

$\sqrt{2}$ times that of the cylindrical part, the end correction is zero and the overtones would be exactly in the harmonic series.

Loudspeaker horns. The sound produced by an unaided sound box is weak, but it can be amplified by the use of a horn. The ideal horn should of course give maximum transmission and minimum distortion, though even if a distortionless horn can be found, the resulting sound heard may be distorted since the drum of the human ear is an asymmetrical vibrator. If a horn is long and narrow, it certainly confines a large volume of air to suffer compression under the action of the vibrating diaphragm; but there is considerable reflection at the open end of the horn and the transmission is reduced accordingly. On the other hand, if the open end is wide, although the energy can get out, the intensity suffers, since the air can easily move away from the diaphragm without being compressed much.

The mechanism of any loudspeaker represents a load, heavy in comparison with the fluid air, and some additional matching device is required to increase the load of the air through which the acoustic energy is to be propagated. One such matching device is the horn, and of course it is essential that this should be as uniformly efficient as possible throughout the range of audible frequencies and that it should not introduce any distortion. It is found that the best shape for a horn is one flared upon an exponential law. A true exponential surface will satisfy the equation $y = E(x)$, where $E(x)$ is the sum of the infinite series



$$1 + x + \frac{x^2}{1\,.\,2} + \frac{x^3}{1\,.\,2\,.\,3} + \dots\,.$$

If $x = 1$, then the sum of the series is 2·7183, and this is denoted by the symbol e; if x is any positive rational quantity (integral or fractional), we have $E(x) = e^x$. The graph representing a true exponential curve is shown in the diagram. The law upon which the design of "exponential horns" is based in practice is

$$A_x = A_0 e^{Bx},$$

where A_0 is the area of cross-section of the horn at the throat, A_x is the area of cross-section at distance x from the throat and B is a constant determining the rate at which the horn flares with distance from the throat. Such a horn is popularly defined as one in which the ratio of areas of cross-section, taken at successive equal intervals along the axis, are at all points equal. That this generally accepted definition is in accord with the more formal law is shown as follows.

Area at distance $x-1$ from throat $= A_0\, e^{B(x-1)}$. ....(1)

" " " x " " $= A_0\, e^{Bx}$. ....(2)

" " " $x+1$ " " $= A_0\, e^{B(x+1)}$. ....(3)

$\therefore$ Ratio of (2) to (1) is

$$\frac{A_0\, e^{Bx}}{A_0 e^{B(x-1)}} = e^{Bx-Bx+B} = e^B$$

and ratio of (3) to (2) is

$$\frac{A_0\, e^{B(x+1)}}{A_0\, e^{Bx}} = e^{Bx+B-Bx} = e^B$$

which is a constant.

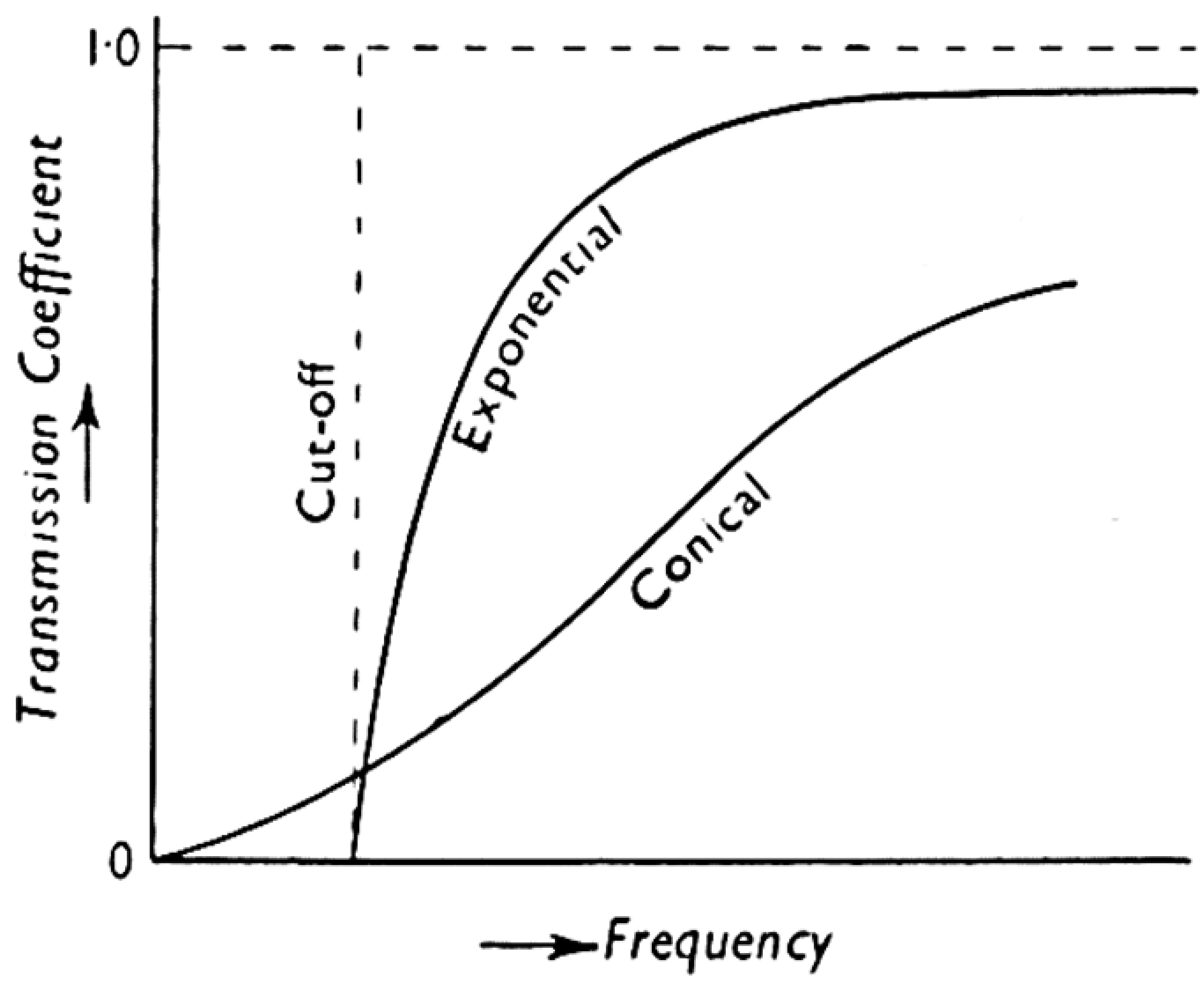
One of the most important features of an exponential horn is that a plane wave-front propagated along its axis suffers no distortion, by reason of the fact that reflections from the walls are in phase with the propagated wave.

In order to convey the entire range of audible frequencies, it is essential that the horn should be long (ideally it should be of infinite length), and in practice this length is determined by several factors. In the first place, every horn has a "cut-off frequency" below which it will not respond, and the lowest frequency which can be transmitted from the mouth of the horn into free air without the incidence of unwanted resonances is dictated by the mouth area A_x. Such resonances are negligible when $A_x = \pi\lambda_c^2/9$ and are not excessive until $A_x < \pi\lambda_c^2/16$, where λ_c is the wave-length corresponding to the cut-off frequency. Thus the size of A_x depends on the cut-off frequency desired.

Also the optimum value for the throat area A_0 has to be decided. The size of the throat controls the loading thrown on to the speaker diaphragm; by decreasing A_0 the effective loading is increased and so the overall matching efficiency. But if A_0 is made small, since A_0 is decided as indicated above, x, the length of the horn, will be increased. Moreover, the smaller the throat, the greater will be frictional losses and also distortion due to

excessive pressures in the throat. The final selection of A_0 is usually a matter of compromise. Having decided the cut-off frequency required and thus the appropriate value of the flare factor B, and having found the most desirable values for A_0 and A_x, substitution in the equation $A_x = A_0 e^{Bx}$ will enable x to be determined.

If x is inconveniently long, it is possible to fold the horn up to make a " folded exponential horn ", but the additional convenience is gained at the price of some distortion, since folding inevitably destroys the exponential flare for some part of the length. The effect of the rim on the outside edge of the horn, which acts as a " baffle " plate, was referred to on p. 60.



Comparison of the frequency characteristics of a conical and an exponential horn of the same overall dimensions.

The oldest type of horn, and until recent years, probably the most widely used, is that of a conical shape. This type is defined by the equation $A_x = A_o\left(1 + \frac{x}{x_o}\right)^2$, when A_o is the area of cross-section of the horn at the small end and x_o is the distance from the small end to the apex of the cone. It is found however, that the transmission of the exponential horn is much superior to that of the simple conical horn, in the low-frequency range particularly. The transmission coefficient of *any* horn may be regarded as a measure of the efficiency of the horn as compared with a simple direct generator of plane waves, and the diagram indicates in a general way the performance of the two types of horn having the same over-all dimensions.

RESONATORS

A resonator is a vessel of any shape containing air with an opening to the external air in the form of a narrow neck or orifice, and a typical resonator is that due to Helmholtz (see p. 131).

The action of a resonator is rather different from that of a resonance tube closed at one end as was described on p. 159. The vibration of the air column in a resonance tube resembles the vibrations of a helical spring with one end fixed, after the spring has been stretched and then released. On the other hand, the vibrations of a typical resonator are analogous to those of a system consisting of a light spring fixed at one end, with a heavy mass suspended from the other end.

Consider a bottle containing air closed by a piston p without fricton in the neck of the bottle. Let v be the volume of the bottle below the piston, m the mass of the piston and A its area of cross-section. Let the piston be originally in the position of equilibrium, and let the pressure outside be p_0 and that inside p.

Then
$$p \,.\, A = p_0 \,.\, A + mg.$$

If now the piston be displaced downwards a distance x so quickly that the change may be regarded as adiabatic, a new pressure p_1 will be generated such that $p_1(v - Ax)^\gamma = pv^\gamma$. This may be written

$$p_1 = p \Big/ \left(1 - \frac{Ax}{v}\right)^\gamma = p + \frac{p\gamma A}{v} \cdot x \quad \text{if } x \text{ is small.}$$

The total force acting downwards now is :

$$mg + p_0 A - p_1 A = (p - p_1) A = -\frac{p\gamma A^2}{v} \,.\, x.$$

But this force is
$$m\,\frac{d^2x}{dt^2}.$$

Thus we have
$$m\,\frac{d^2x}{dt^2} = -\frac{p\gamma A^2}{v} \,.\, x.$$

Hence the motion is simple harmonic motion and the time of vibration about the position of equilibrium is

$$2\pi\sqrt{\frac{mv}{p\gamma A^2}}.$$

The frequency n is therefore given by

$$n = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{p\gamma A^2}{mv}}.$$

In this calculation we have assumed that the pressure is the same throughout the air during the oscillations. This is not true, since time is required for the transmission of the pressure. Also we have assumed that the character of the compression is adiabatic. This is very approximately true, especially as the neck of the bottle is narrow and heat will not easily escape from the bottle or be transmitted to it. The piston therefore will behave approximately according to the formula with a period of oscillation given by the expression above.

In a typical resonator the layer of air in the neck of the bottle takes the place of the piston ; it corresponds to the mass referred to on p. 173, while the air in the cavity corresponds to the light spring. Now the mass of air in the neck is given by $m = \rho l A$, where ρ is the density of the air, A is the area of its cross-section and l is the length of the neck, and since $p\gamma = E$, the modulus of elasticity, we may rewrite the expression for frequency as follows :

$$n = \frac{1}{2\pi}\sqrt{\frac{p\gamma A^2}{mv}} = \frac{1}{2\pi}\sqrt{\frac{EA^2}{v\,.\,\rho l A}} = \frac{1}{2\pi}\sqrt{\frac{EA}{v\,.\,\rho l}} = \frac{V}{2\pi}\sqrt{\frac{A}{v\,.\,l}},$$

since $V = \sqrt{E/\rho}$ where V is the velocity of sound in air. The quantity A/l is known as the conductivity of the orifice, already referred to on p. 161.

The lines of flow of an incompressible fluid (and the air in the neck of the resonator may be regarded as one) through the aperture due to a difference of pressure are of the same form as the lines of flow of an electric current due to a difference of potential in a uniformly conducting medium, if the boundaries of the aperture are non-conductors. Hence the analogy between electrical conductivity and the conductivity of the various forms of apertures in resonators.

It will be seen from the relationship

$$n = \frac{V}{2\pi}\sqrt{\frac{A}{v\,.\,l}},$$

that, assuming V and A/l to be constant,

$$n^2 v = \text{constant}.$$

This can be verified experimentally in the following way. Procure an ordinary good-size medicine bottle or a small Winchester bottle to use as the resonator, and first calibrate it for volumes by pouring in water from a burette to various depths and measuring the height of the water surface above the top of the

bench on which the bottle rests. This should be done for a series of intervals up to the base of the neck. If the bottle is marked in "table-spoons", these marks will serve for heights. Tabulate the results as under:

Height of water	Volume of water poured in	Volume of air above height in column 1
h_1	v_1	$v_n - v_1$
h_2	v_2	$v_n - v_2$
$\vdots$	$\vdots$	$\vdots$
h_n	v_n	0

v_n denotes the capacity of the bottle up to the base of the neck. Draw a graph showing the relation between heights as abscissae and volumes (column 3) as ordinates. From this graph the volume of air in the bottle corresponding to various heights of water can be found. Start the main experiment with the bottle empty, and gradually pour water in until resonance occurs between the air in the bottle and a 128-tuning fork. The position of resonance can be confirmed by blowing across the neck and noting any beats that occur between the two notes. Measure the height of the water, and from the previous graph find the corresponding volume of air. Repeat the experiment with other forks, and then from the results obtained plot v as ordinates against $1/n^2$ as abscissae; this should be a straight line. It will probably be found that the line does not pass through the origin; it will most likely obey the law $n^2(v+e) = \text{constant}$, where e is a correction to be applied to v and may be regarded as a correction for the air space in the neck of the bottle. The value of e should be about one-half of the volume of the neck. It will be found that the tuning is quite sharp when the higher-frequency forks are used, but not so sharp with the lower-frequency forks.

Since the conductivity of the neck is given by the expression A/l, it follows that it has the dimension of length, and Rayleigh showed that for a circular aperture in a *thin* wall, the conductivity is equal to twice the radius of the aperture. The frequency for a circular opening of radius r is therefore given by

$$n = \frac{V}{2\pi}\sqrt{\frac{2r}{v}}\,.$$

Hence for a given frequency the volume v should be directly proportional to the diameter of the opening. This can be verified experimentally by putting plates with circular openings of different diameters in turn on top of the resonator aperture and adjusting the volume in the resonator until resonance occurs when a vibrating fork is held over the opening. It must be pointed out, however, that an aperture in a resonator can never be of *no* thickness, and this has an effect on both conductivity and end correction. Rayleigh showed that if l is very small in comparison with r, the value of the end correction approaches $\pi r/4$, or $0{\cdot}785r$, as a lower limit. The end correction, however, increases with the length of the neck, and the upper limit is about $8r/3\pi$, or $0{\cdot}849r$.

CHAPTER IX

VIBRATIONS OF MEMBRANES, DIAPHRAGMS AND PLATES

ALTHOUGH the terms *membrane* and *diaphragm* are sometimes applied to the same thing, yet there is a fundamental difference between membranes on one hand and diaphragms and plates on the other. A membrane is strictly a very thin film of material in which any transverse vibrations set up are conditioned by an applied tension and are independent of elastic forces; but with diaphragms and plates it is the elastic forces which are important, while the tension can be regarded as almost negligible. Hence the vibrations of stretched membranes are related to those of diaphragms in a manner analogous to that of the vibrations of stretched strings to those of elastic bars (see Chapter V).

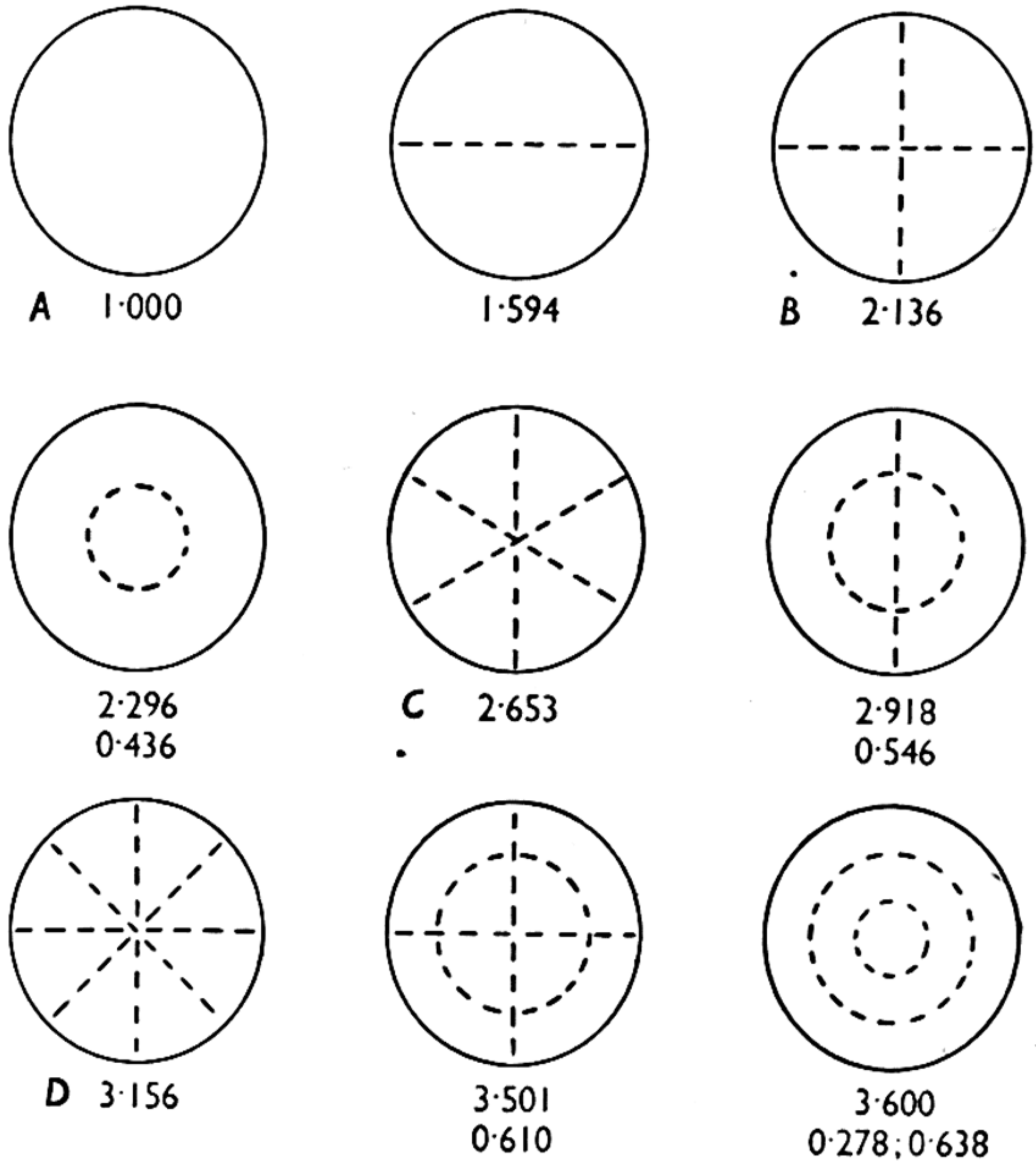
MEMBRANES

In consequence of their flexibility, membranes cannot vibrate unless they are stretched, like the skin of a drum; Savart obtained membranes by fastening gold-beater's skin on wooden frames.

Membranes approximating to the ideal type have been made from soap films or films of thin collodion stretched in a metal ring, and the nature of the vibrations examined by optical means. If it is desired to investigate the effects of tension on the vibrations, then probably sheets of parchment or thin metal will be found more suitable; in Wente's condenser microphone (see p. 23) a highly tensioned steel membrane of fundamental frequency of 10,000 is used. The various modes of vibration of a steel membrane can be conveniently studied by means of a small electromagnet (such as is found in a telephone receiver) and a valve oscillator with a suitable range of frequency control.

Membranes can readily be set in vibration by the vibrations of the air caused by an intense sound, for example, a bell, and they are eminently fitted for this on account of their small mass, large surface and the readiness with which they subdivide.

In actual practice, membranes are used in tambourines, and in various forms of drums where the vibrations are reinforced by the resonating air inside the drum.



In the case of a circular membrane of radius a, Rayleigh calculated that the absolute frequency is given by

$$n = \frac{0{\cdot}765}{2\pi a}\sqrt{\frac{T}{m}},$$

where T is the tension per unit length on the surface and m is the mass per unit area or the superficial density. The membrane can, however, and does, vibrate in other modes, and in these cases nodal rings and diameters are present (see also p. 184). It has been calculated by Bourget (1866) that the frequencies of the next simple tones where nodal circles are concerned are related to that of the gravest (lowest) tone in the ratios 0·765, 1·757, 2·755, etc.

The accompanying diagram represents the more important normal modes of vibration of a circular membrane, and the numbers below the circles give the frequencies referred to the gravest tone as unity, together with the radii of the nodal circles. The tones corresponding to the various modes of vibration of the

circular membrane do not belong to a harmonic scale ; but it is found that the four gravest modes with nodal diameters only, those labelled A, B, C, D in the diagram, would give approximately a consonant chord corresponding to the notes c, f, a, c'.

Any contraction of the fixed boundary of a vibrating membrane causes a rise in pitch, since an additional element of stiffness is introduced ; for example, the pitch of a membrane in the shape of a regular polygon is intermediate between those of the inscribed and circumscribed circles. Further, for different shaped membranes having the same area and vibrating under similar conditions, the circular membrane will give the lowest pitch. Thus, if a square and a circular membrane have the same area, the ratio of the pitches of the two gravest tones is 1·043 : 1·0, the square being the higher.

The theory of the free vibrations of various types of membranes was first successfully considered by Poisson in 1829. He was followed by Kirchhoff and others who dealt with circular membranes, and in 1866 Bourget published his " Mémoire sur le Mouvement Vibratoire des Membranes Circulaires ". In his experimental investigations, Bourget made use of various materials, of which paper proved to be as good as any. The paper is immersed in water, and after the superfluous water has been removed by blotting paper, the paper is put on a frame of wood the edges of which have been previously coated with glue ; the contraction of the paper in drying produces the necessary tension. The vibrations are excited by organ-pipes, of which it is necessary to have a series proceeding by small intervals of pitch, they are made evident to the eye by means of a little sand scattered on the membrane. If the vibration be sufficiently vigorous, the sand accumulates on the nodal lines and the form of the vibration is shown with more or less precision. Bourget concluded from his experiments that a circular membrane cannot vibrate in unison with every sound, though Savart, who also performed experiments on membranes, held that a membrane was capable of responding to any sound no matter what its pitch might be. Other conclusions of Bourget were that nodal lines are only formed distinctly in response to certain definite sounds (Savart supposed there was a continuous transition from one nodal system to another) and that the nodal lines are circles or diameters or combinations of both.

Bourget found a good agreement between theory and experiment so far as the radii of the nodal circles are concerned, but the relative pitch of the various simple tones deviated considerably

from the theoretical estimates. This is explained partly by the want of perfect fixity of the boundary, and also by the fact that theory demands perfect flexibility, a condition which is not closely approached by an ordinary membrane stretched with a comparatively small force. Rayleigh suggested that the most disturbing cause of deviation is the resistance of the air, which acts with much greater force on a membrane than on a string or bar on account of the large surface exposed. The gravest mode of vibration, in which the displacement is at all points in the same direction, might be affected very differently from the higher modes, which would not require so great a transference of air from one side to the other. In the case of kettle-drums, the matter is further complicated by the action of the shell, which limits the motion of the air upon one side of the membrane.

The vibrations of soap-films have been investigated by Melde. The frequencies for surfaces of equal area in the form of a circle, a square and an equilateral triangle, were found to be as

$$1{\cdot}000 : 1{\cdot}049 : 1{\cdot}175.$$

In membranes of this kind the tension is due to capillarity, and is independent of the thickness of the film.

DIAPHRAGMS

As has been said earlier, in the case of diaphragms it is the elastic forces brought into action which are all-important, the tension being negligible in comparison. By a complex analysis Rayleigh calculated that the fundamental frequency of a diaphragm *in vacuo* and clamped around its periphery is given by

$$n = \frac{2{\cdot}96}{2\pi} \cdot \frac{h}{a^2} \sqrt{\frac{E}{\rho(1-\sigma^2)}},$$

where h is the thickness, a is the radius, E is Young's modulus, ρ is the density of the material and σ is the value of Poisson's ratio. He also showed that the addition of a load m to the centre of a diaphragm of mass M lowers the frequency in the ratio

$$1 \Big/ \sqrt{\frac{1+5m}{M}}.$$

Later, Lamb calculated the frequency and damping of circular diaphragms in air and water, and his values agree closely with

Rayleigh's estimate. As an example of the *order* of frequency obtained, we might consider a steel diaphragm of radius 5 cm. and thickness 0·1 cm. Here $E = 2 \times 10^{12}$, $\rho = 7{\cdot}8$ and $\sigma = 0{\cdot}28$.

Hence $$\sqrt{\frac{E}{\rho(1-\sigma^2)}} = \sqrt{\frac{2 \times 10^{12}}{7{\cdot}8(1-0{\cdot}28^2)}},$$

which works out to be $5{\cdot}27 \times 10^5$ cm./sec., which is the velocity of the wave in the diaphragm. For the value of the frequency, we have

$$n = \frac{2{\cdot}96}{2\pi} \times \frac{0{\cdot}1}{25} \times 5{\cdot}27 \times 10^5 = 1{,}000 \text{ approximately.}$$

It will be seen therefore that, by suitably choosing the dimensions of a diaphragm, the frequency of the sound generated may have any value, and a thick diaphragm of small dimensions when excited by, say an electromagnet, provides a very convenient source of high-frequency sounds.

Diaphragms in contact with water. Diaphragms form one of the most convenient means of producing and receiving sounds both in air and in water, and large diaphragms are used as sources of sound for signalling over great distances in these media.

When a diaphragm is in contact with water, two effects are prominent. In the first place, the frequency of the sound is lowered on account of the added mass of water vibrating, and secondly the vibrations are damped owing to the energy radiated in the water. Lamb showed in the case of a diaphragm with one side only in water that the inertia of the diaphragm is increased in the ratio $(1+\beta)$, where $\beta = 0{\cdot}6689\rho_1 a/\rho h$, ($\rho_1$ is the density of the water, ρ the density of the material, a the radius and h the thickness of the diaphragm). The frequency calculated by using Rayleigh's equation (p. 180) must therefore be divided by $\sqrt{1+\beta}$ in this case; when *both* sides of the diaphragm are immersed, the value of β must be doubled. If we consider the dimensions of the steel diaphragm already mentioned above, we find that

$$\beta = 0{\cdot}67\rho_1 a/\rho h = 4{\cdot}3 \text{ approximately,}$$

and using this value in the amended equation, the corresponding frequency of the diaphragm vibrating with one face in the water works out to be 435 approximately as against 1,000.

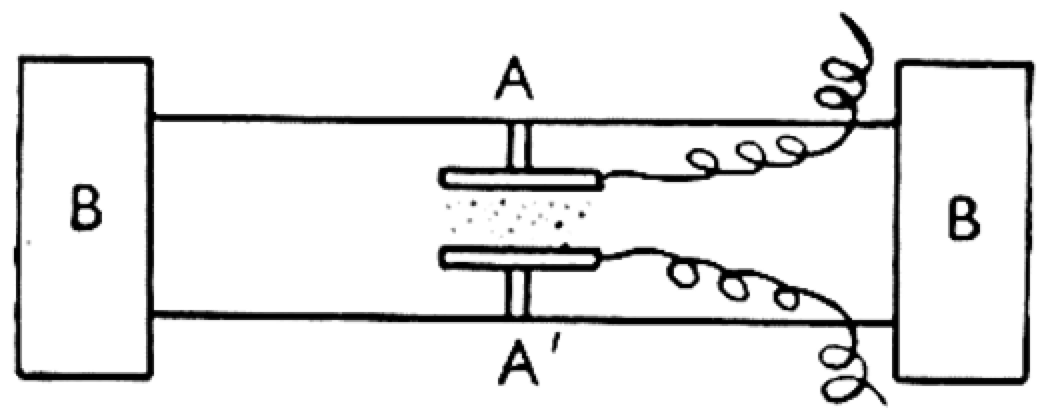
In some forms of echo-sounding apparatus (see Chapter XII) the transmitter and the receiver are fixed *inside* the hull of the ship, so that the sound has to be transmitted through the hull.

This raises the point as to whether the transmission will be affected by the hull. It was stated in Chapter IV that when sound travels from one medium to another through an intervening medium, the transmission does not suffer if certain conditions are fulfilled. But this independence of the transmission of the properties of the intervening medium assumes that this medium is a *true* medium, that is, one which does not vibrate as a whole. The hull of a ship certainly cannot be regarded as a true medium ; but it is an experimental fact that the vibration of the hull is of the same order of magnitude as that of the water. It can be shown theoretically that for the resonance frequency of the hull, the transmission from the water to the air inside or vice versa takes place more or less independently of the existence of the hull, and the general conclusion is reached that the hull will not interfere markedly with the transmission unless it is very thick. Hence the use of the hull for attaching apparatus for the transmission and reception of sound is quite practical.

DIRECTIONAL PROPERTIES OF MEMBRANES AND DIAPHRAGMS

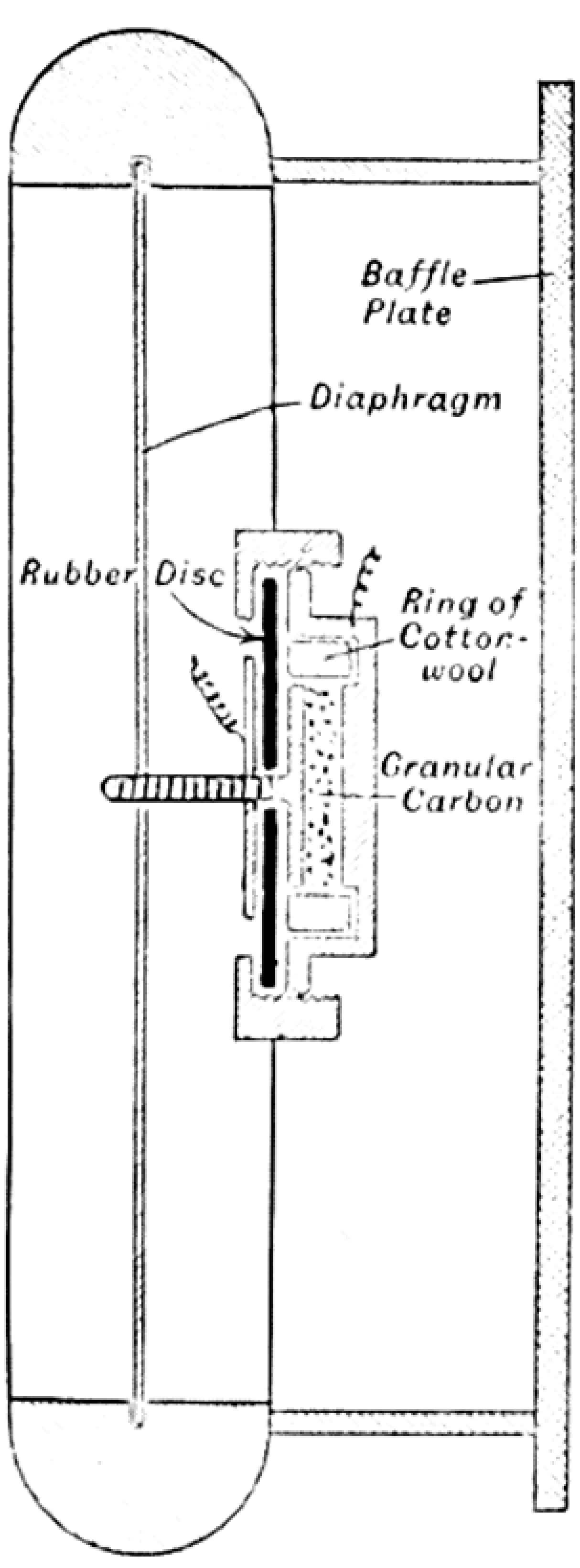
If a membrane or a diaphragm is mounted on an annular ring it will possess definite directional properties when used either as a transmitter or a receiver. When used as a transmitter, the sound emitted from opposite sides of the vibrator will be of the same intensity but opposite in phase ; hence an observer edge-on to the vibrator will hear nothing on account of the consequent neutralisation. The energy propagated from the back of the diaphragm will be partially screened from a listener in front by the ring and by the diaphragm, whereas that from the front is not. Therefore if a transmitting diaphragm is rotated through 360° a listener will hear two distinct maxima 180° apart, separated by two minima. The maxima, however, will be of smaller intensity than that observed when one side of the diaphragm is *completely* screened ; in this case, of course, the diaphragm is non-directional.

The directional properties of an unscreened diaphragm may be shown as a receiver by means of a " button " microphone attached to the centre of the diaphragm. This type of microphone, which is the kind generally used in subaqueous reception, is primarily a " displacement " detector rather than a " pressure " detector like the older types of microphone in which sound pressure produces variations in the resistance of an electric circuit by varying the compression of carbon granules.



One form of "button" microphone consists of two thin membranes A, A', fixed a little distance apart in a massive ring BB. Between the membranes and fixed to the centre of each by a support is the button microphone, which consists of the two small electrodes and the carbon granules between. When a sound wave is incident on the membrane, it shakes the supports and disturbs the granules, so producing the desired alteration in resistance.

If the microphone is turned edgewise to the direction of the sound waves, no sound is heard; if, however, one face A is towards the sound waves, a maximum sound is heard. Such a modified microphone has been used to determine the direction of a sound source under water; in this case, the instrument is called a **hydrophone.** This consists of a heavy metal ring carrying a thin stainless steel diaphragm with a small water-tight capsule attached to its centre. Inside the capsule is the granular carbon microphone. As the effect of the incident sound waves is the same whether the energy falls on the face or the back of the diaphragm, the hydrophone is fitted with a baffle plate on one side a short distance from the back of the diaphragm; hence the instrument is practically uni-directional, and by slowly rotating the hydrophone, the direction of the sound can be determined with considerable accuracy.

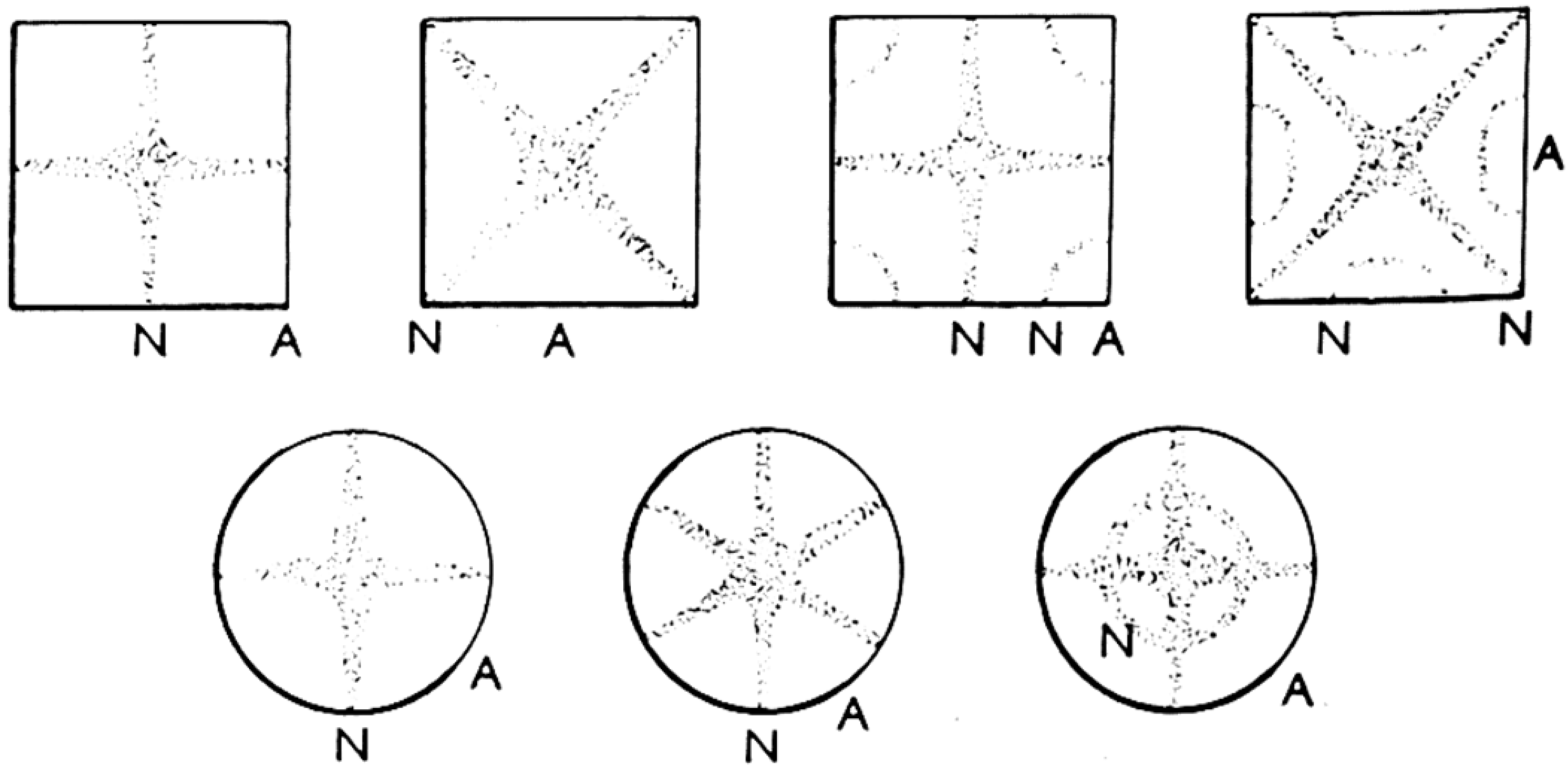


PLATES

Reference has already been made (p. 26) to quartz plates vibrating longitudinally in the direction of their thickness, and their use as a source of ultrasonic vibrations, particularly under water, while in the present chapter the vibrations of circular plates (diaphragms) clamped at the edge have been dealt with.

The experimental study of the vibrations of plates begins with Chladni, who found that when a square plate of metal or glass is

fixed at the centre in a horizontal plane by being screwed to a vertical pillar and the edge is bowed, a note of definite pitch is produced. Moreover, if some sand is sprinkled on the top of the plate, it is found that when the plate is set in vibration, the sand gathers along well-defined lines forming a pattern on the plate. These lines represent the portions of the plate which remain permanently at rest and are called **nodal lines.** The pattern formed, and the note emitted, depend upon where the plate is clamped and where it is excited, and it is obvious that the number of notes from any one plate is practically infinite. If lycopodium powder is used instead of sand, it collects, not along the nodes, but at the points of maximum motion. This is due, according to Faraday, to the formation of small vortices in the air near the plate, just above the loops, which sweep the light powder on to the loops. In a vacuum all powders move to the nodes.



Chladni figures

Chladni also investigated the vibrations of rectangular and circular plates, and it was found that in the latter the nodal lines are either radial lines (*diameters*), or circles or a combination of the two. The radial lines are obtained by fixing the centre of the plate and bowing the edge while two points on the edge are damped. The circular lines are produced by resting the plate on three points on one of the circles and causing the vibrations by drawing a resined string through a hole in the centre; they may also be obtained by fixing the plate by its centre to the end of a rod and making the rod vibrate longitudinally.

Examples of the figures obtained by Chladni are shown in the

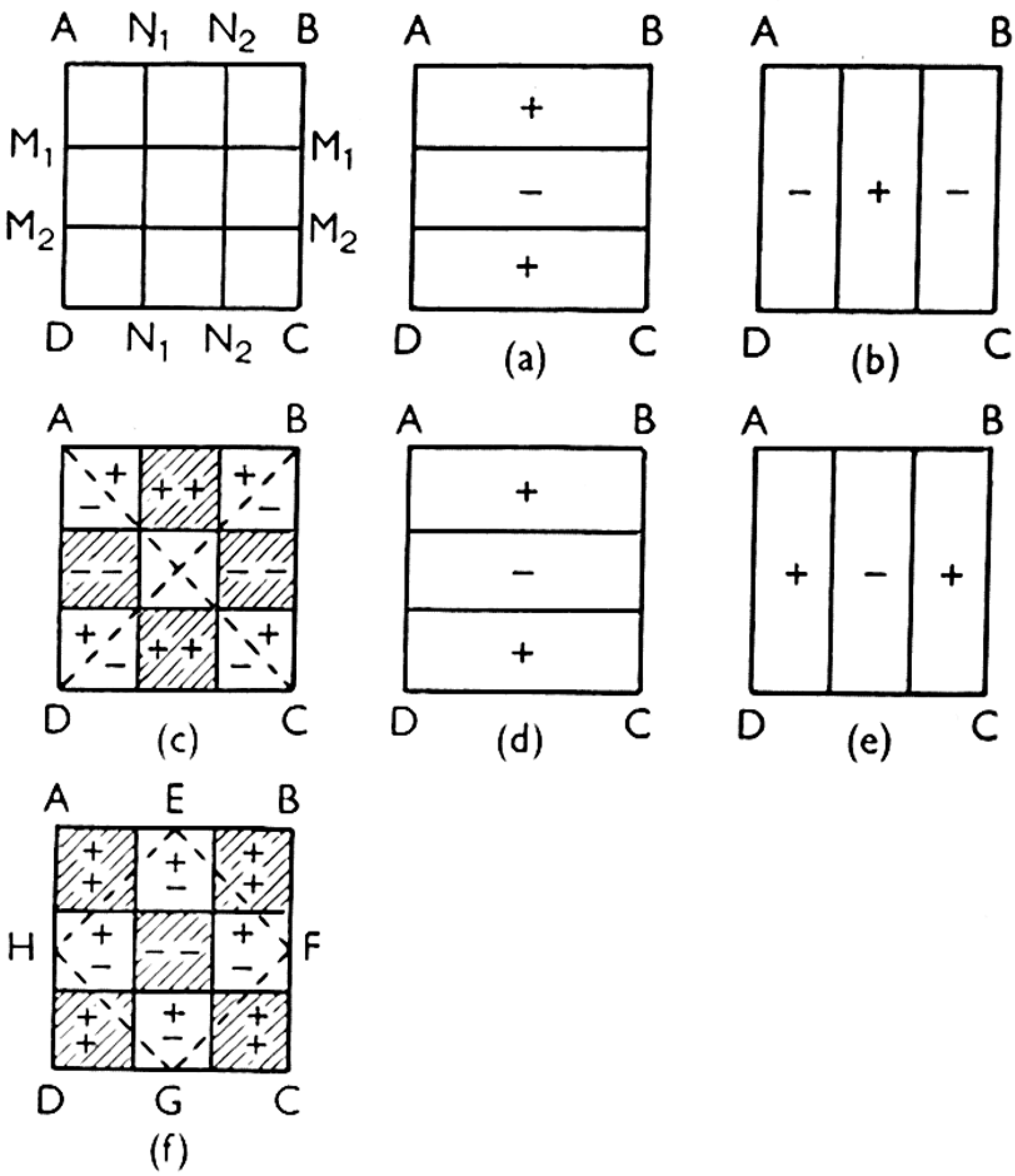
diagram ; in each case the plate is touched or held at the points N and bowed at the points A.

When a circular plate is truly symmetrical, theory indicates, and experiment verifies, that the position of the nodal diameters is arbitrary, depending only on the manner in which the plate is supported and excited. By varying the place of support, any desired diameter may be made nodal. It is generally otherwise, however, when there is any sensible departure from exact symmetry.

In general, Chladni's figures as traced by sand agree very closely with the circles and diameters of theory, but in certain cases deviations occur, which are usually attributed to irregularities in the plate. Rayleigh pointed out, however, that the vibrations excited by a bow are not strictly speaking free, and that their periods are therefore liable to a certain modification. It may be that under the action of the bow two or more normal component vibrations coexist. The whole motion may be simple harmonic in virtue of the external force, although the natural periods would be slightly different. Another cause of deviation may perhaps be found in the manner in which the plates are supported, for the requirements of theory are often difficult to meet in actual experiment.

In the ordinary use of sand to investigate the vibrations of flat plates and membranes, the movement of the nodes is irregular in its character. If a grain be situated elsewhere than at a node, it is made to jump by a sufficiently vigorous transverse vibration. The result may be a movement either towards or away from a node, but after a succession of such jumps the grain ultimately finds its way to a node as the only place where it can remain undisturbed.

Wheatstone's explanation of Chladni's figures. Wheatstone explained the figures obtained by the vibration of plates by considering the plate to be made up of a number of rods parallel to the edges of the plate. Consider the rods parallel to the edge AB (p. 186). These rods could vibrate so as to have all the nodes along N_1N_1 and N_2N_2. Similarly with the rods parallel to AD and, if the two movements go on together, the actual movement of the rods is the algebraic sum of both. The central segments of the two sets of rods may be in the same or in the opposite phase. Consider first that they are in *opposite* phase as shown in (a) and (b), where we are assuming that any given part of the plate above the plane of the plate is represented by the symbol $+$, and below by the symbol $-$. When the two motions are combined, we get the figure shown in (c). In the shaded portions the two displace-



ments assist one another, and in the unshaded parts the upward displacement due to one set of rods is neutralised by the downward displacement due to the other set. Hence the minimum displacement will be along the two diagonals of the plate and these will be the nodal lines. It will be noticed that this agrees with the figure shown on p. 184, where the corner of the plate is damped and the bow used in the middle of one edge.

If the central portions of the two sets of rods are in the *same* phase, as represented in (*d*) and (*e*), the resultant figure obtained is as in (*f*), and the nodal lines are the square $EFGH$. In this case it will be seen that the central part of the plate is vibrating; hence the plate can not be clamped at the centre but at a point on the nodal line.

On the whole, there is good agreement between the theory given above and practice. The slight differences noticed are probably due to the fact that in the theory we have assumed that the amplitude of the motion of the central portions of the rods and that at the extremities are equal. This is not true, for the amplitude at the ends is greater than at the centre. Hence some

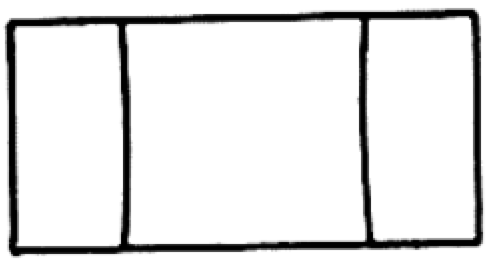
points which in the theory we have taken to be at rest owing to the displacement of the end segment of one set of rods neutralising the displacement of the central segment of the other set, will not really be at rest.

Since a nodal line always represents the line of separation between two parts vibrating in opposite phases, there must always be an *even* number of radial lines in the case of a circular plate. If there were an odd number, then at one line at least the plate on both sides would be vibrating in the same phase.

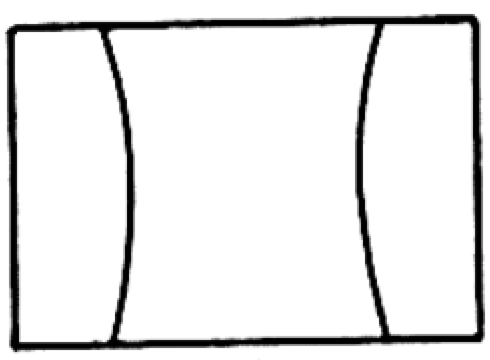
In the more modern investigations into the vibrations of plates, solid carbon dioxide is used as the exciting agent instead of the bow ; it is found that by this method the vibrations are set up more easily. A pointed " rod " of solid carbon dioxide is held in contact with the edge of the plate, and as metal plates are good conductors of heat, the solid is very rapidly vaporised, and the disturbance caused sets up vibrations easily.

Recent work on Chladni's figures. The theory of the vibrations of free bars is well known, and that concerning square and circular plates has also been developed, particularly by Rayleigh ; but the theory of the vibrations of free rectangular plates, of which the square and the bar are limiting shapes, has not been seriously considered until the last decade or so. Chladni observed (1787, 1802) that the normal nodal systems of rectangular plates consist in general of straight lines parallel to the sides, and this has been confirmed by observations and calculations in the present century by Pavlik (1937). Much experimental work has been done still more recently by Dr. Mary D. Waller, of the Royal Free Hospital School of Medicine, London, in connection with rectangular plates, and she approached the problem from the point of view of symmetry. " The symmetry of a rectangle, the sides of which are not equal, is less than that of the square. The latter possesses 90° rotational, the former only 180° rotational symmetry, and whereas there is mirror symmetry about both medians and diagonals in the nodal designs for the square, in the case of the rectangle the diagonal mirror symmetry is lost. It follows from the principle of symmetry that while for the square as many as four nodal lines (medians and diagonals) may pass through the centre, for the rectangle it is not possible for more than two nodal lines (medians) to pass through its centre."

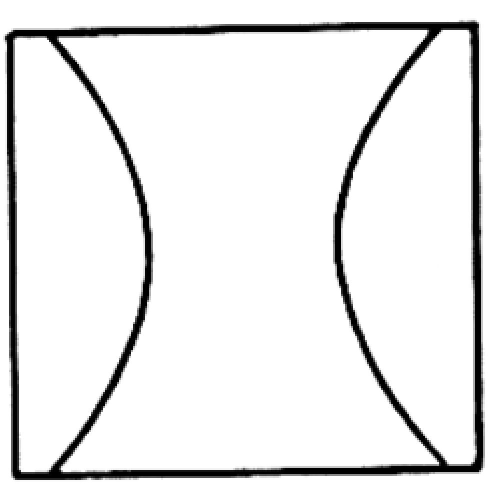
Photographic records of the nodal designs obtained by Waller on narrow, medium and wide rectangles indicate that the prevalent design undoubtedly consists of lines parallel to the sides, and when the plates are narrow there is a very close connection



$\frac{2}{1}$



$\frac{3}{2}$



$\frac{1\cdot09}{1}$

between these and rectangular bars. As the width of the plate increases, pronounced curvatures, either convex or concave in the centre, occur in the nodal lines. The diagrams indicate the difference in the lines shown by three rectangular plates of length/breadth ratios of 2/1, 3/2 and 1·09/1 vibrating in the same mode.

In addition to the curvatures mentioned above, there may also be departures from the normal nodal systems consisting of straight lines, due to a second mode of vibration combining with the principal one. Further, it appears that these compound modes are more prevalent in the case of rectangles than with squares.

In Waller's experiments, she used the solid carbon dioxide sublimation method, supplemented occasionally by the use of the bow, and the vibration frequencies were determined by means of a calibrated valve-operated mains oscillator. Other methods of excitation, such as the piezo-electric method, have been used in connection with the investigation of high overtones, and in these cases the spacing of the nodal lines is much closer together than for vibrations obtained by using carbon dioxide. It is claimed, however, that the sublimation method is unsurpassed for producing powerful and exceptionally free vibrations in metal objects and also in quartz.

In later investigations Dr. Waller noticed while experimenting with a Chladni plate supported horizontally on rubber studs, that striations appeared on some sand which had inadvertently been spilled on the bench just a few millimetres below the plate. That the striations were not caused by vibrations of the bench was proved by supporting the plate independently from above, and then producing the striations on a second stationary plate, or drawing board, which rested on a "Sorbo" mat.

Although the investigations on this new and interesting phenomenon are not yet complete, Dr. Waller suggests it is probable that the striations are formed in the same way as for the Kundt striations which Prof. E. N. da C. Andrade has shown are caused by the oscillatory motion of the air setting up vortex motions around individual particles. In this connection it is significant that the distances between the striations produced,

either by a Kundt tube or by a Chladni plate, are of the same order.

For further information on this subject, the student should consult the original papers by M. D. Waller (*Proc. Phys. Society* and *Nature*).

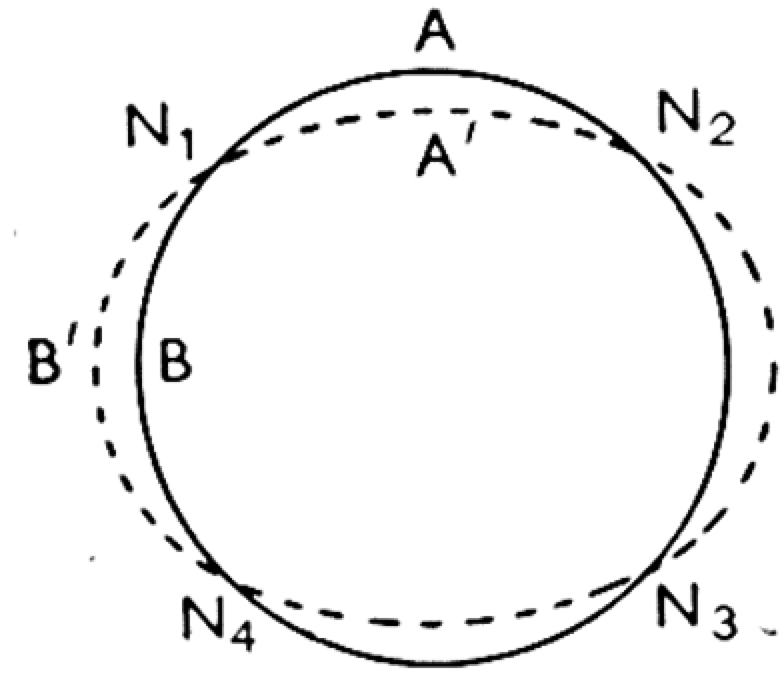
CURVED PLATES, CYLINDERS AND BELLS

The complex problem of the vibration of flat plates is further complicated when the plate is curved. Rayleigh calculated that the fundamental frequency N of a thin cylindrical shell is given by the expression

$$N \propto \frac{h}{a^2}\sqrt{\frac{e}{\rho}},$$

where e is an elastic modulus involving both bulk modulus and rigidity, h is the thickness, a the radius and ρ the density of the material.

From a practical point of view, a curved plate in the shape of a bell is perhaps the most important, and a bell may be regarded as a progressive development of a curved plate. When a bell-shaped object is sounded by a blow, the point of application of the blow is a place of maximum normal motion of the resulting vibrations. It is important to notice, however, that the vibrations are not entirely radial but must also be tangential as well. Let the circle in the diagram represent the bell before it is struck, and let the dotted line represent one extreme position of the vibration. As in circular discs, there must always be an even number of nodal lines, the portions of the bell on opposite sides of each line vibrating in opposite phase. Now the simplest form of vibration is that in which there are four nodal lines which can be represented by N_1, N_2, N_3 and N_4. Although these nodal lines are places of no radial motion, they must be positions where there is maximum tangential motion. For when the rim on one side of a node is outside the mean position, the rim on the other side is inside, and the length of the rim intercepted by adjacent nodes is greater when this portion of the rim is outside than when it is inside the mean position. Hence to allow for the changes in the length of the rim, a motion of the rim in its



own plane takes place at the nodes ; in other words, the nodes must have a small vibration in a circumferential manner. This explains why glass tumblers and wine glasses can easily be thrown into regular vibration by friction with the wetted finger rubbed around the circumference. The effect of the friction is in the first instance to excite tangential motion, and the point of application of the friction is the place where the tangential motion is greatest and where the normal motion vanishes.

Rayleigh made a particular study of church bells, and to pick out the various tones and ascertain the number of nodal meridians he used resonators of the Helmholtz pattern. He found that from a bell of 6 cwt. made by Mears and Stainbank, six tones could be obtained, namely :

e'	c''	$f''+$	$b''_\flat$	d'''	f'''
(4)	(4)	(6)	(6)	(8)	

The pitch of this bell as given by the makers was d''', so that here it is the fifth in the above series of tones which characterises the bell. The figures in brackets indicate the number of nodal meridians in the various components ; the mode of vibration of the highest tone, f''', could not be fixed satisfactorily, since it was difficult to observe this note.

Other bells were experimented with in a similar way, and the results are summarised in the following table, no attention being paid to the question of the octave :

Bell	*Normal pitch*	*Actual pitch of tones given by bell.*				
5	$f\sharp$	$g-3$,	$g'-4$,	$a'+6$,	$d''-3$,	$f''\sharp-2$.
4	$g\sharp$	$a+3$,	$g'\sharp-4$,	$b'+6$,	$d''\sharp-e''$,	$g''\sharp-6$.
3	$a\sharp$	$a\sharp+3$,	$a'+6$,	$c''\sharp+4$,	$e''+6$,	$a''\sharp$.
2	b	$d'-6$,	$d'\sharp-5$,	$d''+8$,	$g''\sharp+10$,	$b''+2$.
1	$c\sharp$	$d'+2$,	$b'+2$,	e'',	$g''\sharp+4$,	$c'''\sharp+3$.

Thus in every case it is the fifth tone which characterises the nominal pitch of the bell and further, the overtones of the bell do not form a harmonic series. It will be agreed that it is not easy when listening to the sound of a bell to fix its pitch, for in addition to the nominal pitched tone and the overtones, beats arising from the various tones frequently occur, thus complicating the issue ; one has only to listen to the striking of Big Ben, as broadcast by the B.B.C., to realise the complexity of the problem.

The names given by bell founders in England to the five chief tones in a church bell, reckoning from the highest pitch, are the *nominal*, the *fifth*, *tierce*, *fundamental* and the *hum-note* which is the lowest. By a suitable distribution of metal in the bell, the founder very often aims at making the hum-note, the fundamental and the nominal successive octaves, but in practice this is difficult to achieve.

MICROPHONES

In view of the importance of microphones in broadcasting and modern methods of communication, these instruments must be discussed and this chapter seems to be the appropriate place. As is well known, the function of any microphone is to convert the minute fluctuations of air pressure, which reach its sensitive surface and constitute the sound waves, into corresponding electric currents which can be amplified by suitable means and used to modulate, perhaps, a broadcasting transmitter. It is clear that if faithful reproduction is to be attained, the electric currents must be precisely similar in form to the changes of pressure with which they correspond. Hence a microphone must possess certain definite characteristics.

One requirement is a *good frequency characteristic*, which means that the microphone response must be sensibly the same over the whole audible range of frequency; failure to fulfil this condition involves unnatural tone in the reproduction. A second characteristic is *linearity of response*, meaning that the magnitude of the currents produced by the microphone must always be strictly proportional to that of the original sound pressures; otherwise the type of distortion known as "blasting" occurs, resulting in rough, harsh reproduction characterised by very disagreeable combination tones.

Other requirements of an efficient microphone are freedom from hiss or other background noise, adequate sensitivity and ease of maintenance and reliability of operation, while directional effect and dependence upon frequency have also to be taken into account.

Carbon microphones. The first carbon transmitter was constructed by Edison in 1877; he used a metal plate in loose contact with a carbon button. By means of a battery, a current was made to pass from the plate to the button, and under the influence of sound vibrations the electrical resistance of the contact varied and thus the current in the line was made representative of the sound. About the same time, Hughes invented his microphone.

in which he used a carbon rod with pointed ends resting loosely in sockets bored in two carbon blocks. The working principle is the same as in Edison's instrument, but its sensitivity was greater.

These forms of transmitter led to the granular carbon type which is the well-known carbon microphone of the present time as used in telephones. For communication purposes, particularly in broadcasting, considerable care is necessary to avoid " blasting ", owing to the limited linearity of these microphones, though it is found that their frequency characteristic is fairly good. Such microphones are no doubt capable of improvement to overcome the fundamental difficulties of non-linearity and background noise.

Condenser microphones. One form of this instrument, devised by E. C. Wente, has already been described on p. 23.

Another form which has been used for broadcasting purposes is known as the **stretched-diaphragm** microphone. This consists of a rigid metal back-plate in front of which, at a distance of about one-thousandth of an inch, is tightly stretched a very thin metal diaphragm, usually made of duralumin. This diaphragm is insulated from the back-plate and thus forms with it an electrical condenser, the capacity of which is altered by the variations of air pressure due to the sound-waves. The changes of capacity can be made to cause corresponding voltage changes on the grid of an amplifying valve and so generate the " speech " currents.

The great advantage of this condenser microphone over the carbon type is its almost complete linearity and consequent freedom from " blasting ", as well as freedom from background noise; its frequency characteristic, however, is very similar to that of the carbon instrument.

A newer type of microphone is known as the **slack-diaphragm** type. In one form it consists of a central pillar a few inches long and of elliptical section. This has a conducting surface and takes the place of the back-plate of the stretched-diaphragm microphone, forming one electrode of the condenser. Over this is a thin film of insulating material and around the whole is wrapped a piece of aluminium foil of about one-thousandth of an inch thick; this forms the second electrode of the condenser. The foil is stretched just tightly enough to keep it in place, and is not kept under tension as in other types. The frequency characteristic of this type of microphone is much flatter than in the case of the stretched-diaphragm instrument, and its response is much more uniform over a wide range of frequencies. Maximum response in the stretched type, which occurs at about 4,000 cycles.

is due partly to the resonance of the tightly stretched diaphragm itself, partly to the resonance of the air column formed by the cavity in the face of the microphone and partly to what is known as " obstacle effect ". This last effect is due to the fact that the waves of the higher frequencies, the wave-lengths of which in air are smaller than the actual dimensions of the microphone, are completely reflected by the latter. Their effective pressure on the microphone is therefore doubled by comparison with that of waves of greater wave-length. It is found that the slack-diaphragm microphone is effectively free from the effects mentioned above ; on the other hand, however, it suffers from a lack of sensitivity, compared even with other types of condenser microphone, and great precautions have to be taken when using it.

From its construction it will be seen that the slack-diaphragm instrument possesses one definite advantage from the point of view of studio technique in certain types of broadcasts, in that it is practically non-directional in a plane at right angles to the principal axis of the instrument. It is possible for a number of actors to be grouped around a single microphone, or such a single instrument may be used in connection with concerts involving artists and orchestra. In both cases, if the microphone is put in a suitable position, the works are correctly reproduced and suitably balanced.

So far as broadcasting is concerned, the position of the microphone and its type, whether directional or not, are important factors in determining the degree of reverberation associated with any performance. The effect of reverberation as heard by the listener can be varied at will, within certain limits, by attention to microphone position. Further, the use of a directional or a non-directional microphone increases the latitude in this direction, since it is the relative importance of direct sound from the source, as compared with the sound reflected from the walls of the studio, which determines the effect of reverberation. A non-directional microphone tends to respond to the reflected rays of sound reaching it from all directions, whereas a directional one tends to concentrate more upon the direct rays. Between the limits, therefore, represented by a directional microphone close to the source of sound and a non-directional instrument situated some distance away, a whole variety of acoustical effects can be obtained.

A variation of the stretched-diaphragm microphone is known as the **baffled microphone,** in which the small microphone is surrounded by a large rigid baffle board. This type certainly does

avoid the results of "obstacle effect" by extending it to the whole of the audible frequency range, thus improving the frequency characteristic. But the directional properties of the microphone are also rendered more marked, thus limiting its use, while its large size is a disadvantage for some purposes.

Electrodynamic microphones. One type of instrument of great sensitivity is the ribbon or band microphone of Gerlach and Schottky. This consists of a light metallic ribbon suspended in a strong magnetic field. The vibration of the ribbon due to an incident sound wave causes an induced E.M.F. corresponding to the undulations of the wave, and for frequencies below about 4,000 cycles the ribbon follows very closely the motion of the air particles in the sound wave.

Another form of instrument is the **moving coil** microphone. In this, the sound waves set in motion a light diaphragm, to which is attached a coil of wire, situated in a strong magnetic field provided by a suitable permanent magnet. The currents which are thus generated by the vibrations are amplified in the usual way. This type of microphone gives a good technical performance, including a level response over a particularly wide band of frequencies, also a reasonable sensitivity. It is simple to instal and maintain, it is small and inconspicuous and it is completely free from inherent background noise.

Crystal microphones. A later type of instrument depends for its operation on the piezo-electric effect and is known as the crystal microphone. In one form the instrument consists of two crystals of Rochelle salt clamped together as a double layer, and to this is attached the diaphragm, which, when set in motion, causes one crystal to be compressed and the other extended. Metal foil is cemented to the surfaces between and on the outside of the sandwich, and the potentials developed can be applied to the input circuit of an amplifier.

In order to avoid the undesirable resonances peculiar to a diaphragm, the latter is sometimes dispensed with, and the sound waves impinge directly on the crystal surfaces. The output of this form of microphone is, however, less than that when a diaphragm is used.

When Rochelle salt is employed, care must be taken to prevent dehydration of the salt and also to keep temperatures moderately low; other more stable crystals are now available.

Some kinds of crystal microphones reveal a flat frequency characteristic over a wide frequency range, and although their sensitivity is not high they are very free from background noise.

In the above discussion on microphones, it is not intended to suggest that any one type has superseded any other. There is probably no such instrument nowadays which can be regarded as a " general purpose " microphone, particularly in the sphere of broadcasting. The tendency would seem to be towards the development of a number of different types of microphone, each useful in its own sphere and with its own special advantages.

CHAPTER X

DETERMINATION OF FREQUENCY

FREQUENCY is such a fundamental property of wave-motion, especially in connection with music, that this chapter will be devoted to a consideration of experimental methods of determining the frequency of sounds. It will be remembered that the pitch (frequency) of organ pipes varies rapidly with temperature and the pressure of the wind, and that of strings with the tension, which can never be kept constant for long. But a tuning fork usually retains its pitch with great fidelity, and for this reason these instruments are invaluable as standards of pitch. Hence the methods dealt with here will be mostly in connection with the frequency of forks.

The human ear can, of course, sometimes give the approximate frequency of sound. A good musician can identify a certain sound as being a definite note in the musical scale with its characteristic frequency. It is perhaps easier for the musician to compare the frequencies of two musical notes sounding simultaneously from his knowledge of musical intervals, while of course even the non-musical ear can give an approximate comparison by listening for beats. The method of beats is dealt with later in the chapter.

USE OF SONOMETER

It is clear from the relationship established in Chapter V for the frequency of vibration of a stretched string, namely,

$$n=\frac{1}{2l}\sqrt{\frac{T}{m}},$$

that a sonometer can be used to determine the frequency of a sound emitted, say, by a fork. The wire is stretched by a known load and is tuned by altering its length until it is in unison with the fork. When the mass per unit length of the string is obtained, the frequency can be calculated from the formula. The frequency of the note given by a string and the character of the fundamental vibration were first investigated on mechanical

principles by Brook Taylor in 1715, and he deduced the above formula. By applying this formula, Seebeck obtained some very accurate results as follows. The tension was produced by a suitable weight, and in order that the whole of the tension should act on the vibrating segment, no bridge was interposed, a condition only to be satisfied by suspending the string vertically. After the weight was attached, a portion of the wire was isolated by clamping it firmly at two points, and the length l measured. The mass of the wire per unit length (m) refers to the stretched state of the string and can be found indirectly by obtaining the length of the unstretched wire corresponding to the length l of the stretched string, and weighing a known length of wire in its normal state. After the clamps are secured, care is needed to avoid changes of temperature which of course might seriously affect the tension.

Scheibler (see also p. 206) used a sonometer to determine the absolute frequency of a fork, and his method depended on deducing the absolute frequencies of two notes from a knowledge of both the ratio and the difference of their frequencies. The lengths of the sonometer wire when in unison with a fork and when giving with it four beats per second were carefully measured. The ratio of the lengths is the inverse ratio of the frequencies, and the difference of the frequencies is 4; from these data the absolute frequency of the fork can be calculated. For, let n be the frequency of the fork, and l_1 and l_2 the two lengths of the sonometer wire, where $l_2 < l_1$. When the fork and wire are in unison we have $n \propto 1/l_1$; also when there are four beats per second, we have $n+4 \propto 1/l_2$. Therefore, $n/(n+4) = l_2/l_1$, from which

$$n = \frac{4l_2}{l_1 - l_2}.$$

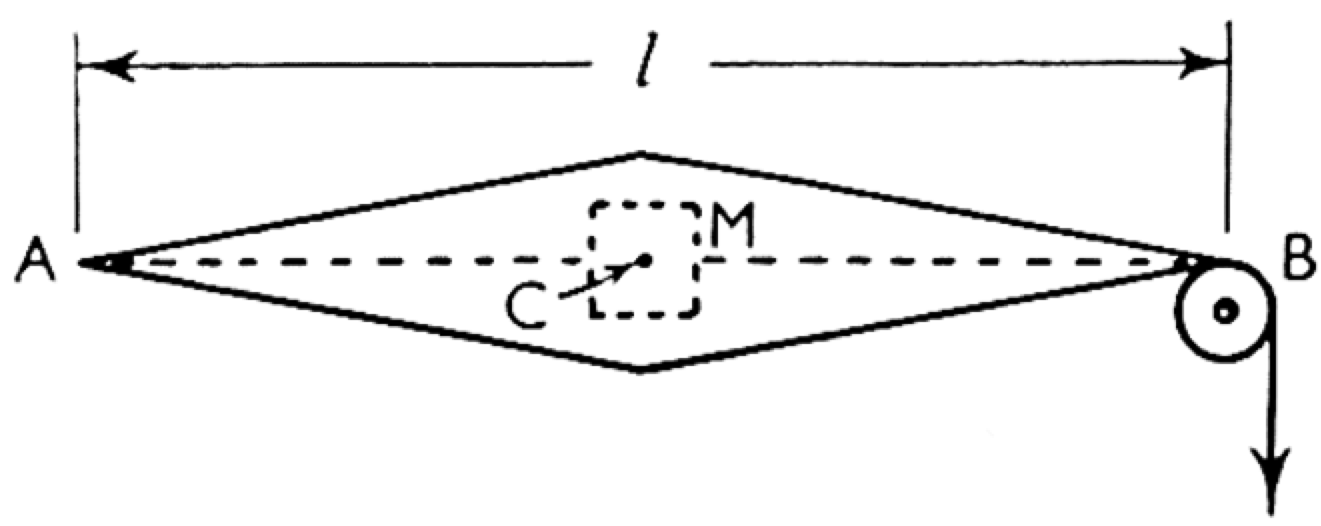
In connection with sonometer methods of determining frequency, it should be noted that, unless a thin wire is used, the frequency obtained may not be the true frequency of the fork, for the rigidity of the string will produce an extra restoring force which will bring the vibrating wire more quickly to its normal position; hence the frequency will be increased. In musical instruments, however, the tension is usually so great as to render the effect of rigidity negligible.

Incidentally, the equation

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

can be used to find the density of the material of a wire if a standard fork is available. Also the sonometer can be employed to determine the frequency of other forks by comparing the lengths of the wire in unison with the forks and a standard fork.

Let us now consider from a fresh angle how vibrating strings might be used for the measurement of frequency. Suppose a wire of length l is stretched between the fixed points A and B, and that a mass M, large in comparison with the mass of the string, is attached to the centre c. When M is pulled aside and then released, it executes simple harmonic vibrations. If the amplitude of vibrations is small, any variations in tension in the string as it passes from one extreme through the normal position



to the other extreme can be neglected. By considering the equation of motion, it can be shown that the periodic time is given by :

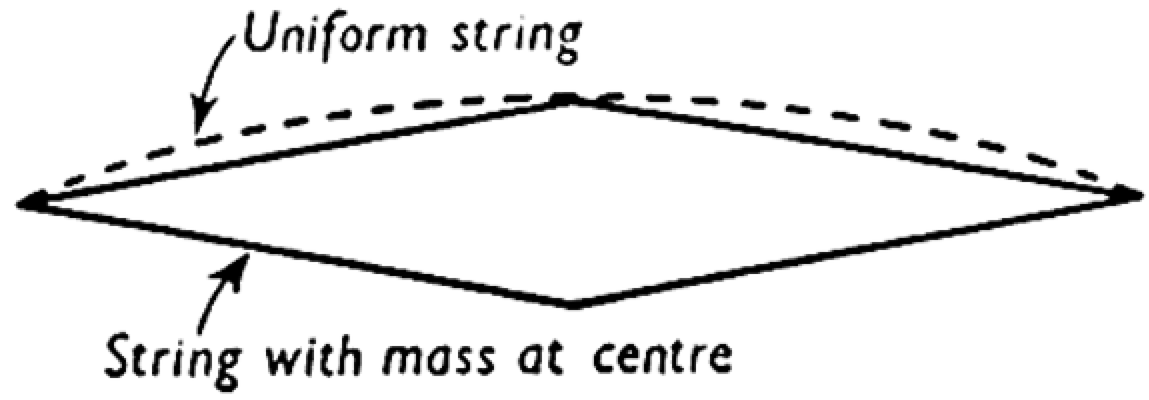
$$t = \frac{2\pi}{\sqrt{4T/lM}}.$$

The stretching can be done by attaching a weight W over a pulley, and the value of t will be given by :

$$t = 2\pi\sqrt{\frac{lM}{4Wg}},$$

from which the frequency can be found. One difficulty in carrying out this experiment is to make M sufficiently large in relation to the mass of the wire without at the same time lowering the pitch of the note too much.

The above experiment does not present the same problem as an ordinary musical string, where the mass is uniformly distributed over its length. In this case, the different parts of the string, at the moment of passing through the normal position, have different velocities, increasing from either end towards the centre, and if we attribute to the whole mass the velocity of the centre it is clear that the kinetic energy on which the equation of



motion partly depends will be considerably over-estimated. Further, at the moment when the string is in the extreme position, it is stretched more than in the example when M is at the centre; hence the potential energy of the uniform string will be greater. As a result of the differences in the energies in the two cases, the periodic time of the vibrating uniform string will be less than that given by the above relationship.

USE OF SIREN

Disc siren. The siren is an instrument which can be used for several purposes. In its simplest form, it consists of a disc having two circles of holes in it and capable of being rapidly rotated, and a tube which is connected to a bellows so that a current of air can be directed against the holes. When the rotation is very slow, a puff of air passes through each hole as it comes opposite the tube, and the ear will hear the separate puffs. On increasing the speed the puffs blend into a note, the pitch of which rises with increasing speed (refer to diagram on p. 121).

This type of siren is usefully employed for showing that the relation between frequency and pitch is of a quantitative nature. Suppose there are twice as many holes in the outer ring as in the inner ring. When the siren is working, there will be an easily recognisable relation between the pitches of the notes produced when the jet is directed against each set of holes in turn; in this case the note from the outer ring will be the octave above the other. Moreover, this relation is true no matter what the speed of the disc, showing that one note is the upper octave of another if it corresponds to twice the frequency. By having other series of holes in the disc, the method can be extended to the case of musical intervals, when it can be shown that the interval between two notes is determined by the *ratio* of the frequencies and not by the absolute frequencies.

Cagniard de la Tour's siren. In this instrument, which is used to determine the frequency of a note, air is blown through a tube at the bottom into a cylindrical wind-chest, the plate forming the top of which is pierced with a ring of holes at equal distances apart. Above and nearly in contact with the perforated plate is a

disc pierced with a corresponding ring of holes and mounted on a vertical spindle so that it can rotate freely. The holes are cut obliquely, those in the disc and wind-chest slanting in different directions. A screw thread cut in the spindle engages a cog-wheel and works a revolution counter at the top, and pointers move around two dials, one showing the number of revolutions up to 100 and the other the number of complete hundreds. In some instruments means are provided for throwing the counting mechanism into and out of gear at will. The siren is connected either directly or through a larger wind-chest with bellows.

To find the frequency of a tuning fork, the fork is struck or bowed, and when the fundamental tone of the siren reaches that of the fork the blast of air is regulated to keep the two notes as nearly as possible in unison. If the note of the siren rises above or falls below that of the fork beats will be heard, and the blowing must be regulated until they disappear. This is no easy matter, since the listener often cannot tell whether the note of the siren has become too high or too low. When, however, it is judged that the two notes are in unison, the time is noted as the pointer on the counter passes one of the hundred marks on the dial. After two or three minutes the time is again noted as the pointer again passes a mark on the dial, and the number of revolutions (N) of the disc is found. Suppose there are n holes in the disc and an equal number in the top of the chest; the holes in the disc will come over those in the wind-chest n times in each revolution. If t seconds be the time occupied by the N revolutions, Nn/t puffs of air issue through each hole per second. As the puffs issuing from the different holes are simultaneous, the intermittent current of air gives rise to Nn/t impulses per second and Nn/t disturbances per second are propagated outwards. This produces a note the fundamental tone of which has a frequency Nn/t, and this is of course the frequency of the fork.

The difficulty of preventing variations in speed when the disc is driven by the air current limits the accuracy of the determinations. More accurate results can be obtained if the disc is driven independently of the air blast by an electric motor, the speed of which can be regulated by a resistance; in this case the holes should be cut normally to the disc to avoid air pressure in the direction of rotation.

The sound given by a siren is not a pure tone, a large amount of harmonics being present; in any experiment, therefore, it is necessary to pick out the fundamental. The siren is particularly useful for calibrating organ pipes; many other methods, which

give more accurate measures of frequency, are unsuitable for this purpose.

Incidentally, it may be mentioned that a siren is very useful in producing loud sounds such as might be required for fog horns. When horns are used with a siren, it has been found to be important that the frequency of the note given by the siren should coincide with that of the fundamental tone of the horn ; that is, there should be resonance between the two sounds.

Lord Rayleigh has also shown that the shape of the mouth of the horn is important, and that this should be elliptical, the shortest diameter of the ellipse being one quarter of the longest one ; also that the mouth should occupy such a position that the long axis is vertical. Moreover, he considered that the short axis should not exceed half the wave-length of the sound being emitted. With a horn-mouth of such a shape, the sound is prevented to some extent from being projected up and down, but is diffused better laterally—a result which is desirable in coastal sound-signals.

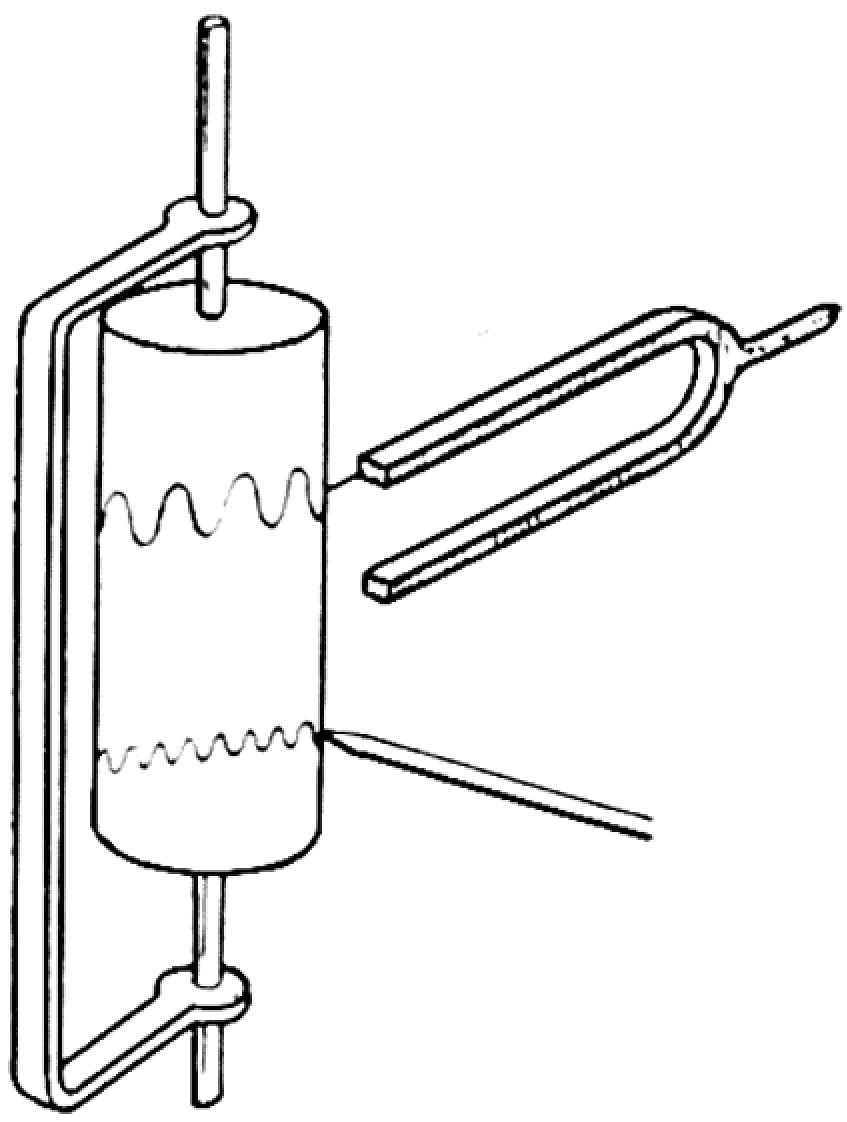
Efficiency of a siren. The acoustic efficiency of any sound generator is the ratio of the acoustic output (the rate of energy flow in the wave from the generator) to the mechanical output. The case of the siren and other compressed air generators was studied by L. V. King, and he found that the efficiency could be expressed by the equation :

$$\text{Efficiency} = \frac{T_1 - T}{\{1 - (p_0/p_1)^{(\gamma-1/\gamma)}\}\, T_1},$$

where T and T_1 are the absolute temperatures of the air on the low-pressure and high-pressure sides respectively, p_0 and p_1 are atmospheric pressure and operating pressure respectively, and $\gamma = 1{\cdot}41$. The efficiency therefore increases with the temperature difference between the two sides and decreases with the operating pressure.

GRAPHICAL METHODS

The frequency of a fork can be determined to a certain degree of accuracy by causing the vibrating fork, to which is fixed a short style, to trace its vibratory path on a suitable surface fastened on a rotating cylinder. This was the method used by Duhamel to find the frequency of a vibrating steel rod. The apparatus consists of a wood or metal cylinder A fixed to a vertical axis O and turned by a handle. The lower part of the axis is a screw working



in a fixed nut so that as the handle is turned the cylinder is raised or lowered. Round the cylinder is rolled a sheet of paper covered with a film of lampblack. The steel rod is held firmly at one end, and the other end, which carries a fine point, just touches the surface of the cylinder and thus produces an undulating trace. This trace is compared with a similar one traced out simultaneously by a standard fork of known frequency, and the frequency of the steel rod can be calculated.

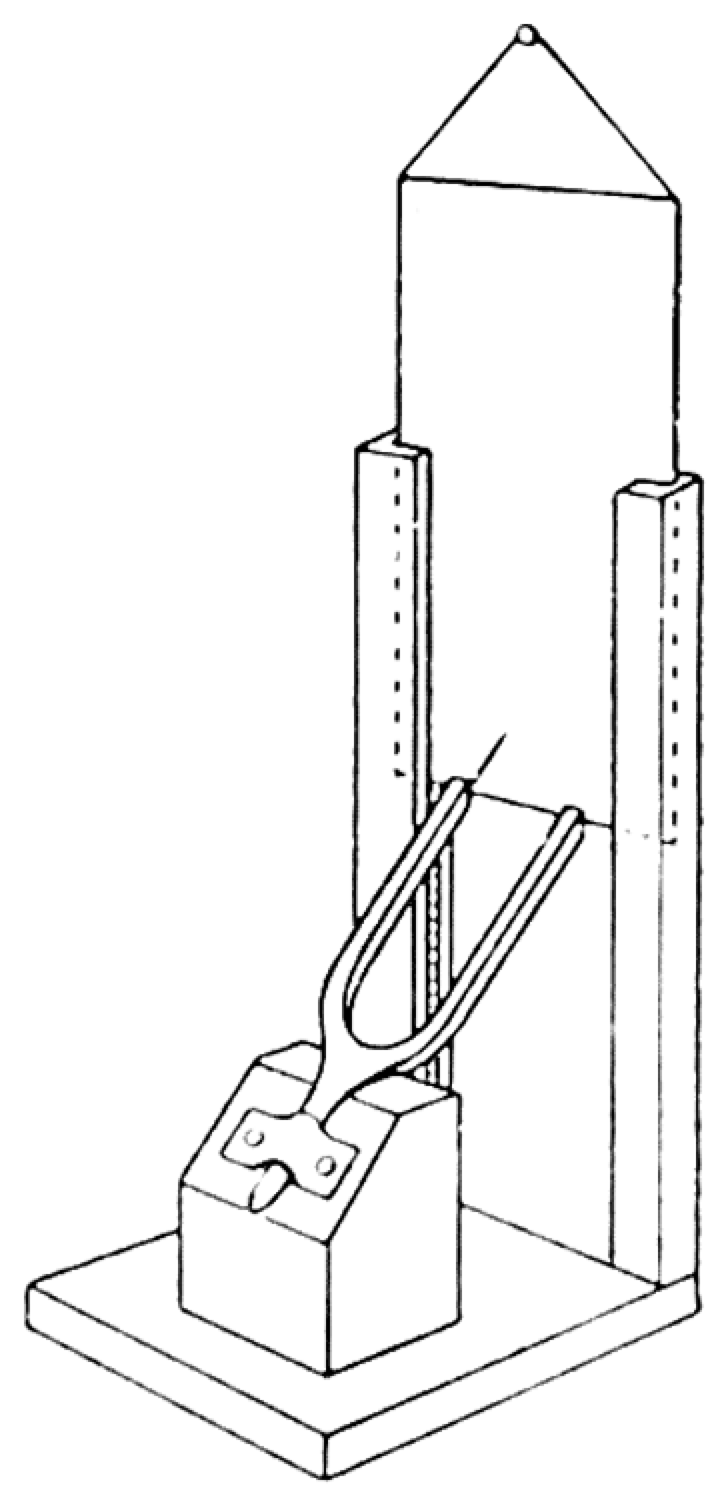
Since this method involves only a comparison of frequencies, it is immaterial whether the rotation of the cylinder is steady or not, but a source of inaccuracy is due to the fact that the standard fork has a style fixed on it and this slightly reduces its normal frequency. This alteration would of course be very small if a large fork is used, but with a small fork a correction should be made. To find the correct frequency of the loaded fork, another standard fork of about the same frequency should be obtained. Sound the two forks together and count the number of beats per second. Let the number of beats be n and the frequency of the unloaded standard fork N. Then the frequency of the other fork is $N \pm n$, which of course reduces to $N - n$ if the two standard forks are nominally of the same pitch. If they are not, it will be necessary to determine which of the two has the lower frequency when they are producing the beats. To do this, load either of them with a small quantity of wax and again count the number of beats. If this number is greater than n, then the fork just loaded probably had the lower frequency originally, for the load has only served to increase the difference in frequency between the forks. It is, however, possible that the load has reduced the frequency of the higher fork A to such an extent that it is now less than that of the other fork B by a greater number than that by which B was originally less than that of A. It is safer, therefore, always to adjust the load so that its effect is to *diminish* the number of beats, and to do this the load must have been put on the fork which was originally of the higher pitch.

The Chronograph. A more refined form of Duhamel's apparatus for finding frequency is an arrangement called a chronograph, in which the undulating trace of a vibrating fork is produced side

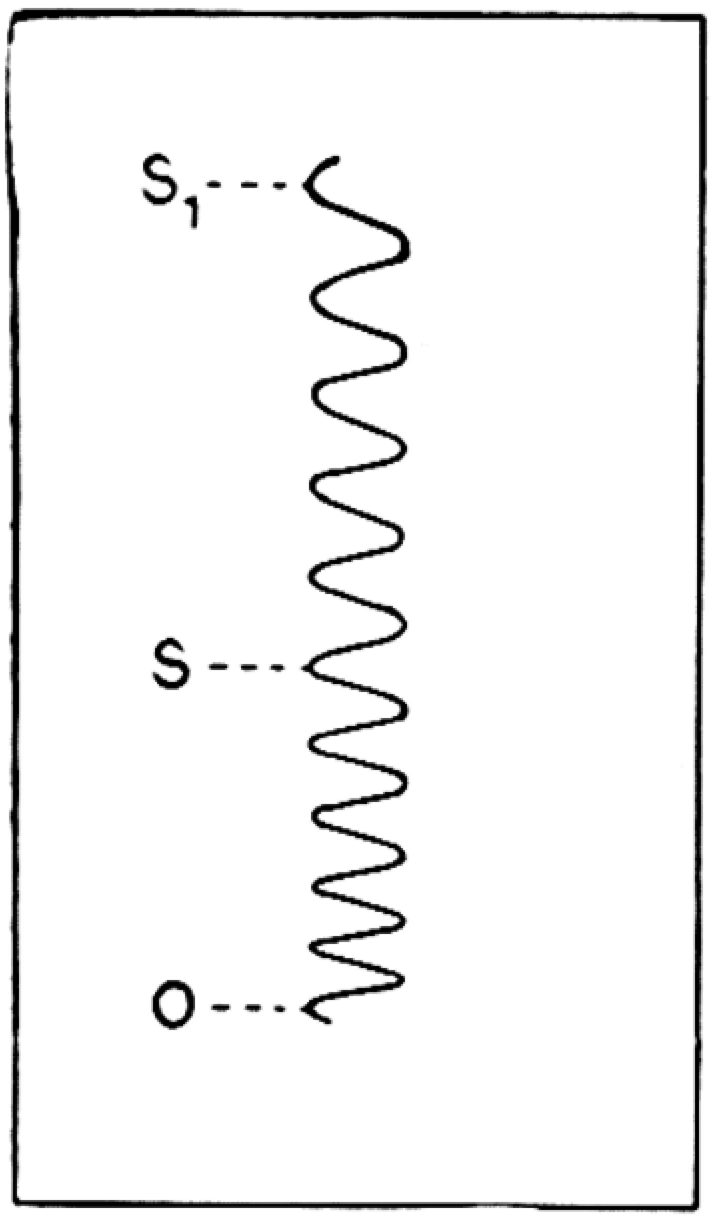
by side with a time-trace. A piece of blackened paper is fastened round a cylinder which can rotate about an axle on which a screw thread of large pitch is cut. When the handle is turned, the cylinder advances as it rotates, and the trace of the tuning fork, which is fitted with a style, can be obtained on the paper. To mark the intervals of time on the paper, a small electromagnet is provided with a style attached to its armature. The current can be supplied by a battery, and in series with it is a make-and-break actuated by a standard clock the pendulum of which beats half seconds. Thus, every half second, when the pendulum is at the bottom of its swing, contact is made and the style makes a slight movement at right angles to the normal trace, so marking the time intervals. By comparing the two traces, the frequency of the fork can easily be found.

A method similar to the above was used by McLeod and Clarke in 1880 in their investigation of the effect upon previously existing vibrations of bowing a fork, while in the chronographic method used by Prof. A. M. Mayer in 1884, the fork under investigation was fitted with a triangular piece of thin sheet metal, about a milligram in weight, which traced the vibrations upon smoked paper. In this case the time was recorded by small electric discharges from an induction apparatus under the control of a clock, and delivered from the same tracing point.

It will be noticed that in the methods above when the time-trace and the vibration-trace are recorded simultaneously on the paper, it is not necessary for the cylinder to have uniform rotation. It will also be seen that the chronograph lends itself with facility to the measurement of small intervals of time, if a standard tuning fork of known frequency is available.



Falling plate. In this method, it is an advantage to use an electrically driven fork with a light style or bristle attached to one prong. A smoked glass plate is suspended vertically by a piece of thread passing over one, or two, hooks as indicated in the diagram. The plate can be made to slide in a groove in a wooden framework to give its vertical fall, or two screws can be inserted through the



wooden upright so that the tips are against the back of the plate, and these can be adjusted so that the plate in falling is always in contact with the style on the fork. In order to prevent breakage when the plate reaches the bottom, the base of the wooden framework is padded.

The fork is set in vibration and then allowed to fall by burning the thread; owing to the combination of the two motions, a wavy line is traced by the style similar to that shown.

A point O is chosen just clear of the indistinct portion traced when the plate was moving down in its first stage and consequently before its velocity had sufficiently increased to open out the waves. Count a number of waves, n, between O and S and the same number between S and S_1, and let the distances OS and SS_1 be s and s_1 respectively.

If t is the time required for the plate to fall through these distances, and N is the frequency of the fork, then $n = Nt$. Now, if u is the velocity of the plate at the instant corresponding to point O, we have:

$$s = ut + \tfrac{1}{2}gt^2,$$

and since the total time taken is $2t$,

$$s + s_1 = 2ut + 2gt^2.$$

Hence $$s_1 - s = gt^2, \quad \text{or} \quad t = \sqrt{\frac{s_1 - s}{g}};$$

and so $$N = n \,.\, \sqrt{\frac{g}{s_1 - s}}.$$

The distances s and s_1 should be measured by means of the travelling microscope. It should be noted that the value of N obtained here is that for the fork vibrating with the load and affected by friction as the curve is being traced. To allow for this, take a second fork of approximately the same frequency as the first, but of slightly *higher* pitch, and carefully load this fork to bring it into unison with the first. Then, when the first fork is loaded with its style, etc., and the style is touching the plate, again sound the two together and count the beats per second.

This number gives the number of vibrations lost per second on account of loading and friction, and when added to N will give the corrected frequency.

The above apparatus can, of course, be used to compare the frequencies of two forks. The forks, fitted with bristles, are arranged side by side and set into vibration, and when the plate falls each style draws its particular trace. By counting the number of vibrations between the same two points in each trace, the ratio of the frequencies can be found.

METHOD OF BEATS

The phenomenon of beats has been referred to several times in this book, and the student must have recognised that this provides an excellent method of finding frequency. It is now well known that the number of beats heard in a second is the difference of the frequencies of the two sounds which produce them, and making use of this fact it is possible to copy a standard tuning fork with great precision. But before dealing with this problem, a few observations will be made concerning slow and rapid beats.

It is sometimes supposed that rapid beats have the advantage of admitting of greater relative accuracy in counting. It must be remembered, however, that in a comparison of frequencies, it is the *absolute* and not the *relative* accuracy of the counting which is important, for if the number of beats, say in a minute, is miscounted by one, it makes just the same error in the result whether the total number of beats in the time is large or small. As a matter of fact, if the two sounds are pure tones, it is advisable to use beats slower even than four per second. It is possible by choosing a suitable position to make the intensities at the ear equal, thus causing the phase of *silence* to be extremely well marked, and slow beats may be counted with great accuracy by observing the time which elapses between the periods of silence.

If the phases of maximum sound are used to count the number of beats when the beating is slow, a difficulty arises owing to the uncertainty whether a falling-off in the sound is due to interference or to the gradual dying away of the vibrations, and in his method of copying a standard fork, Scheibler adopted a somewhat modified plan. He took a fork slightly different in pitch from the standard—it is immaterial whether higher or lower—and counted the number of beats, about 4 per second, when they were sounded together. The fork to be adjusted is then made slightly higher than the auxiliary fork if this is lower than the

standard, and tuned to give with it exactly the same number of beats as did the standard. To facilitate the counting of the beats Scheibler used pendulums the periods of vibration of which could be adjusted, and in this way a highly accurate copy of the standard is obtained.

Scheibler's Tonometer. Scheibler also used the method of beats to determine the absolute frequency of his standards, the instrument devised by him for this purpose being called a tonometer. A set of tuning forks extending through an octave are arranged in ascending order of frequency, each of which gives the same number of beats with its neighbour. Hence the various frequencies increase by equal steps, and they are arranged so that the frequency of the highest is exactly twice that of the lowest. In Scheibler's instrument the consecutive forks gave 4 beats per second, so for the complete octave 65 forks would be required to bridge over the interval from c' (256) to c'' (512).

The first step is to find the absolute frequency of the lowest tone, and this is done by counting the number of beats between the sounds from the consecutive forks in the series.

Suppose there are $(k+1)$ notes, thus giving k intervals, and let the frequencies be represented by N_1 (lowest), N_2, N_3, ... , $N_{(k+1)}$ (highest). If the number of beats observed between all the successive notes be denoted by n_1, n_2, n_3 ..., n_k, we have

$$\begin{aligned} N_2 - N_1 &= n_1, \\ N_3 - N_2 &= n_2, \\ \downarrow \quad & \quad \downarrow \\ N_{(k+1)} - N_k &= n_k. \end{aligned}$$

Adding both sides, we get :

$$N_{(k+1)} - N_1 = n_1 + n_2 + \dots n_k.$$

But

$$N_{k+1} = 2N_1.$$

$$\therefore \; N_1 = n_1 + n_2 + \dots n_k.$$

This relationship gives the absolute frequency of the lowest tone, and the others can be found from this.

Although the method is somewhat laborious, it is undoubtedly a very accurate one, though it is essential to its success that each of the sounds should be of definite pitch, and that the number of vibrations of any fork should be constant whether that fork is sounding with its neighbour above or below. These conditions probably hold when independent forks are used, but a set of

reeds mounted side by side on a common wind-chest will introduce an error owing to a disturbance of pitch by mutual interaction.

A tonometer such as is described above but ranging over a series of octaves could be taken into a belfry to find the exact frequencies of the overtones of any bell. It could also be used for tuning a note to any desired pitch, and in his standard book "The Theory of Sound", Lord Rayleigh describes a method of tuning pinaofortes or organs by its use.

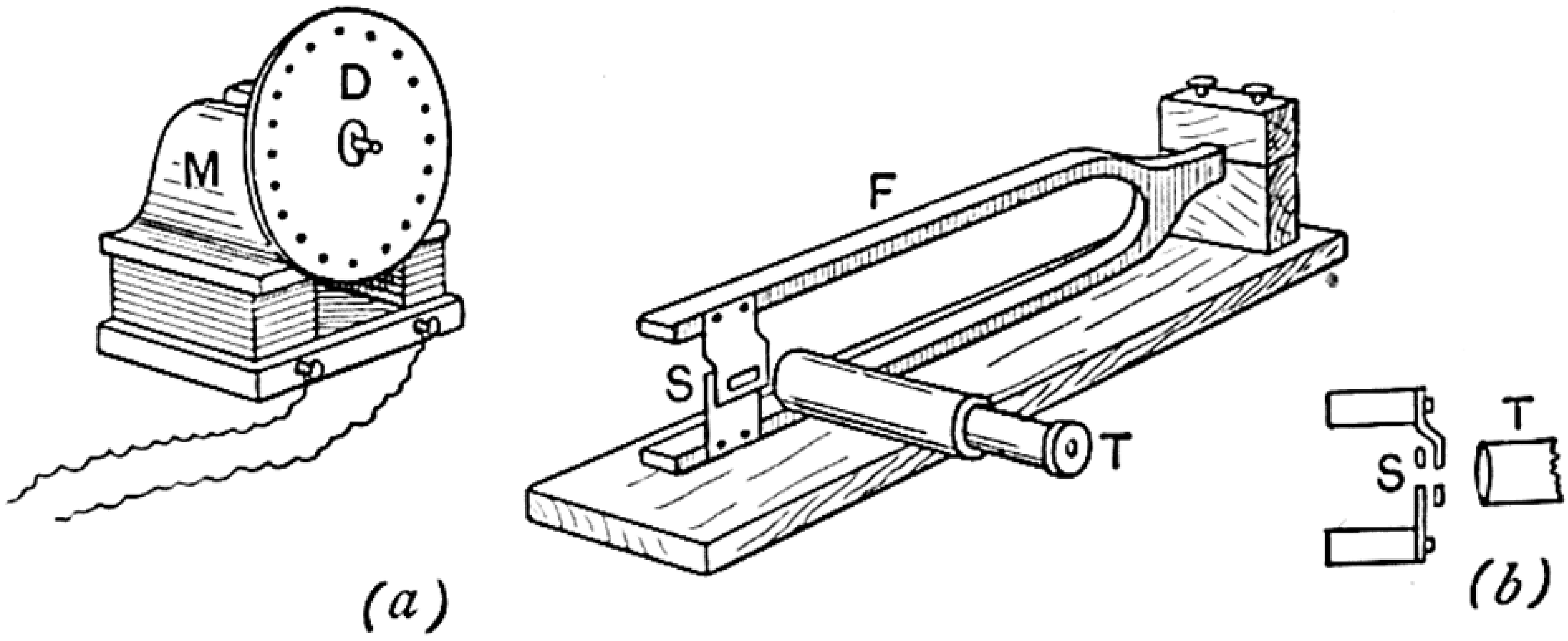
A set of twelve forks may be used giving the notes of the chromatic scale on the equal temperament (see Chapter XI), or any desired system. The corresponding notes of the forks and the piano are adjusted to unison, and the others tuned by octaves. It is perhaps better to prepare the forks so as to give four vibrations per second less than their normal frequency. Each note on the piano is then tuned a little higher than the prepared fork so that when the two notes are sounded together exactly four beats per second are heard. It is the usual practice to start from the note *a′* and get these notes in unison, and then determine the others by estimation of fifths. It will be seen in the next chapter that a fifth on the scale of equal temperament is slightly flatter than a *true* fifth, for twelve true fifths are slightly in excess of seven octaves, so that there is an inevitable, though very small, error in the tuning of the notes between the octaves, which of course are all tuned true. In violins and instruments of that class, tuning is done by true fifths from *a′*.

STROBOSCOPIC METHODS

A stroboscope is a device by means of which a moving object can be made to appear stationary. The underlying principle is that a rotating or vibrating object illuminated intermittently appears to be at rest when the frequency of illumination is the same as the number of revolutions per second made by the object, or as the frequency of the vibration. Thus a wheel making 10 revolutions per second illuminated 10 times per second appears to be stationary. Also the wheel will appear at rest if each spoke moves during the interval between the flashes into the exact position previously occupied by another spoke. If the spokes move either not quite so far or a little farther in the interval, the wheel will appear to be rotating slowly backwards or forwards. The effect is often seen in the cinema, when a wheeled vehicle is shown starting or stopping; the intermittent illumination is

caused by the movement of the camera shutter (16 times a sec.). It is clear that if the rate of the intermittent illumination can be both controlled and measured, the rate of rotation of the wheel or the frequency of vibration of a fork can be measured.

It is worth noting that as early as 1836 Plateau investigated the motion of a vibrating object by means of intermittent illumination. If, for example, an object vibrating with a certain frequency is intermittently illuminated say by a series of electric sparks occurring at the same rate, the object must appear at rest because it can be seen only in one position. If, however, the period of vibration of the object differs slightly from the rate at which the sparks occur, the object will appear to vibrate slowly with a frequency which is the difference between that of the spark and that of the object. Thus the type of vibration can be examined with facility.



The frequency of a fork can be found in the following way. In the diagram, the disc D is driven by the motor M and is viewed through the telescope T. The disc has a circle of dots equally spaced, and between the disc and the telescope is situated the fork of unknown frequency, which should preferably be electrically driven. Attached to the prongs of the fork are two light metal pieces S shown in section in diagram (b). Each metal piece has a slot in it, in such a position that the telescope and slots are in line with the dots on the disc when the fork is at rest. On causing the fork to vibrate, the slots S will pass each other, and allow the circle of dots to be seen by an observer looking through the telescope, twice in every complete vibration of the prongs. The speed of the motor can be regulated so that each time the circle of dots is seen, the dots appear to be in the same position, each dot having taken the position occupied by the dot in front of it when last seen. The dots will then appear to be stationary. If n is the number of revolutions per second of the disc and d the

number of dots in the circle, then $n \times d$ is *twice* the frequency of the fork, the dots being seen twice in each vibration of the fork.

Sometimes for convenience the disc has a number of rings of equidistant dots.

If the speed of rotation of the disc is slightly greater than that for which the circle of dots appears stationary, then each time a glimpse of the disc is obtained a given dot will have slightly passed the position occupied by the preceding dot at the preceding glimpse. Hence the dots will appear to be rotating slowly in the same direction as that in which the disc is rotating. During the time the dots appear to advance through the distance between two dots, one more dot will have passed any point than the number of glimpses. Hence if x dots appear to pass in 1 second, we have:

$$2N = nd - x,$$

where N is the frequency of the fork. Similarly, if the disc is rotating too slowly, the dots will appear to rotate slowly in an opposite direction to that in which the disc turns, and in this case we have:

$$2N = nd + x.$$

The value of the frequency obtained by this method is not strictly the absolute frequency of the fork on account of the metal pieces on the prongs; hence a correction would have to be made if the absolute value is desired. It is, however, possible to avoid this correction by making one prong bright over a small area and by rotating a disc provided with several series of concentric holes in front of it. If the fork is well illuminated and the disc carefully mounted, it will happen that for some particular speeds the fork will appear stationary when viewed through the holes.

An alternative stroboscopic method of finding the frequency of a fork is by the use of a neon lamp which, as is well known, lights up immediately without any appreciable lag when a voltage is applied, and is extinguished immediately when switched off. The fork is put in the same circuit as the primary coil of an induction coil, and the current in the primary is thus made and broken once per vibration of the fork. The neon lamp is put in the secondary circuit of the induction coil; hence it is caused to flash once per complete vibration. The lamp is used to illuminate a rotating disc provided with a series of dots on a white background, and the speed of rotation of the disc is varied until the dots appear to be stationary. When this is the case, one dot just moves up to

take the place of a dot in front of it during the interval of darkness between the flashes of the lamp, that is, during the period of vibration of the fork.

A stroboscopic method is used for timing gramophone turntables. A paper disc with evenly spaced radial markings is illuminated by a neon lamp connected to alternating current mains. If the electric supply alternates at the rate of 50 cycles per second, the neon lamp will flash 100 times per second, and the speed of rotation of the disc is increased until it appears at rest when illuminated by the lamp. Suppose there are 80 sectors on the disc. When it appears to be stationary, the disc turns through 1/80 revolutions in 1/100 second. Hence the rate of rotation of the disc is $1/80 \times 100 \times 60 = 75$ per minute. Careful regulation of the disc speed has become very important with the introduction of long-playing records.

A similar method is used for checking the rate of rotation of a gyroscope disc, which should rotate at a constant rate. The rotating disc is illuminated by a neon lamp which is caused to flash at a varying rate by means of an alternating supply, the frequency of which can be adjusted. The adjustment is made until, on viewing the disc, it appears to remain stationary, and on noting the frequency of the supply at this stage the rate of rotation of the disc can be obtained.

A modern application of the stroboscopic principle. Since the action of a stroboscope is to cause a moving object to appear stationary, it can be extended to investigate rapidly-moving parts of machinery, and actually to photograph parts of the machinery at those moments at which the observation is desired. In the case of high-speed machines, speeds up to 100 revolutions per second are not exceptional, and if it is desired to fix accurately the momentary situation of a part of such a machine the observation will not last longer than a very small fraction of a second and the moment of observation must also be fixed accurately. The light from an electric spark is very suitable for illuminating purposes and by this means it is possible to reduce the exposure time in photography to 10^{-5} or 10^{-6} sec. Careful synchronisation is required to fix with the same precision the moment at which the exposure takes place.

Messrs. Philips of Eindhoven, Holland, have designed an apparatus suitable for stroboscopic examination of rapidly moving machines. The apparatus is designed to give a single flash or periodically repeated flashes. The time interval between the flashes can be adjusted within wide limits and the generator

which excites the flashes can be synchronised with the part of the machine to be observed. The flash of an electric discharge is used and the spark gap is constructed as a discharge lamp filled with argon at high pressure. The tube of the lamp consists of quartz and is mounted in a nitrogen-filled bulb, the rear of which is covered on the inside with a mirror to concentrate the light beam in a relatively small solid angle. The flash-time of such a lamp is about 10^{-5} sec.

The generator consists essentially of a condenser which is gradually charged through a resistance up to 600 volts, and then discharged through a relay valve, which controls the moments at which the discharges are required, and the flash lamp. According to the employment of the stroboscope, there are different ways, both electrical and mechanical, of synchronising the voltage impulses which actuate the relay valve ; in the case of non-periodical phenomena, use may be made of a microphone or a photo-cell with an amplifier for synchronising purposes.

In order to be able to control accurately the number of flashes per second furnished by the stroboscope, the apparatus is provided with a frequency meter, and with the help of this the frequencies of vibrations or other periodic movements can be determined.

With such a stroboscope interesting phenomena which occur in a certain phase of the motion being investigated can be recorded photographically, while photography is also capable of furnishing information about events which occur only once.

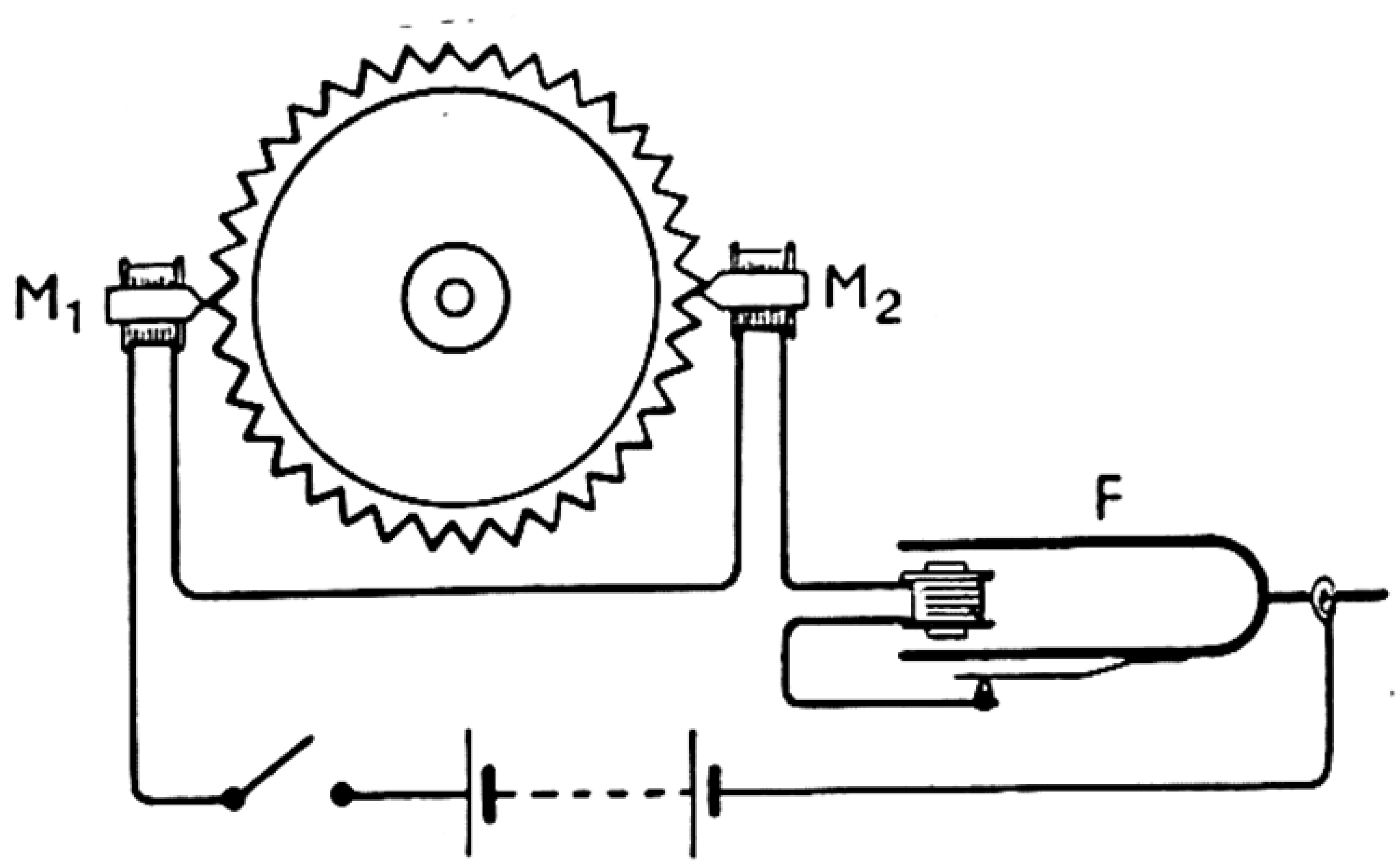
The following examples will indicate the scope of the instrument (see Plate 6 facing p. 263). In order to study the phenomenon of *cavitation* due to bubbles of gas or vapour in the water and which may result in serious erosion of the material of a ship's screw, models of screws are observed in a tank. The tank is provided with glass windows through which it is possible to illuminate the model screw stroboscopically and to observe and photograph the cavitation effects. Thus it is possible to modify the dimensions and shape of the screw in order to avoid cavitation.

A further application is found in the investigation of jets of liquids in the lubrication of machines for metal working. In the photograph of the lubrication of a centreless grinding machine which is reproduced, it can be seen that the lubricant is blown aside by the wind from the grindstone, and thus does not flow over the grindstone as was intended. This observation led to a change in construction.

Another example of the use of the stroboscope is suggested by the illustration of the collision between a tennis ball and a racket, while the possibility of photography at exactly the right moment is suggested by the illustration of a hammer smashing an electric light bulb.

OTHER METHODS OF DETERMINING FREQUENCY

Phonic wheel. This device, which was invented independently about 1878 by Lord Rayleigh and M. la Cour, can give a very accurate determination of the frequency of a fork. The apparatus may take various forms, but the essential feature is the approxi-



Phonic wheel.

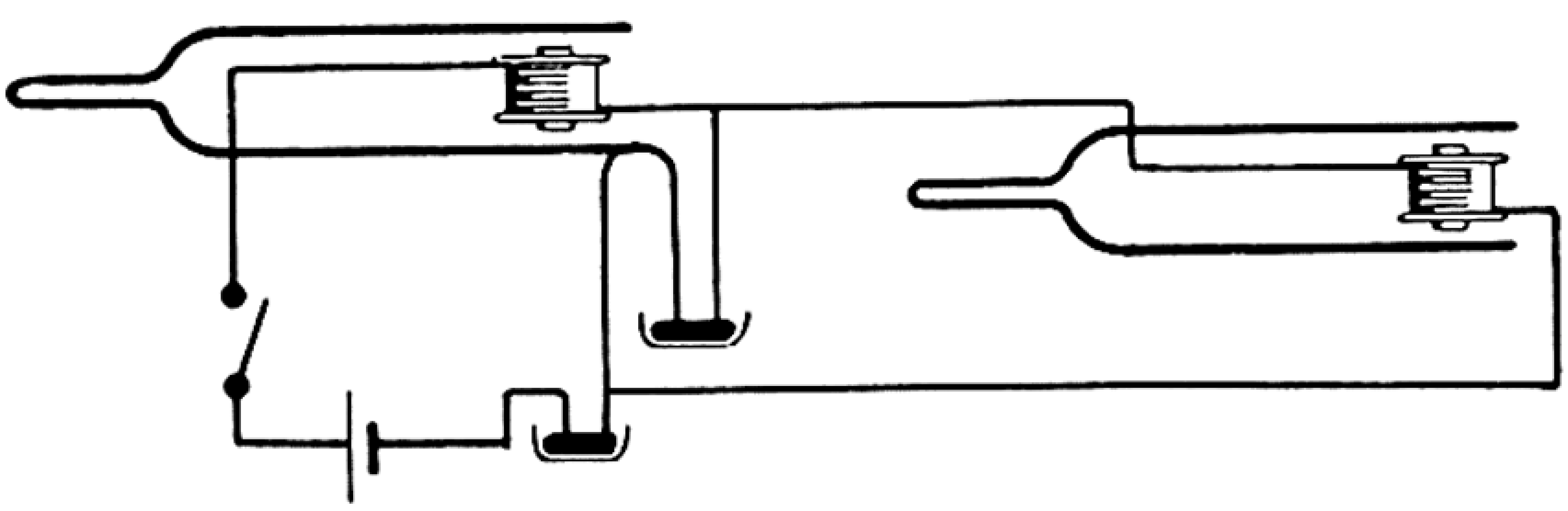
mate closing of the magnetic circuit of an electromagnet fed with an intermittent current by one or more soft-iron armatures carried by the wheel and arranged symmetrically around the circumference.

The phonic wheel consists of an iron wheel a few inches in diameter having equidistant studs or cogs on its periphery and capable of rotation about a horizontal axis. Two electromagnets M_1 and M_2, placed as shown so that the cogs almost touch the cores of the magnets, are excited by the intermittent current from an electrically maintained fork F. The wheel is first made to rotate by hand, but at a certain speed the wheel will continue to run of its own account. This is due to the fact that the frequency of excitation, that is, the frequency of the fork, is then equal to the number of cogs passing per second. Hence, if the number of cogs is known and the rotation of the wheel timed, the frequency can easily be calculated.

In the course of the rotation of the wheel, if the passage of a cog opposite the electromagnet synchronises with the middle of

the time of excitation, the electromagnetic forces acting on the cog during its advance and its retreat balance one another. If, however, the wheel be a little in arrear, it will be found that the acting forces encourage the rotation, while if the phase of the wheel be in advance the motion will be retarded. By a self-acting adjustment the rotation settles down into such a phase that the driving forces balance the resistances.

Thus, in addition to its use as a means of finding frequency, the phonic wheel is an example of an intermittent current being used to regulate the speed of a rotating object.



Helmholtz fork-interrupter with a driven fork (right).

Apart from valve-maintained forks, the usual method of obtaining an intermittent current to actuate an electrically driven tuning fork is by using a metallic make and break as described on p. 102. Reference must also be made to the **fork-interrupter**, invented by Helmholtz. This may consist of a tuning fork with the usual electromagnet between the prongs. The wires of the magnet are connected one with one pole of a battery and the other with a mercury cup as indicated in the diagram ; the other pole of the battery is connected with a second mercury cup. A U-shaped rider of insulated wire is carried by the lower prong so that the ends are over the cups and at such a height that during the vibration the circuit is alternately made and broken by one end of the rider in and out of the mercury ; the other end of the rider may be permanently immersed in the mercury in the other cup. By means of the periodic force thus obtained, the vibrations of the fork are permanently maintained.

To understand fully the mode of working of such an interrupter, it is necessary to consider what work is available to compensate for the effect of frictional forces during the motion, and to do this, account must be taken of the retardation of the current due to irregular contact and to self-induction. When the point of the rider first touches the mercury, the electric contact is imperfect, probably on account of adhering air ; while when it leaves the

mercury, the contact is prolonged on account of adhesion of the mercury to the wire. Thus, in both cases, the current is retarded behind what would correspond to the position of the fork. The effect of this retardation, together with that due to self-induction, is that more work is gained by the fork while the rider is leaving the mercury, than is lost during its entrance, and thus a balance remains to overcome the friction.

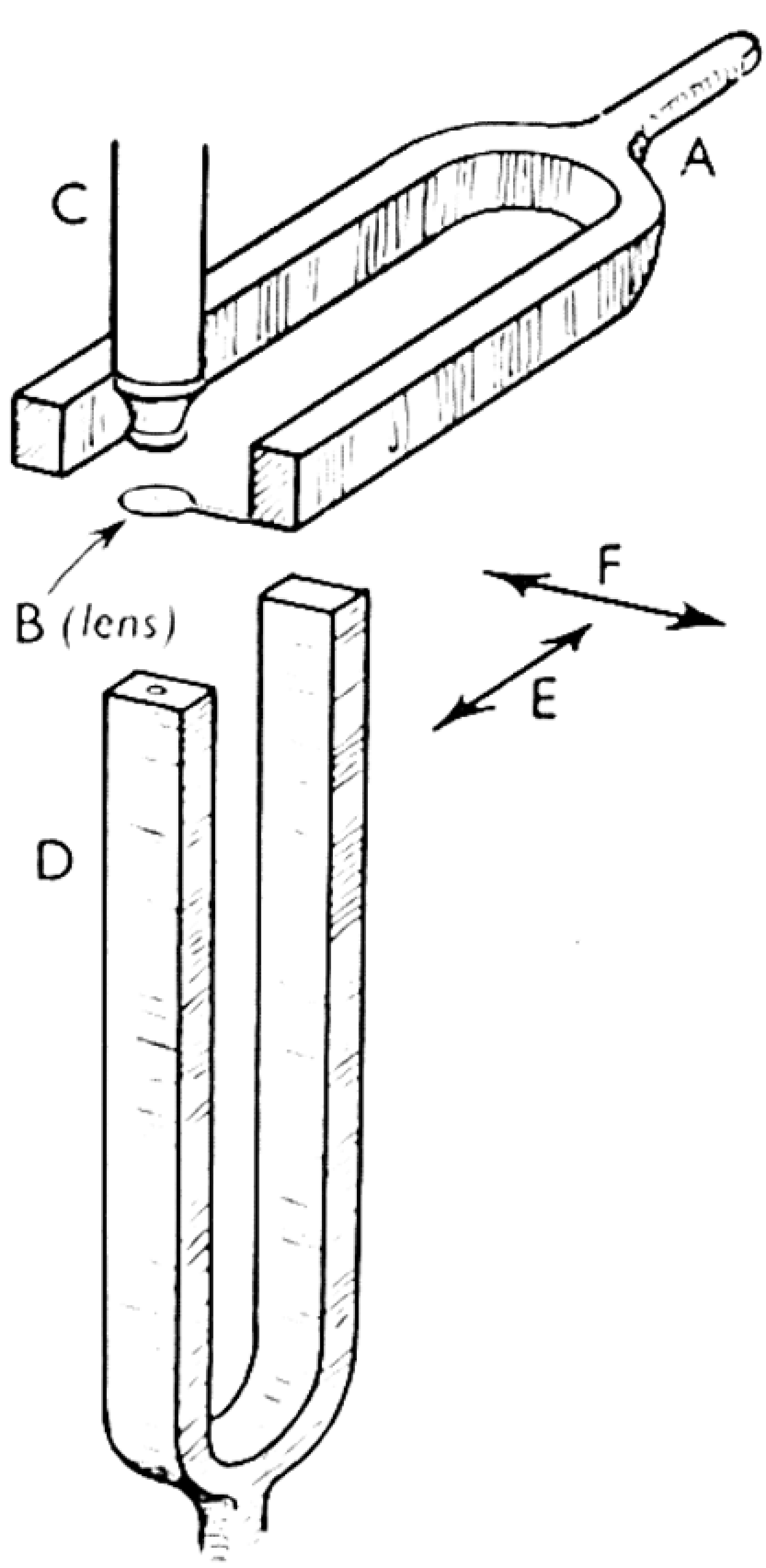
The fork-interrupter can of course be used to set another fork into forced vibration, and the diagram shows the necessary connections.

Lord Rayleigh's method. The late Lord Rayleigh performed many experiments on the determination of frequency, and two of his methods will be briefly described. In the first method, an electrically maintained interrupter fork of frequency 32 was employed to drive a dependent fork of frequency 128, exactly four times as great. This apparatus was used to test the accuracy of a standard fork of nominal frequency 128, and this fork can of course be readily compared by beats or by optical methods with the dependent fork. Therefore if the exact frequency of the driver fork (32) can be obtained, the frequency of the standard fork is easily determined.

The driver fork was compared with the pendulum of a clock, the rate of which was known, and the comparison could be direct or by the use of a phonic wheel ; Rayleigh used the latter method. The pendulum was provided with a silvered bead on which a light was concentrated, and in front of the pendulum was placed a screen perforated with a narrow vertical slit. The bright point of light reflected from the pendulum was viewed through a hole in the phonic wheel which was arranged so that one revolution corresponded to four complete vibrations of the interrupter ; thus there were eight views of the pendulum per second. Now, any deviation of the period of the pendulum from an exact multiple of the period of intermittence shows itself as a cycle of changes in the appearance of the flash of light, and the duration of this cycle was observed. Let a be the number of cycles per second between the wheel and the clock. The period of the cycle is the time required for the wheel to gain, or lose, one revolution upon the clock ; hence the frequency of revolution is $8 \pm a$, and the frequency of the driver fork is $32 \pm 4a$. This makes the frequency of the dependent fork $128 \pm 16a$, and if there are b beats per second between this fork and the standard, the frequency of the latter is $128 \pm 16a \pm b$. This is on the assumption that the clock is quite correct ; if it is not, any error can be allowed for.

The second method involves the use of only a harmonium and a watch, and the principle of the method is that the absolute frequencies of two musical notes can be deduced from the interval between them, and the number of beats produced when both notes are sounded together. If x and y are the frequencies of two notes giving an interval of a major third on the equal temperament scale, it is known that $y = 1{\cdot}25992x$ (see Chapter XI). Now the major third on the tempered scale is slightly different from the same interval on the true scale, and if the two notes are sounded together, the number of beats heard in a second depending on the deviation of the third from true intonation is $4y - 5x$. Hence, from the two equations, the values of x and y can be found.

Vibration microscope. As the changes which take place in Lissajous' figures afford such an accurate method of adjusting the frequencies of two forks to certain fixed ratios, the method is of much use in adjusting the pitch of forks. Since, however, ordinary forks are not fitted with a mirror and also the addition of a mirror would alter the pitch, the arrangement described previously in Chapter VI would not be applicable. A modification of the arrangement was invented by Lissajous and is called the vibration microscope. A large fork A, which is the standard, carries a small lens B attached to one of its prongs. This lens forms the objective of a small microscope C supported in a separate stand. Fork D which is being adjusted is arranged so that its prongs vibrate at right angles to those of A. If the microscope is focussed on a small dot on the end of one of the prongs of D and the standard fork alone is sounding, the dot will appear to be drawn out into a line parallel to line F owing to the to and fro motion of lens B. If A is at rest and D is vibrating, the dot will appear as a line parallel to line E. When both are vibrating the



pattern traced out will be the Lissajous figure appropriate to the relative frequencies of the forks. The frequency of D can be adjusted to the value required by filing the prongs either near the extremity to raise its pitch, or near the stem to lower it.

CHAPTER XI

MUSICAL SCALES: TEMPERAMENT: INSTRUMENTS: RECORDING

ALTHOUGH the music of different nations shows many striking and characteristic differences, a fundamental similarity is that each uses a definite scale or series of notes, and its music proceeds from note to note by determinate steps, though the selection of notes is varied in detail. The octave is an interval that is universally used, and the fourth and fifth are extremely common; and it is worth remembering that these intervals are the ones which gives the most perfect consonances. It must not be thought, however, that the notes of the scale were deliberately chosen so that they might be used for harmony, for even as late as the fifteenth and sixteenth centuries the principles of harmony were unknown, the purely melodic music being predominant.

The rapid growth of harmony and tonality was largely due to the Reformation, for it was a Protestant principle that the congregation should do its own singing, and as all voices could not sing the same notes without strain, this led to a repetition of the melody a fifth or a fourth above or below. From this beginning the scale was gradually developed to include notes derived from the more consonant intervals, the chief being the octave, fifth, fourth, major sixth and major third. If C is taken as the starting point, the notes defined by these intervals are C, G, F, A, E, and arranging them in ascending order of pitch, we get $C - E\ F\ G\ A - c$. Now, the first and the last intervals between these notes are much larger than the others, and eventually two notes D and B were found to fill the gaps, thus giving the major diatonic scale.

Diatonic scale. The diatonic scale is really developed from one of the old Greek scales and afterwards modified by the principle of tonality. The Greek scales were developed with the aid of the tetrachord to give a series of eight notes, and it seems that the Greek system arrived at maturity in the stage in which a range of sounds extending only for two octaves was mapped out into a series of seven modes.

The modern diatonic scale has already been referred to in

Chapter VI, but for the sake of reference the chief features will be repeated here. The scale consists of eight notes comprising an octave, and there is a definite frequency-ratio between the notes as shown.

$$\begin{array}{cccccccc} c & d & e & f & g & a & b & c' \\ 1 & \frac{9}{8} & \frac{5}{4} & \frac{4}{3} & \frac{3}{2} & \frac{5}{3} & \frac{15}{8} & 2 \\ | & | & | & | & | & | & | & | \\ 24 & 27 & 30 & 32 & 36 & 40 & 45 & 48 \end{array}$$

The intervals between successive notes are found by dividing the number representing each note by the number representing the one immediately below. Thus the interval from d to c is given by 27/24 or 9/8, and we obtain the following ratios for the intervals between successive notes :

$$c \underbrace{\ \ }_{\frac{9}{8}} d \underbrace{\ \ }_{\frac{10}{9}} e \underbrace{\ \ }_{\frac{16}{15}} f \underbrace{\ \ }_{\frac{9}{8}} g \underbrace{\ \ }_{\frac{10}{9}} a \underbrace{\ \ }_{\frac{9}{8}} b \underbrace{\ \ }_{\frac{16}{15}} c'$$

Notice that these intervals are of three different sizes ; the largest, 9/8, is a *major* tone, the next ratio 10/9 is a *minor* tone and the smallest 16/15 is a *semitone.* It will also be noticed that the following relations exist between certain notes in an octave.

$$c : e : g = 4 : 5 : 6,$$
$$g : b : d' = 4 : 5 : 6,$$
$$f : a : c' = 4 : 5 : 6.$$

These particular sets of notes are called **harmonic triads;** the first is the **tonic** triad, the next the **dominant** triad and the last the **sub-dominant** triad. In all three cases the effect which is produced when the notes are sounded simultaneously is pleasing to Western ears.

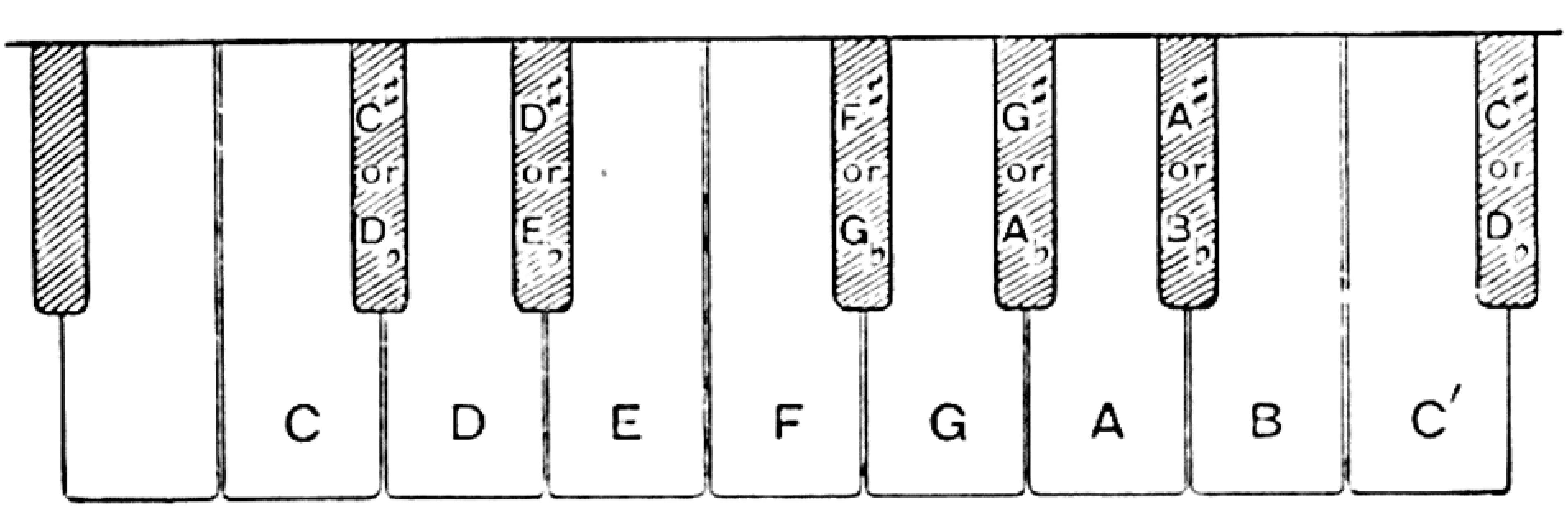
Chromatic scale and temperament. In ancient music, the intervals of tones and semitones were differently arranged in the different modes or scales to infuse variety into the music ; but by the time the diatonic scale was completed the old modes had disappeared and the only ones left were the major and minor modes, which of course limited variety. Furthermore, since the human voice and many other musical intervals have a limited compass, it became necessary to extend the choice of a key-note to any note of the scale, so that the song or composition could be brought within the limited compass. So long as the diatonic scale of C is retained, the seven notes of the scale are all that are required in a musical composition ; but as stated above, it is

frequently desired to change the key-note, and another diatonic scale would be required with the new key-note as *doh*. For example, suppose it is required to transpose a composition from the original in C to the key of G ; this process is called *modulation.* It is now necessary to find what notes would be required in the new scale, and this is done by raising each note of the original scale by a fifth by multiplying its frequency by 3/2. Hence we have (refer to table at top of p. 218) :

Key C		Key G
D	$\frac{9}{8} \times \frac{3}{2} = \frac{27}{16}$	
E	$\frac{5}{4} \times \frac{3}{2} = \frac{15}{8}$	B
F	$\frac{4}{3} \times \frac{3}{2} = 2$	giving c
G	$\frac{3}{2} \times \frac{3}{2} = \frac{9}{4}$	giving d an octave higher than D
A	$\frac{5}{3} \times \frac{3}{2} = \frac{5}{2}$	giving e an octave higher than E
B	$\frac{15}{8} \times \frac{3}{2} = \frac{45}{16}$	

It will be seen that, of the original notes in key C, use can be made in the new scale of the notes B, C, D and E, together with, of course, the new key-note G. But instead of A with a frequency of 5/3, there is a note slightly different with a frequency of 27/16, the interval between these two, represented by 81/80, being called a *comma*. Also, instead of F, with a frequency of 4/3, there is a note of frequency 45/32, almost midway between F and G. Hence for the new scale two new notes have to be introduced, and if this has to be done for each modulation, it is obvious that the total number of notes required in the octave will become very large. For voices and some stringed instruments this does not matter, but for keyed instruments it is of the greatest importance.

The requirement of seven notes to the octave as represented by the diatonic scale has now been extended and the number of notes increased to twelve, and a glance at the keyboard of a piano will show that the octave comprises seven white and five black notes as shown. It must be noted, however, that some



sacrifice of true intonation has to be made in order to limit the the notes to twelve, and this compromise is termed **temperament.**

Various systems of temperament have been used, chief among which are the Pythagorean system, mean tone temperament and equal temperament. The simplest and that now universally adopted, is the last one. On referring to the numbers representing the frequencies for the diatonic scale, it will be seen that the intervals from c to d, d to e, f to g, g to a and a to b are nearly the same, being represented by 9/8 or 10/9, while the intervals from e to f and b to c, represented by 16/15, are about half as much. In the system of equal temperament, the octave is divided into twelve exactly *equal* semitones, and this gives complete freedom of modulation. In addition, as all the intervals are exactly equal, it is just a matter of convenience which note is chosen as the key-note. From the twelve notes, which comprise the **chromatic** scale, the diatonic scale belonging to any key can be selected according to the following rule. Taking the key-note as the first, fill up the series with the third, fifth, sixth, eighth, tenth, twelfth and thirteenth notes counting upwards. As an example, consider the scale of G. The notes of the chromatic scale can be written as follows :

G	$G\sharp$	A	$A\sharp$	B	c	$c\sharp$	d	$d\sharp$	e	f	$f\sharp$	g.
1		3		5	6		8		10		12	13

Thus the notes required for the scale of G are

$$G, \quad A, \quad B, \quad c, \quad d, \quad e, \quad f\sharp, \quad g.$$

Other scales may be dealt with in a similar way ; but it must be noted that $f\sharp$ (f sharp) is in between the notes f and g, and is a note higher in pitch than f but lower than g. It could equally well be written $g\flat$ (g flat), and indeed it is sometimes more convenient to regard these *accidental* notes, as they are sometimes called, as flats rather than as sharps. But whichever way they are regarded, on the tempered scale $g\flat$ and $f\sharp$ and similar pairs are identical in pitch, for they are really the same note on a keyed instrument.

As has been stated earlier, the advantages of the equal temperament scale are obtained at a sacrifice of true intonation, and in the first place it is easy to show that the *tempered* semitone is not identical with the *true* semitone, which interval is 16/15 or 1·067. To find the value of the tempered semitone, it must be remembered that, repeated twelve times, it doubles the frequency of the

note. Let x be the ratio of the frequencies for the semitone. We have $x^{12}=2$, whence $x=2^{1/12}=1{\cdot}0595$. Further examples of the difference between the two scales are as follows. The tempered third, being the third part of an octave, is represented by the ratio $2^{4/12}:1$ or 1·2599, while the true third is represented by 1·25. This difference, of course, is not of much consequence in quick music played on a piano, but if the notes are sustained as in a harmonium or an organ, the consonance of chords is slightly impaired. Again, the tempered fifth is obtained from the ratio $2^{7/12}:1$ or 1·4983, since seven semitones make a fifth and there are twelve in an octave. This interval is therefore lower than a true fifth in the ratio 1·4983 : 1·5 or approximately 881 : 882, which also is very small. Finally, from the accompanying table it will be seen that the intervals of the harmonic triads in the tempered scale are 1 : 1·2599 : 1·4983 instead of 1 : 1·25 : 1·5 for the true diatonic scale.

Table Comparing the True Diatonic Scale with the Scale of Equal Temperament

Note	Frequency-ratio	Frequency-ratio	Interval in cents		Frequency with $a'=440$ (Standard)	
	True Scale	Tempered scale	True	Tempered	True	Tempered
c'	1·0	1·0	0	0	264	261·7
d'	$\frac{9}{8}=1{\cdot}125$	$2^{2/12}=1{\cdot}1246$	204	200	297	293·7
e'	$\frac{5}{4}=1{\cdot}25$	$2^{4/12}=1{\cdot}2599$	386	400	330	329·7
f'	$\frac{4}{3}=1{\cdot}333$	$2^{5/12}=1{\cdot}3348$	498	500	352	349·2
g'	$\frac{3}{2}=1{\cdot}500$	$2^{7/12}=1{\cdot}4983$	702	700	396	392
a'	$\frac{5}{3}=1{\cdot}667$	$2^{9/12}=1{\cdot}6818$	884	900	440	440
b'	$\frac{15}{8}=1{\cdot}875$	$2^{11/12}=1{\cdot}8878$	1088	1100	495	493·9
c''	$2=2{\cdot}0$	$2=2{\cdot}0$	1200	1200	528	523·3

MUSICAL INSTRUMENTS

Probably the first musical instrument ever invented was the drum, perhaps a development of the hollow tree which primitive men found emitted a sound when struck on the outside. In any event, the drum is certainly the ancestor of all percussion instruments.

So far as wind instruments are concerned, these probably have

their origin in the sounds produced in hollow bamboo pipes when the wind blows across them, and a development of this, *Pan's pipes*, blown with the mouth, is the ancestor of modern wind instruments; one of the earliest of these instruments was the *ocarina*, which was certainly known to the ancient Chinese. The forerunner of the stringed instruments seems to be the *lyre*, which of course was played in olden times, most probably to accompany singing and not as a solo instrument.

It is not the intention here to discuss all the musical instruments, ancient and modern, but rather to consider how the acoustic principles dealt with in earlier chapters are applied to typical instruments.

The most general and broad classification is to divide all instruments, as suggested above, into the three classes, stringed, wind and percussion instruments, but of course there are other ways in which instruments can be distinguished from each other. In the first place, one could investigate how the sounds are produced in the various instruments, and in this connection, the

Features of instrument	Instrument	Characteristic
Compass of instrument	Harmonium Concertina	large compass small compass
Scale in use	Bugle Cornet	harmonic series chromatic series
Power of sound	Trombone Flute	powerful sound feeble sound
Nature of sound	Trumpet Clarinet	declamatory smooth
Persistence of sound	Harp Violin	sounds quickly die away sounds are sustained
Quality	Oboe French horn	penetrating quality muffled quality
Capacity for melody or harmony	Cello Piano	melody (generally) harmony

exciter which causes the vibration, the vibrating system including the actual vibrator and any resonator present, and the manipulative mechanism for the production of scales, expression, etc., would all have to be considered. Then again, different instruments possess definite characteristics which distinguish them from other instruments, such as compass, the scale used, whether the sounds are powerful or feeble, and so on, and the table opposite indicates some of these distinguishing features.

Finally, musical instruments may be classified on a more scientific basis by considering the overtones in the sounds produced. In most instruments, the full harmonic series of overtones is present ; but, as has been seen earlier, there are several in which only the odd harmonic series is found. In church bells, bars and gongs, the overtones are inharmonic, that is, the frequencies are inexpressible by small whole numbers, while of course the tuning fork is practically without overtones. The human voice, which will be dealt with more fully later in the chapter, is rather unique as a musical instrument, inasmuch as some of its harmonics near fixed pitches are specially favoured whatever the pitch of the note.

The rest of this section will be devoted to considering certain typical instruments in the three broad classes mentioned above. These are :

(1) **Stringed instruments,** in which a stretched string is made to vibrate (*a*) by means of a bow, as in a violin, cello, etc. ; (*b*) by striking with a padded hammer as in the piano ; (*c*) by plucking with the finger or a plectrum of metal or bone as in the harp, etc.

(2) **Wind instruments,** which are sub-divided into two classes : (*a*) brass instruments in which the air columns are set into vibration by the players' lips, as in the trumpet, etc. ; (*b*) wood-wind instruments, where the vibrations are caused by reeds as in the case of oboes, clarinets, etc.

(3) **Percussion instruments,** such as drums, cymbals, tambourines, bells, etc., in which the vibrations are caused by striking with a hammer a stretched membrane or a metal plate or rod.

STRINGED INSTRUMENTS

(*a*) **Violin.** The essential features of this instrument are well-known. In classical form it consists of a wooden box of characteristic shape, composed of a back, belly and ribs. These are shaped out of thin wood, the belly generally being made of pine, and maple is used for the rest. A neck or handle is affixed to one

end, and a tail-piece, to which the strings are fastened, to the other.

The four strings, giving as their prime notes the pitches of g (lowest), d', a', e'' (highest), are made of gut obtained from the intestines of lambs, and the g string is wrapped round with fine wire. This increases the mass per unit length of the string without unduly interfering with its flexibility. Nowadays, solid metal strings (steel and aluminium) are sometimes used. The strings are tuned in perfect fifths by adjusting the tension by means of the pegs provided (or in the case of metal strings by other devices), and they are made to give the various notes of the scale by "stopping", that is, by pressing them down on to the finger-board with the finger. Since shortening the vibrating part of a string always raises the pitch of the note, it is clear that the compass of the violin is limited at its lower end by the prime tone of the g string, but the upper limit depends almost solely on the skill of the performer. The skilled violinist has also great control over the quality of the notes, and can determine the overtones which shall accompany the fundamental by altering the point on the string at which the bow is applied, and by lightly touching it without "stopping". In general, the special quality of tone characteristic of the violin is associated with a very complete set of overtones, but there seems no doubt that the tone also depends upon the wood chosen, the shape and even the varnish used.

The bow, consisting of stretched horse-hair, is usually applied to a point of the string about one-twelfth of the length from the bridge, and of course the motion of the bow should be across the string. Any motion along the string sets up longitudinal vibrations, and this usually accounts for the high-pitched squeaks produced when the instrument is in the hands of a beginner.

The vibrations of the strings themselves would give a weak and poor sound if they were unable to communicate their vibrations to some resonating surface, and this is achieved by the bridge, which transmits the vibrations to the belly of the instrument. Inside the body is a piece of wood called the sound-post, wedged between the belly and the back and sustaining the pressure of the bridge, and this touches the belly at a point near the foot of the bridge to play its part in transmitting the vibrations. It is the foot of the bridge near the less tightly stretched g string which plays the principal part in the transmission of the vibrations, which are then transmitted to the back chiefly by the sound-post, but also partly by the sides.

The vibrating of a violin string when bowed is due to the fact

that when the bow is firmly applied and then drawn, the friction between the two surfaces is sufficient to cause the string to be displaced while still in contact with the bow. But eventually the restoring force overcomes the force of static friction and the string slips past the bow towards its normal position. It becomes displaced in the opposite direction for probably half a complete vibration, when it is again gripped by the bow and the action repeated a number of times. The nature of the vibration, however, requires a little investigation. Since the note produced is musical, it is to be inferred that the vibrations are periodic, giving a regular displacement diagram, which in the ordinary simplest case of harmonic motion is a sine curve. The note elicited by the bow has practically the same pitch as the natural note of the string. Hence the vibrations, although forced, are in some sense free; they are wholly dependent for their maintenance on the energy drawn from the bow and yet the bow does not determine or scarcely modify their periods.

It must not be concluded, however, that because the string vibrates with its natural periods the displacement diagram is of the usual sine form. Helmholtz investigated this problem, and from the indications of theory supplemented by experimental observation succeeded in determining the principal features of the case. In his experimental work, he used a vibration microscope to obtain a view of the curve representing the motion of the point of the string under observation; but a device used by Krigar Menzel and Raps will be described here. In this arrangement, a brightly illuminated vertical slit was placed behind a horizontally stretched string at the point where the motion was to be observed. An image of the slit was thrown on to a photographic plate by a lens and of course it showed up as a bright slit with a dark spot where it was crossed by the string. If now the string is bowed so as to set it vibrating in a vertical plane, the spot will appear to move up and down, following the motion of the string. Generally, the vibrations will be much too rapid for the spot to be visible, but if the plate is moved horizontally at the same time, a curve will be traced on it, which in fact is the displacement diagram of the point of the string under observation.

It was found that, although the form of the curve depends to some extent on the point of observation and the point at which the bow is applied, the most usual form was the straight-line type of displacement indicated in the diagram, where AB represents a complete period, the abscissae corresponding to time and the ordinates corresponding to the displacement of the point. The

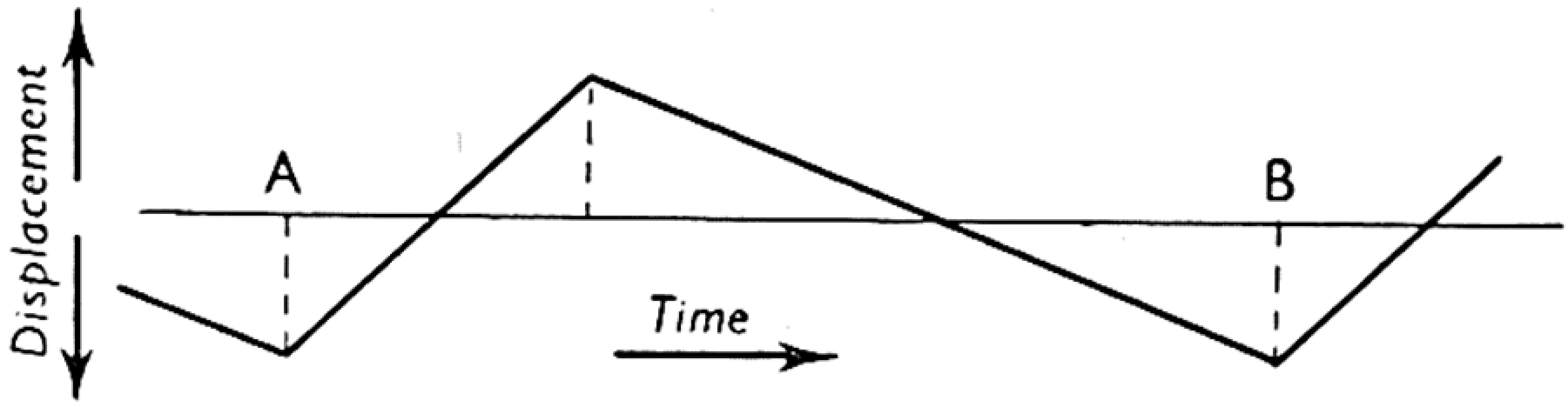


diagram shows that the whole period may be divided into two parts, during each of which the velocity of the observed point is constant, but the velocities to and fro are unequal. According to Lord Rayleigh, the simplest results are obtained when the bow is applied at a node of one of the higher components of the note, and the point observed is one of the other nodes of the same system of vibrations.

It will be seen from the above that the harmonic content of a note played on a violin is a very complex function, depending on a number of factors ; but a further complication is found when the harmonic structure of the sound from each string is analysed, for each string behaves differently as regards both the intensity and the number of harmonics. In the e'' and g strings the fundamental is relatively weak, especially in the latter string, but in spite of this, the characteristic pitch is usually associated with the lowest frequency. This is due to that property of the ear which enables it to associate the whole harmonic series with the mathematical fundamental, even though the latter may be weak or indeed missing altogether. In the a' and d' strings the fundamental is the most intense.

(*b*) **Piano.** In this instrument, a set of wires of different lengths is stretched on an iron frame ; this frame may be vertical as in the *upright* piano, or it may be horizontal, as in the *grand* piano. The longest wires are on the left-hand side of the instrument ; and each of these is a single thick wire loaded with a closely wound spiral of thick copper wire wrapped around it ; these wires when struck emit the low-pitched notes. After these wires, about twenty in all, is a set of pairs of thinner wires which are not loaded ; there are perhaps thirty of these " double " wires. Beyond these on the right-hand side are the remainder of the wires, of gradually decreasing length, arranged in groups of three, the wires of each group giving the same pitched note when struck. The purpose of the " doubling " and the " trebling " of the upper wires it to give a more or less uniform intensity of sound, for if all the notes had single wires the upper notes would sound weak in comparison with the bass notes.

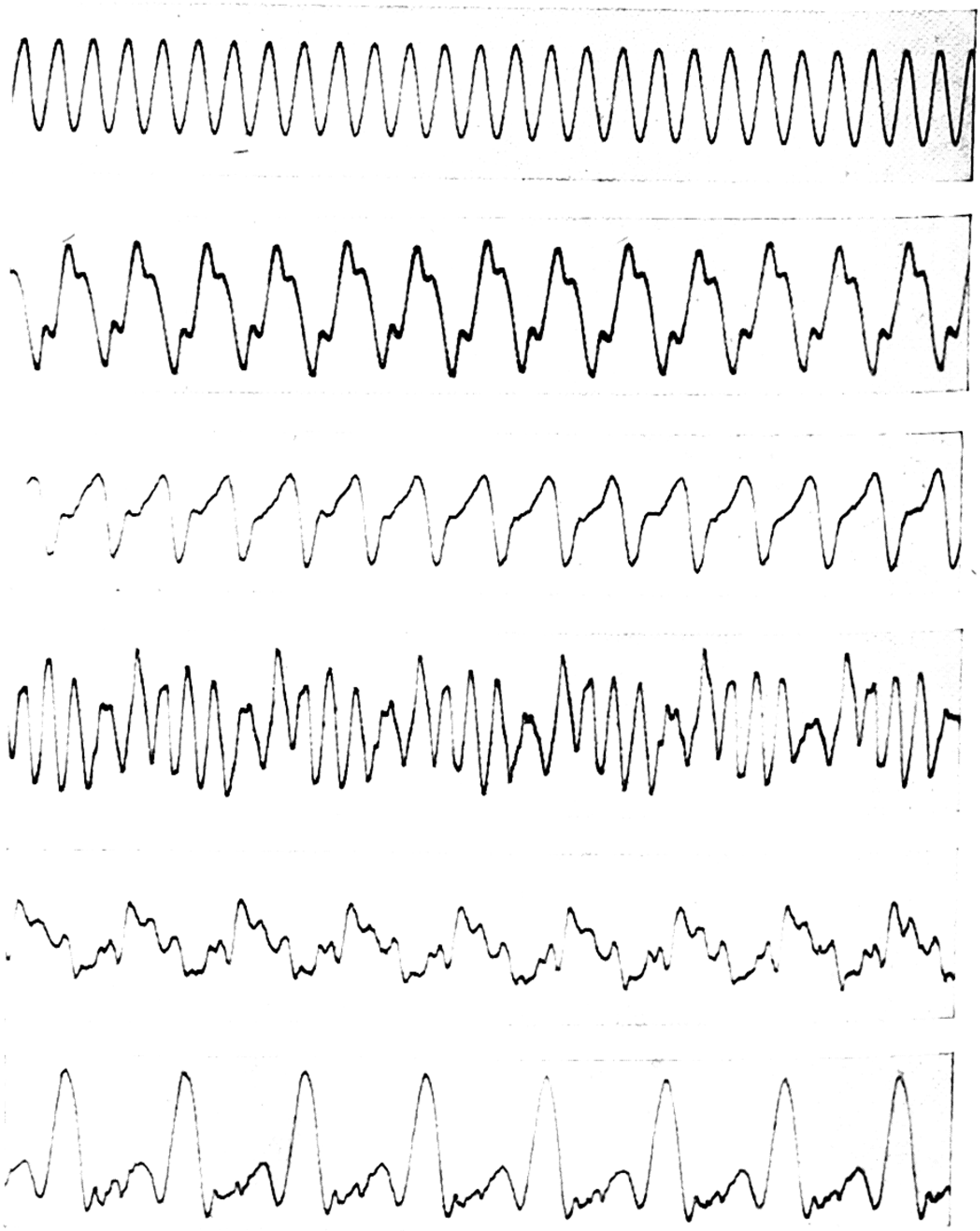
At the bottom of the piano are two *pedals* for use by the feet

of the pianist. When neither pedal is depressed, an arrangement is used for automatically damping the vibration of the wire immediately after it has been struck and the keyboard "note" released; this ensures that the note is sounded for only an instant. Such an action is necessary when the pianist is playing a tune, otherwise one note would be sounding when the next one was struck. If the left pedal, known as the *soft pedal*, is depressed, the rack supporting the hammers is moved nearer the wires, or a strip of felt is interposed between hammers and wires; hence the blow given to the wire is not so strong. If the right pedal, known as the *sustaining pedal*, is depressed, the damping arrangement does not function; hence the wires continue to vibrate, giving a sustained note. A sounding board is fixed to the back of the frame of the piano to increase the intensity of the sounds.

The wooden hammers with which the wires are struck are covered with felt, and strike the wire at a point about one-seventh of the length of the wire from one end, and this causes the seventh, eighth and ninth overtones to be rather weak. The characteristic quality not only depends upon the overtones present, but also on the hardness of the hammers, on their time of contact with the wires and on the sounding board. If the hammer surface is sharp and hard, the upper overtones become more prominent and the resulting quality is brilliant and even harsh. On the other hand, if the hammers are flat and soft, the tone becomes dull; the quality desired between these extremes can be obtained to some extent at any rate by teasing out the felt. In the design of a piano it is important that the strain on the frame should be uniform; the tension therefore of all the wires must be approximately the same.

(*c*) **Harp.** In a piano, the sounds may be regarded as *constant*, inasmuch as each note requires a separate string, while in instruments such as the violin and guitar, the sounds are *varied* by the fingering and can therefore be produced with fewer strings. The harp is a kind of transition from the instruments with constant sounds to those with variable sounds. Its strings correspond to the natural notes of the scale, but by means of pedals the lengths of the vibrating parts can be changed so as to produce the sharps and flats. The sounds are produced by plucking the strings with the finger, which is relatively broad and soft, and this gives to the harp its characteristic sweet and soft tone. It should be noted here that the ivory plectrum used in a mandoline gives a harsh metallic quality. The number of overtones sounding when a harp is played is about the same as in the case of a piano; but

(*a*)
(*b*)
(*c*)
(*d*)
(*e*)
(*f*)



Oscillograms of some musical instruments. (*a*) Tuning fork; (*b*) recorder; (*c*) flute; (*d*) oboe; (*e*) clarinet; (*f*) saxophone. From Winstanley: *Textbook on Sound* (Longmans, Green), by permission of C. B. Daish and D. H. Fender.

there is a marked difference in the relative intensities of the overtones in the two cases. In the piano, the first overtone is quite strong and the subsequent higher ones comparatively weak, whereas in the harp, the first overtone, although strong, has a lower relative intensity than the corresponding one in the piano, and there is a much more gradual falling off in intensity among the subsequent overtones.

The sound from a harp is strengthened by the sounding-box and also by the vibrations of the other strings harmonic with those played.

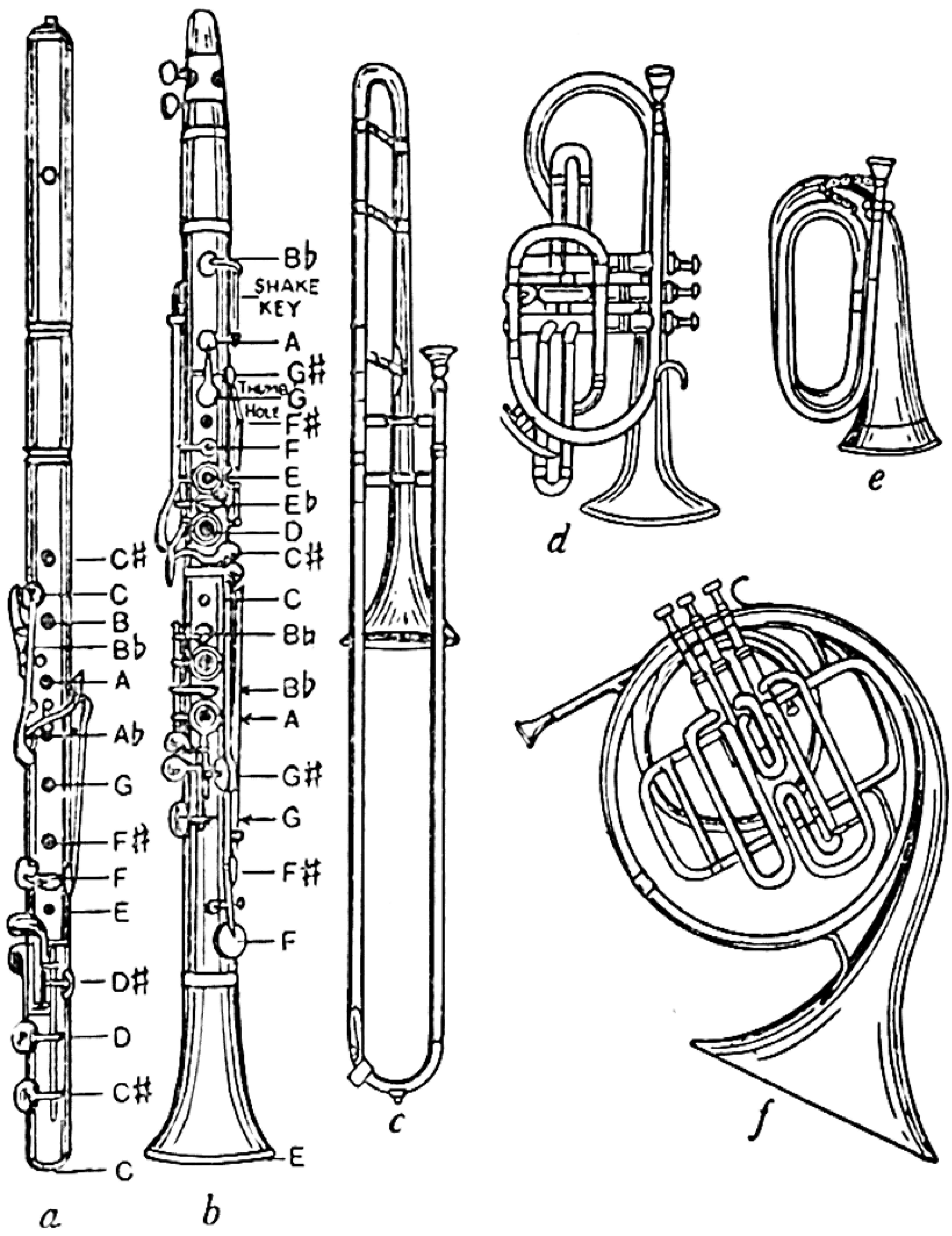
WIND INSTRUMENTS

Wood-wind. The chief instruments of this type used in orchestral work are the flute, oboe, clarinet and bassoon, and a feature common to all of them is that, to give the various notes, they are all provided with holes, some of which are covered by keys and the rest by the fingers.

The **flute** is a special adaptation of the flue pipe of an organ, and is excited by a blade-like stream of air being blown across the mouthpiece aperture. The modes of vibration of the air in the tube and the pitch of the resulting note are controlled by opening and closing the holes at different points. In this respect it is similar to a tin-whistle, though the mechanical method of exciting the vibrations is different in the two cases, and whereas the tin-whistle usually produces the notes of the diatonic scale, the flute gives the chromatic scale with a range of about an octave. The resonances of a flute are similar to those of a pipe open at both ends, and both even and odd harmonics may be present.

The **oboe, clarinet** and **bassoon** are definitely reed instruments, and are excited by small single or double-cane reeds in the mouthpieces. To re-state here what was said in Chapter VIII, the oboe is a conical tube whereas the clarinet is a cylindrical tube. Hence, although in both instruments one end is open and the other closed by a reed, the whole harmonic series of overtones may be present in the oboe but only the odd overtones in the clarinet.

Another important difference between the clarinet and the oboe is in the relative energies of the fundamental and the various harmonics present. In the former instrument, the fundamental is rather weak and most of the energy is in just a few of the higher harmonics ; but in the oboe the fundamental is practically missing and about two-thirds of the total energy is in the 4th and 5th harmonics. In the case of the flute the fundamental is



(*a*) Flute; (*b*) clarinet; (*c*) trombone; (*d*) cornet; (*e*) bugle; (*f*) French horn. From McKenzie, *Sound* (Camb. Univ. Press).

strong, and very little energy is in any of the harmonics except the 2nd.

In the smaller reed instruments like the clarinet the reed is tuned to a frequency considerably higher than the fundamental frequency of the air column. This means that for the lower notes the frequency of the reed is controlled by the natural frequencies of the air column, but for the higher notes, the natural frequencies of the reed are more of a controlling factor, and the reed resonances strengthen the higher harmonics of the instrument.

Brass instruments. In these instruments, chief among which are the bugle, trumpet, cornet, trombone and French horn, the lips of the player act as a double membranous reed and are the controlling cause of the vibrations of the air inside the instruments. The essential feature of all these instruments is a metal

tube, narrow at the mouth-piece and opening out at the other end. As was stated in Chapter VIII, the exact shape of the tube is carefully adjusted so that the natural modes of vibration of the air may give a series of overtones forming an exact harmonic series with the fundamental.

A limitation of these instruments is that it is only when the frequency of vibration of the lips of the performer coincides with the fundamental or one of the overtones of the instrument that any musical note will result. It will be remembered that if the fundamental is, say, c, the harmonic series of overtones will be c', g', c'', e'', g'', etc., and these are the only notes obtainable in brass instruments, unless some special provision is made for altering the effective length of the tube. In the **bugle** there is no mechanism whereby the length can be varied ; hence the range of possible tones is confined to the harmonic series.

In the **trumpet, cornet, French horn,** etc., there are valves or pistons which, when depressed, add an extra length of tubing to the instrument and hence lower the pitch of the note. There are as a rule three such valves, which lower the scale of the instrument by a semitone, a tone and three semitones respectively, and as they can be used simultaneously the scale can be lowered by six semitones. If these instruments are to be used with others it is of course necessary that some provision should be made for tuning. This is very often done by a small adjustable crook which alters the scale by about a semitone, but care has to be taken to see that any alteration in length brought into action by the use of the valves or pistons produces the same effect for one position of tuning-crook as for another. To give facilities for adjusting the valves for any position of the tuning-crook, each is fitted with a small slide, and in some instruments compensating valves are used.

In the **trombone** the length of the tube is varied by means of a U-shaped sliding crook which can move in or out. There are seven positions for the crook, and a change from one position to the next changes the scale by a semitone, so that the scale of the instrument can be changed by six semitones as in the other instruments mentioned above. Clearly this instrument is not a suitable one for playing rapid passages in music.

Effect of temperature on wind instruments. It was seen in Chapter VIII that the natural frequency of vibration of a column of air of given length is related to the velocity of sound in the air ($V = n\lambda$) ; also that this velocity is greater in warm air than in cold, and greater in moist air than in dry. Since the human

breath is warmer and moister than ordinary air, it must follow that the note obtained on first blowing into a cold instrument is somewhat flatter in pitch than that obtained after blowing some time. The performer therefore has to warm up his instrument before starting to play by gently blowing into it.

The pitch must also depend somewhat on the temperature of the air in the room in which the instruments are being played. For small instruments like the flute and oboe which are easily warmed by the player's breath, the variation in pitch is slight, and according to D. J. Blaikley's tables the percentage increase in the frequency of these instruments for a rise of temperature of 10° F. is 0·31 per cent. For the trombone and French horn, the increase is 0·60 per cent. and for organ flue pipes 1·05 per cent.

Reed instruments. It was stated on p. 165 that there are two kinds of reeds used in musical instruments, namely, beating reeds and free reeds. The former type are found in organ pipes, but the rather cutting quality of tone and disagreeable harshness which would be produced as the reed strikes the edges of the opening at each vibration is somewhat modified by the vibration of the air column in the pipes. Very often, to minimise the harshness further, the reed in an organ pipe is curved and made to roll over the opening, the lower end of the reed closing over the aperture before the tip, thus making the stoppage of the air-blast more gradual.

There are some reed instruments, however, such as the mouth-organ, concertina, American organ and harmonium, where there are no pipes to modify the quality of the notes emitted, and in these instruments, free reeds are used. In the American organ, the bellows create a partial vacuum, and the pressure of the atmosphere drives air through the reeds; but in the harmonium the bellows drive the air through the reeds directly. It is of course the natural frequency of the free reed which determines the pitch of the sound emitted in these instruments, and this can be altered if necessary by filing the free end to increase the frequency or filing the fixed end to flatten the tone. It is clear that free-reed instruments, unlike say, the piano, require only occasional tuning.

In the harmonium, above each reed is a small air chamber, the shape and size of which determine the quality of the notes, and generally there are eight sets of these controlled by stops, four for the treble and four for the bass. The pressure in the wind-chest is regulated by a valve, but an " expression " stop can be used so that the pressure in the chest is completely controlled by the pedals, and the sound can be made to swell or diminish as desired.

THE HUMAN VOICE

The mechanism of the human voice may be regarded as a type of double-reed instrument. In a cavity in the air passage from the lungs to the mouth, situated at the top of the wind-pipe and marked in position by the projection in the throat called " Adam's apple ", are two horizontal, stretched, greyish-white membranes. The cavity is called the **larynx** and the membranes the **vocal cords**, between which there is a narrow adjustable slit called the **glottis**. The outer edges of the cords are attached to the walls of the larynx while the inner edges are free, and in repose the cords meet at a fairly acute angle. The sound—the voice—is caused by a stream of air from the lungs, which act as a kind of bellows, on its way to the nose and mouth. In the production of a sound the edges of the cords are brought parallel and practically into contact, thus closing the larynx and preventing the passage of air. Hence a pressure is built up below the cords and this displaces them to allow some air to escape. The elasticity of the cords brings then back into position and the process is repeated while the stream of air continues, thus causing the cords to vibrate with a frequency depending mainly on their natural length and tension.

The vocal cords can be observed in action by means of an instrument known as the **laryngoscope**. In this a powerful beam of light is thrown down into the larynx by means of a mirror held at the back of the throat. This lights up the vocal cords, etc., and the reflection in the mirror can be seen and examined.

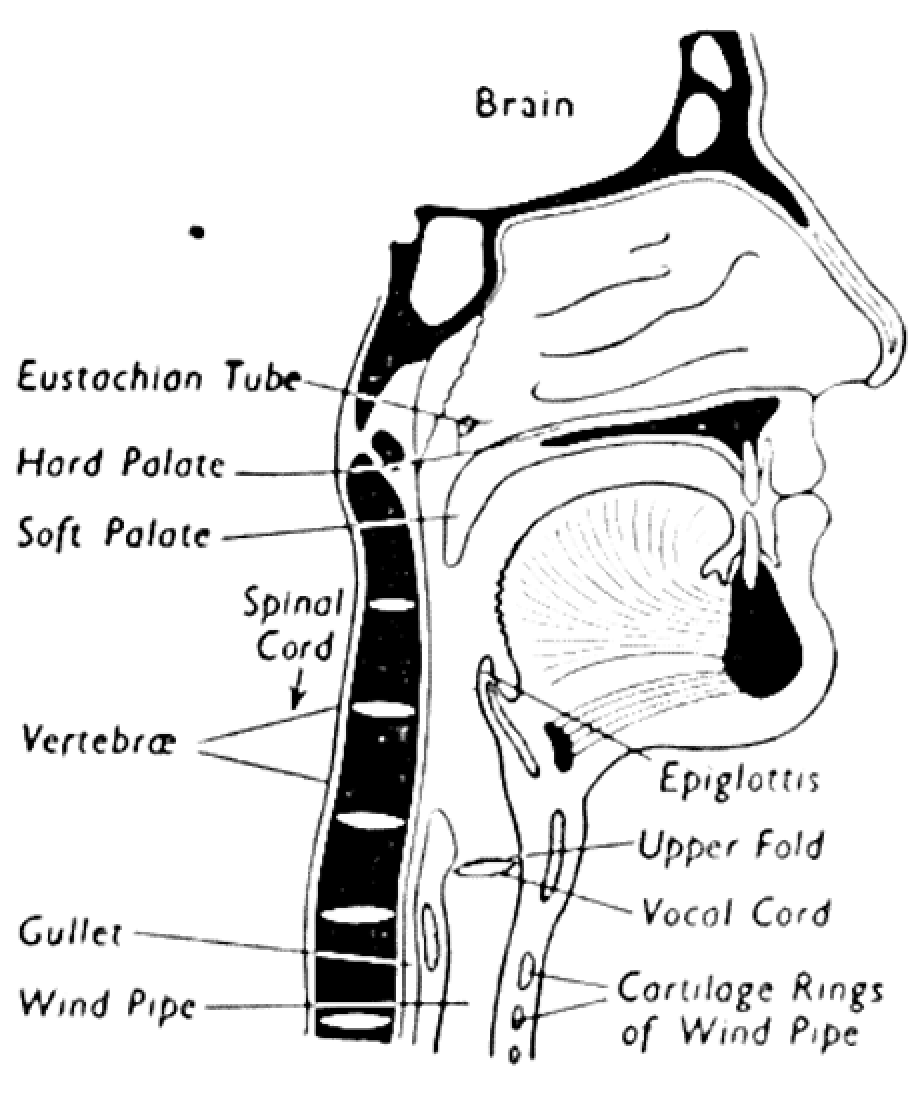


Diagram of mouth and throat in the human being. From Winstanley: *Text Book of Sound* (Longmans Green).

Helmholtz put forward the theory that the pitch of the note emitted depends on the tension of the cords, and the quality on the resonating chamber, which comprises the pharynx, the mouth and the nose. This theory is supported by the fact that the vocal cords of men are greater than those of women, and that tenors have shorter

cords than basses. The "breaking" of a boy's voice is due to the natural growth of the larynx and the vocal cords. Also it is easy to demonstrate that if one sings up the scale using the same vowel sound, say, *ah*, for each note, it will be be found that the setting of the resonator, including the lips, tongue and palate, remains unchanged, the differently pitched notes being obtained by an adjustment of the vocal cords.

Although the resonator cannot modify the pitch of the note, it can alter the quality by strengthening some of the overtones of the note relative to others. It has been observed that when a particular vowel is sung or spoken, overtones of frequencies in the neighbourhood of certain definite values characteristic of the vowel are emphasised whatever the frequency of the note may be. Results of this kind, in which the resonator reinforces one or more overtones of definite pitch, always the same for the same vowel quite irrespective of pitch, suggested the "fixed pitch" theory. Against this was placed a "relative pitch" theory, which suggested that it is the relation between the fundamental and the upper overtones which is effective, and that when the vowel is sung at a higher pitch the whole series of overtones rises, the distribution of intensity between them remaining approximately the same. The true explanation probably lies between the two views.

The human voice is generally divided into three *registers*, the low or chest register, the middle register and the head register. According to Behnke, the first is produced by vibrations of the cords as a whole; in the second, only the inner edges take part in the vibration, while in the case of the head register the vibration is confined to the central portion of the inner edges of the cords.

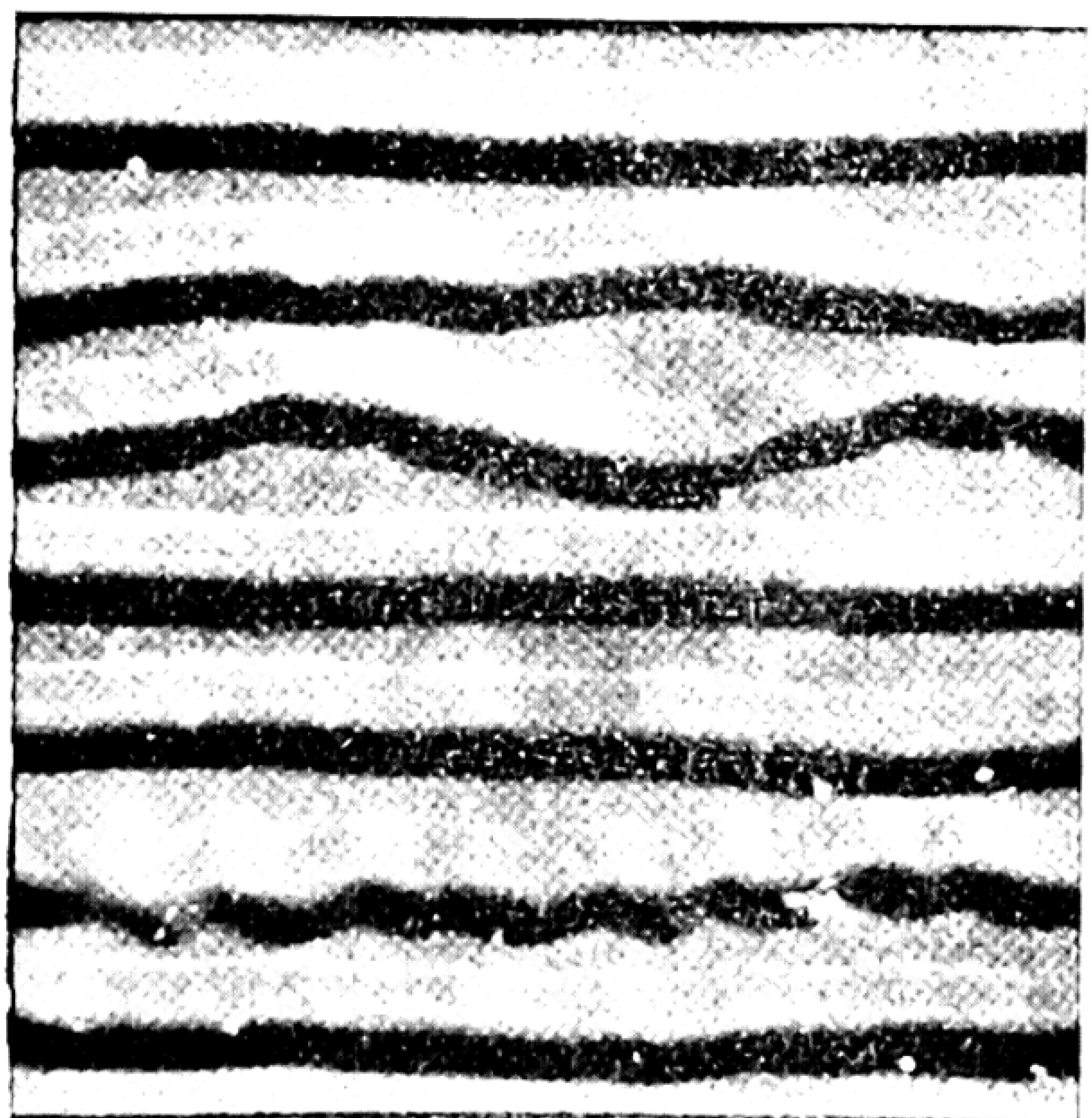
Many singing instructors encourage their pupils, especially boys, to use the head register as freely as possible, and suggest that in their scale practice they should start on a high-pitched note and sing *down* the scale, thereby bringing the "head" notes as low as possible. Whatever may be the nature of the vibrations of the vocal cords in the various registers, it is certain that singers trained on the above plan produce more pleasing sounds with less strain than the rough and raucous sounds sometimes heard.

SOUND RECORDING AND REPRODUCTION

This topic was mentioned in a general way in Chapter I, and now more detailed information of instruments and methods used will be given

THE GRAMOPHONE

Development. The recording of sound was first demonstrated in 1859 by Leon Scott de Martinville, who used an apparatus which he called the *phonautograph*. This consisted of a cylinder mounted on bearings with a handle attached to one end of the spindle. The cylinder was coated with lamp-black, and when it was rotated and sound uttered in front of a mouth-piece, lateral traces of the vibrations were produced on the blackened surface by a bristle attached to a parchment diaphragm. It was not



By courtesy of the Gramophone Co., Ltd.

A photo-micrograph of the surface of a record. There are as many as 130 grooves to the inch.

thought then that these traces could be reproduced into audible sound, and it was nearly twenty years before this was accomplished by Thomas Alva Edison, who substituted a rigid point for the flexible bristle and covered the brass cylinder with tinfoil instead of lamp-black. He found that when the needle was passed over the indentations it had made when " recording ", the sounds were given back again. Edison's machine was called a *phonograph*, and it should be noted that the indentations made on the tinfoil were " up-and-down " and not lateral, so that the groove was of uneven depth.

In 1887 Emile Berliner of Washington, after experimenting on the lines of Scott's phonautograph, conceived the idea of recording on discs instead of cylinders, and considered that the lateral

cuts with constant depth were superior to the "up-and-down" indentations of Edison. In course of time he passed from engraving to the method of etching by means of acid on a zinc plate, but on account of "needle-scratch" due to the roughness of the etched groove, he discarded this method and adopted the practice of cutting in wax; thus was born the **gramophone**, as Berliner called his apparatus.

The disc records used at that time were five inches in diameter and the turntable was rotated by hand; but by 1897 the instrument had advanced to the spring-driven type with a steel needle. As time went on, records grew in size to first seven and then ten, to twelve inches; and gramophones were devised with internal horns. In 1924 a new type of reproducing instrument was introduced in which the horn and sounder were replaced by a large pleated diaphragm. This was invented by Lumière, a French scientist, but it had a short life, for in 1925 came electrical recording, which required a new type of gramophone to handle efficiently the more intense vibrations. Thus came the instruments with logarithmic horns and sound boxes to track the new records without injuring the grooves, and then the "re-entrant" gramophone based on the electrical principle of matched impedance, with its very light diaphragm in an all-metal sound box and its large folded exponential horn.

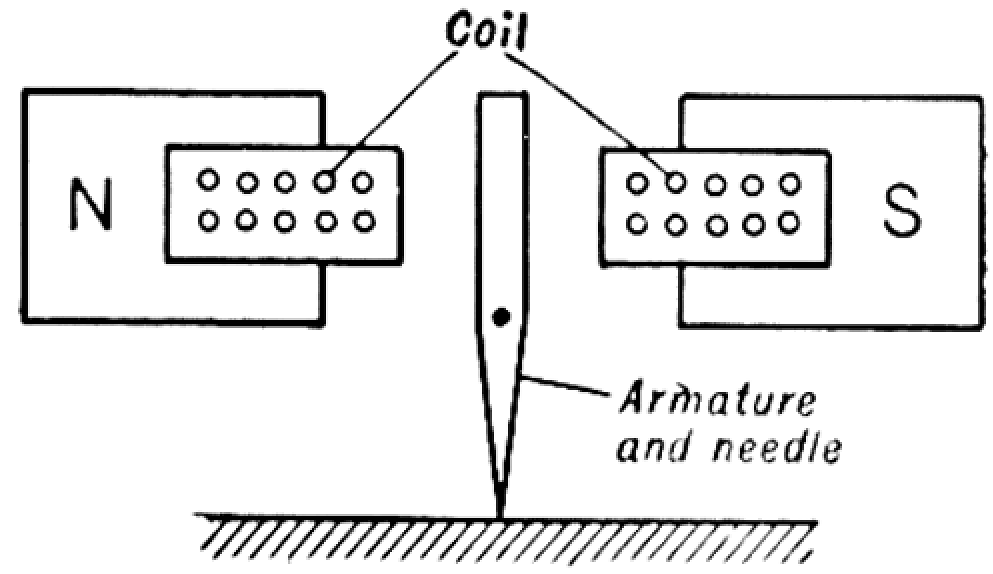
It is seen that many advances have been made in the science of sound reproduction, first in the field of mechanical reproduction by sound boxes and horns, and afterwards through electrical technique using pick-ups, amplifiers and loud-speakers, which comprise the modern **radiogram.**

Many of the older types of instruments operated up to a frequency of only about 8,000 c.p.s.; but it has been found that to re-create perfectly all musical sounds it is necessary to utilise frequencies in the range 30–15,000 c.p.s., even though the latter figure is beyond the range of audibility of the normal listener. But it must be remembered that a mere response to the frequencies existing in a sustained note is not the only factor of importance in obtaining realistic production. Much of the special quality associated with particular instruments depends on the initial pulses of sound containing a large number of higher-frequency constituents which they give out at the instant when a new note begins to sound. These pulses are known as **transients,** and they are of particular importance in instruments of the percussion type, among which must be classed the piano and the string section when playing *pizzicato*, and also in a large

orchestra. By increasing the range of frequency as suggested above, the transient response is much improved.

In one of the latest models, known as the *electrogram*, perfect transient response is claimed, and there are other features which indicate the big advance made in recent years. The whole instrument, which operates on a frequency range of 30–15,000, comprises two units connected by cable, one containing the amplifier and gramophone mechanism and the other containing the three speakers ; thus the listener can sit with the control cabinet at his side, and obtain the reproduction through the speakers at a distance away. Apart from the normal " volume " control, there are two other important controls called the " bass balance " and the " noise filter ". The former enables the bass to be increased or decreased to satisfy individual interpretation of music, and the second one reduces surface noise and brings out the top register ; while ingenious methods have been devised to prevent the introduction of any extra tones such as beat-note effects and intermodulation not present in the original sound.

Electrical recording. The general method of acoustic recording has been described above, but, as has been stated, this method has been superseded by an electrical method. In this method great attention is paid in the first instance to the acoustical conditions of the studio where the recording takes place, for no outside sounds must be heard, and, in addition, reverberation times (see Chapter XIII) must be considered. The general principle of electrical recording is as follows. The sounds fall on a microphone (the usual types are the moving coil, ribbon and condenser), and are there converted into corresponding varying electric current. The fluctuating current is amplified, and if several microphones are used, a " mixer " is necessary to mix the outputs of the microphones in any desired proportions. Afterwards the total output is further amplified to a value suitable for driving the recording head. In principle, this is an armature held so that its upper end moves in the space between the poles of an electromagnet. The fluctuations of magnetisation in the armature give to it a vibratory motion which corresponds to the fluctuations of the current traversing the coil. A sapphire-tipped needle fixed in the lower end of the armature traces out an indented spiral groove in



the surface of a wax slab which is made to rotate at a uniform speed under the cutter.

Recording heads may also be of the moving coil and piezoelectric types, but the moving iron instrument, as briefly described above, is the most popular because of its relative simplicity and robustness.

Size of disc and turntable speeds. The diameter of the disc used for recording largely depends on the time required for reproduction, and generally requirements are met by using discs of different sizes, the smallest being 10 in. in diameter and the largest $17\frac{1}{4}$ in. Twelve-inch discs give a playing time of about 4·9 min. with a turntable speed of 78 r.p.m.; but for programmes of 15 min. or more, $17\frac{1}{4}$ in. discs are used at $33\frac{1}{3}$ r.p.m.

The fidelity of a reproducing disc largely depends upon the cutting speed during recording, and there is a minimum cutting speed below which the recording of the higher frequencies is ineffective. In turn, the cutting speed depends upon the speed of the turntable and the diameter of the groove. It is obvious that for a given turntable speed cutting speed must decrease as the diameter of the groove decreases, and for a specified minimum cutting speed the lower the turntable speed the greater must be the minimum groove diameter. What is required is a turntable speed to give maximum recording time for a specified minimum cutting speed. As has been seen above, with a turntable speed of 78 r.p.m. a recording time of 4·9 min. is obtained with a 12-inch disc, but for the same minimum cutting speed (and the same fidelity) a time of about 5·5 min. can be obtained using the same disc with a turntable speed of 54 r.p.m. Hence although a speed of 78 r.p.m. has been universally adopted, it is not the most economical speed for a 12-inch disc.

In connection with this, it should be noted that it is not possible to design a stroboscope having an integral number of segments for a speed of 78 r.p.m. as will be seen from the following. Suppose the frequency of the mains operating the lamp of a stroboscope is n c.p.s. The number of light pulses per second is $2n$, and the interval between two successive impulses is $1/2n$ sec. In this time the turntable describes $1/2n \times N/60$ of a revolution, where N is the turntable speed in r.p.m. Hence the number of stroboscope segments is $120n/N$.

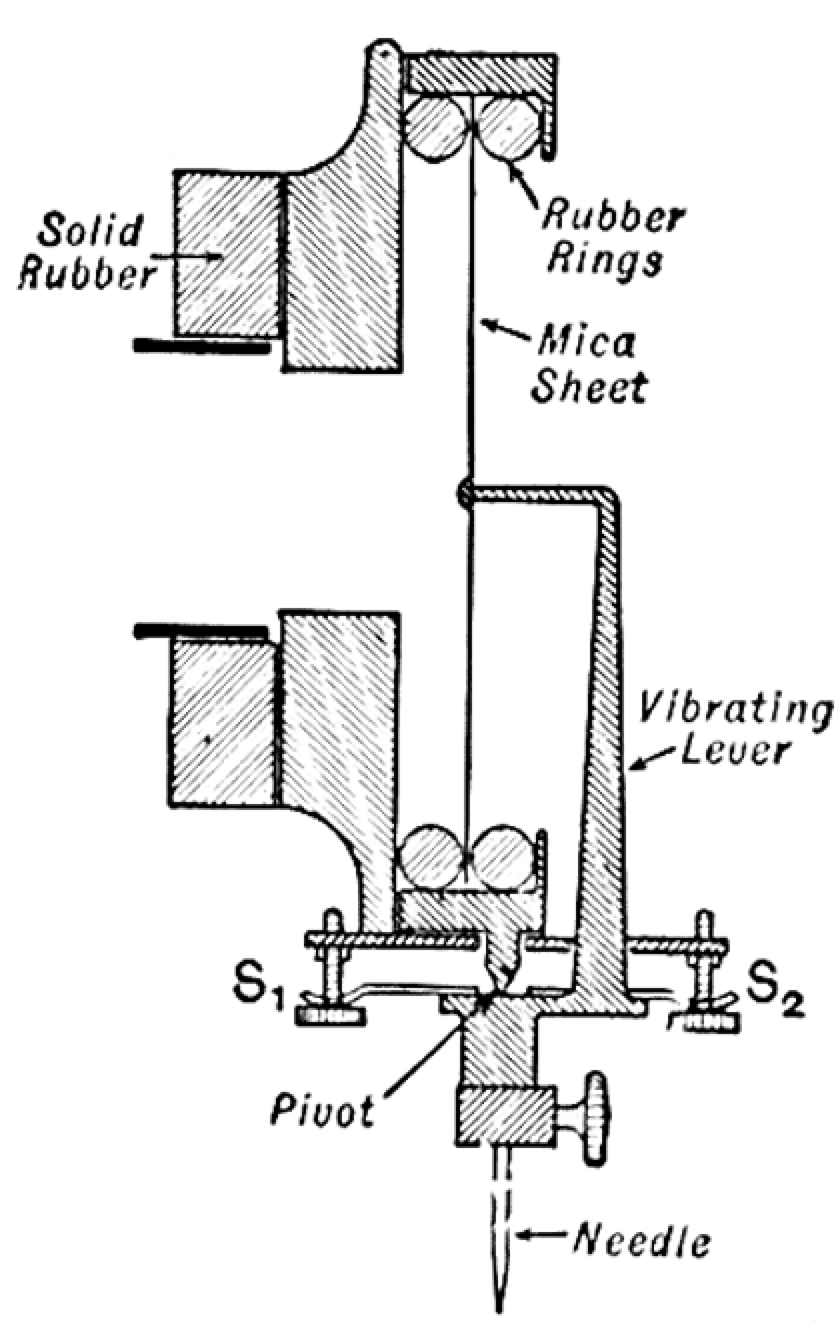
If $n = 50$ and $N = 78$, the number of segments works out to be 76·92; in practice, the stroboscope has 77 segments, and when the pattern appears stationary, N is 77·92 r.p.m.

In the United States n is usually 60 and a stroboscope with 92 segments is used, giving a turntable speed of 78·26 r.p.m.

With a turntable speed of $33\frac{1}{3}$, the above problem does not arise, for a stroboscope with 180 segments operated with 50 c.p.s. mains, and one with 216 segments on a 60 c.p.s. supply both give the speed correctly.

The intensity of the sound produced by a gramophone is proportional to the velocity of the point of the recording stylus, and if this intensity is to be independent of the frequency of the note being played, the velocity of the moving stylus must also be independent of frequency. Now when the stylus moves laterally with a sinusoidal note of amplitude a and frequency n, its maximum velocity is $2\pi na$, and for the intensity to be independent of frequency the product na must be constant ; hence it follows that if the maximum velocity of the stylus is constant, a constant output is obtained for all frequencies. This is what is referred to as **constant velocity recording**, and the amplitudes of notes of the same intensity are inversely proportional to their frequencies ; consequently the amplitudes of the grooves of the low notes will be greater than those of the high notes.

Reproduction. The general principles underlying the ordinary gramophone are well known. Its most important part is the small round sound box, to which the steel or thorn needle is attached. The needle is clamped into the lower end of a metal lever which is pivoted near the same end, and its movements to either end are controlled by two springs S_1 and S_2. The upper end of the lever is attached to the centre of a circular disc of thin mica, supported round its edge between two solid rings of rubber. As the upper arm of the lever is much longer than the lower arm, any vibrations sideways of the needle cause magnified, but similar, vibrations in the mica disc. These vibrations set up waves in the air which imitate in all details the vibrations of the needle.

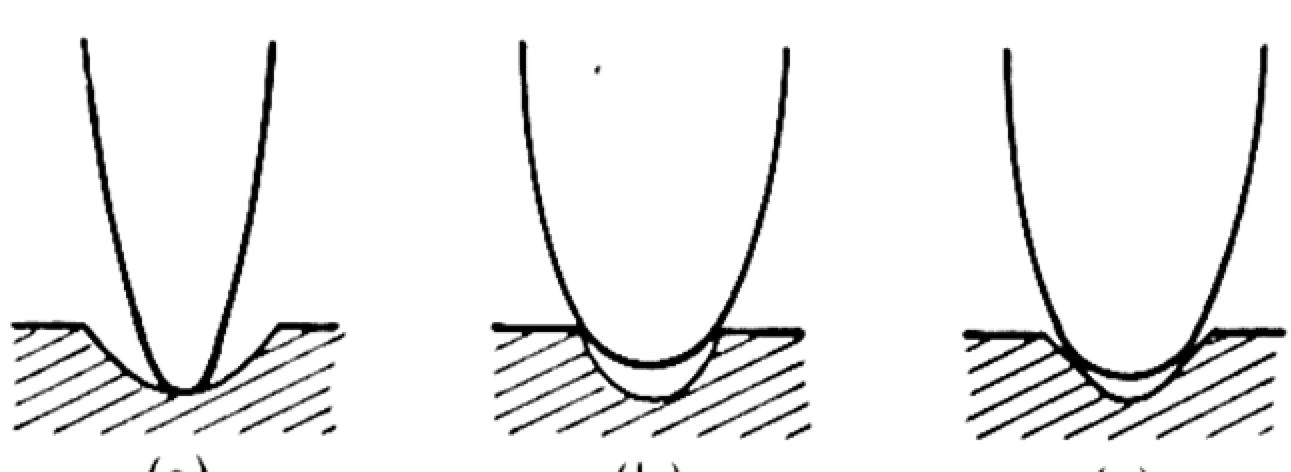


When the gramophone is working, the needle rests in the groove in the recording disc, and as the disc rotates, the wave-form cut in the groove compels the needle to move to-and-fro sideways in exact imitation of the waves, and these movements are transmitted by the lever to the

mica diaphragm of the sound-box. For pure reproduction the diaphragm should behave like a piston alternately compressing and rarefying the air next to it, and ought not to vibrate in any of its natural frequencies. To achieve this, so far as possible, the diaphragm is often connected to the lever by a number of feet known as a " spider " instead of in the middle only.

The above type of reproduction may be regarded as the acoustic method. Nowadays, however, the preference is for electrical reproduction as given in the radiogram, in which the recorded vibrations are converted into electrical impulses and amplified by valves to be reproduced through a moving coil loud-speaker. This involves the use of a pick-up to replace the ordinary sound box. The pick-up, which comprises the reproducing head and the arm on which it is mounted, may be of the moving iron, moving coil or piezoelectric type, and in all these the lateral motion of the needle causes alternating electric potentials to be set up at the terminals of the instrument.

The dimensions of the groove cut in the disc and those of the needle point are of great importance for faithful recording and reproduction. The three diagrams show the point of the needle



resting in the groove of the record. In (*a*) the area of contact is so small that the pressure exerted would be distinctly damaging to the record, and of course there would be excessive noise. In addition, as the point is not firmly wedged between the two walls of the groove it would skate about across the bottom of the groove and cause the reproduction to be distorted. In (*b*) the other extreme is indicated, and the point is so large that it straddles across and destroys the tops of the walls of the groove. Diagram (*c*) indicates a mean between the two extremes and is recommended as a correct setting.

Commercially, there are many types of needles in use and of various dimensions, and the general aim now seems to be to get a closer degree of common-usage among all makers of disc records and of reproducing-points as far as possible all over the world.

For this purpose the following dimensions have been suggested as a basis for greater uniformity :

For the groove : radius of bottom of groove, 0·0015–0·0018 in.
angle between sides, 85°–90°.
width across top, 0·0067 in.

For the needle : radius of bottom of tip, 0·0022–0·0032 in.
angle of tip, 40°–45°.

Making disc records. After the wax disc has been engraved, it is prepared for electro-typing by dusting the surface with metallic powder so fine that it does not affect the delicate sound traces. It is then mounted and left revolving for some hours in the electrolytic bath, where a thin but strong " negative " shell of copper is deposited on the wax. This shell, known as the " master ", is

By courtesy of E.M.I. Factories Ltd.
Removing the copper shell (" master ") from the wax.

very carefully stripped from the wax, and, of course, instead of grooves in its surface it bears the sound waves from the wax in the form of ridges. Records could be duplicated at once from this, but after a limited time the " master " would begin to wear. So the negative " master " is again immersed and a second copper shell called the " mother " is grown on to it. This is a positive and of no use for pressing records ; hence it is put back

into the plating bath and a third and final shell deposited. This is called the "working matrix", and these are taken from the "mother" as often as is necessary. The "master" is then filed and stored in a fireproof vault, and is only withdrawn when damage to a "mother" shell necessitates the making of a new one. The "working matrix" is now backed with a steel plate and the centre hole drilled and then sample records are pressed from it. These samples undergo various tests for wear, musical quality, etc., and if these prove satisfactory the record is put into production.

Pressing the records. The material used for the records is a mixture of shellac, resin, copal and a number of other ingredients; it becomes plastic when hot but is hard at normal temperature. The material is mixed in machines which reject automatically any gritty particles, and the refined material is passed through heated rollers which grade it to uniform thickness and mark the sheets into uniform sizes suitable for 10-inch and 12-inch records. When the sheet becomes cold and hardens, these pieces, known as "biscuit", are broken off. The biscuit is taken to the pressing room where the operator has already fixed two matrices in position to make a double-sided record. He puts the requisite label in the centre of each matrix, and from a heated slab by the press he takes one of the pieces of biscuit, rolls it into a ball and puts it on the centre of the lower matrix. The hinged plates bearing the matrices are then closed with a force of nearly 100 tons. As the pressure is applied, steam is circulated behind the matrices and this is followed by water cooling. The press then is opened to reveal the familiar disc, which is removed and the surplus material broken from the edges. All that now remains is to buff the edges and polish the disc.

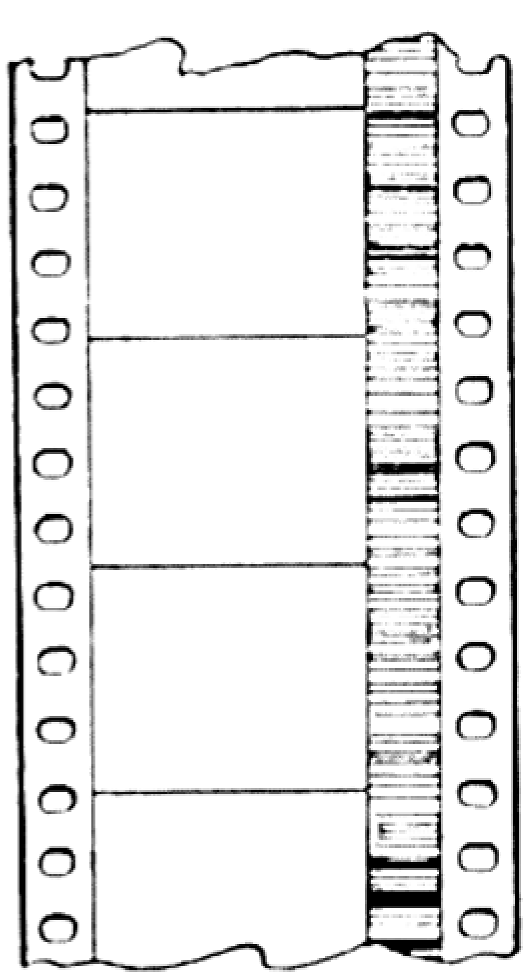
FILM RECORDING AND REPRODUCTION

In 1927 discs of 16-inch diameter, recorded with the "lateral" cut but revolving at a slow speed, were mechanically coupled to the drives of cinema projectors to produce the first commercially successful talking-motion pictures. Such a possibility had been foreshadowed in 1887 by Edison and his English collaborator, Dickson, who made photographs on a glass cylinder coated with a sensitive emulsion and synchronised with a cylinder sound record. Although technically the idea of coupling a record to a projector seemed quite satisfactory, there were practical difficulties; for example, it was difficult to retain the synchronisation if the film broke and a portion had to be cut out. Hence work

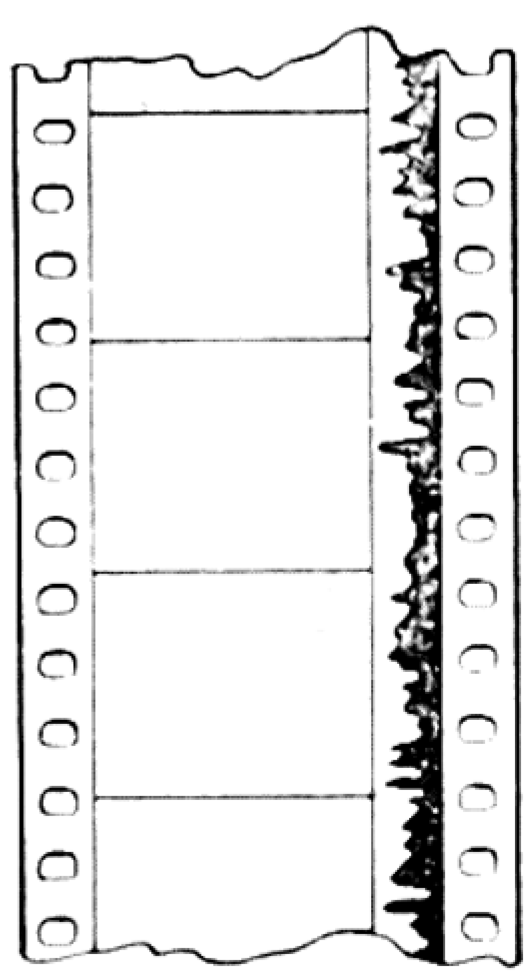
was intensified on a method whereby the sound was recorded photographically and printed along the edge of the picture film.

One of the early inventors of photographic film recording was W. D. B. Duddell. His method was to convert sounds by means of a microphone into an electric current and to record the variable current using an oscillograph. The record was made on a moving film in such a way that the width of the part, exposed or not exposed, was determined by the deflection of the oscillograph, and from the original record others could be made. To reproduce the sound a photo-electric cell was used. This was the beginning of what is known as the **variable area system.** The most important item in the equipment is the oscillograph vibrator, which consists of a loop of thin metal tape suspended vertically, under tension, between the poles of an electromagnet so that the magnetic field is parallel to the plane of the loop. The ends of the loop are connected to the amplifier and microphone, and a very small mirror is attached to the loop. A beam of light is directed on to the mirror and the reflection is focused on to a narrow slit in a screen fixed vertically in front of the travelling film. The fluctuating phonic currents through the loop cause the mirror to turn in a vibratory manner, and this causes the beam of light to extend to varying distances across the slit and so across the film. The intensity of the sound is recorded by the magnitude of the changes in the ratio clear/opaque section across the sound track, while the pitch is indicated by the number of " peaks " per unit length of the film.

In another system of photographic recording, a narrow streak of light extends across the full width of the sound track, and its



Variable intensity recording



Variable area recording

intensity is caused to vary by the phonic currents. This is done either (i) by making the currents control the brightness of the source of light, the " glow-lamp " method, or (ii) by making them regulate the amount of light admitted to the film by means of a " light-valve ".

In the first method, which is generally used with news-reel cameras, the fluctuating phonic currents are superimposed on a steady current which is causing a rarefied gas to glow in a quartz tube, and so the brightness of the glow responds exactly, both in intensity and frequency, to the fluctuating current. Thus there will be a series of light and darker bands on the sound track, the difference in shade being a measure of the intensity of the sound and the number of lines per unit length of the film a measure of the frequency ; for example, since the standard speed at which the film is moved past the slit is 18 in. per second a note of frequency 256 will produce 14·2 lines per in.

In the second method, employed in talking-film studios, a steady beam of light is focused on to a narrow slit in a vertical screen, and immediately behind the slit is a loop of duralumin tape stretched over two bridges, the two arms being separated by a gap of about 0·002 in. This loop constitutes the light-valve, for the width of the gap determines the amount of light passing to the film. Another lens is adjusted to throw an image of the gap 0·001 in. wide on to the sound track of the film. The light-valve is put between the poles of an electromagnet so that the field is at right angles to the plane of the loop, and the ends of the loop are connected to the microphone through the amplifier. When the current passing through the loop is increased, the two arms of the loop move so that the gap is widened and vice versa. Hence the width of the gap is determined by the phonic currents, and the number of vibrations of the light-valve per second will correspond to the various frequencies of the original sound wave. A continuous series of parallel lines of varying thickness and number per unit length will be photographed. This syste known as **variable density recording**, in contrast to the variable area system.

One side only of the sound wave is photographed on the sound track, as shown in the diagram. In the projection machine the variations in light transmitted through the sound track produce corresponding variations of current in a photoelectric cell. This current is amplified and delivered to a loud-speaker behind the screen. Either type of sound motion-picture film may thus be run in the same projection machine.

Both systems have their particular difficulties. Variable-area records require more exactness in exposure and development in order to preserve a clear and true outline of the wave, while variable-density records are subject to a wave-shape distortion which becomes appreciable at very high frequencies. The most recent advances in both systems are concerned with removing certain causes of noise and distortion, particularly those which arise because the film does not move past the photoelectric cell with a perfectly uniform motion, and methods devised for this purpose have reduced the defect to an almost negligible quantity. It may be that in future sound-motion pictures the sound may be given a spatial effect so that the audience may hear the sound or speech coming from the place where it is produced, either to the left or right, or at the front or back of the stage ; this advance would certainly produce greater realism.

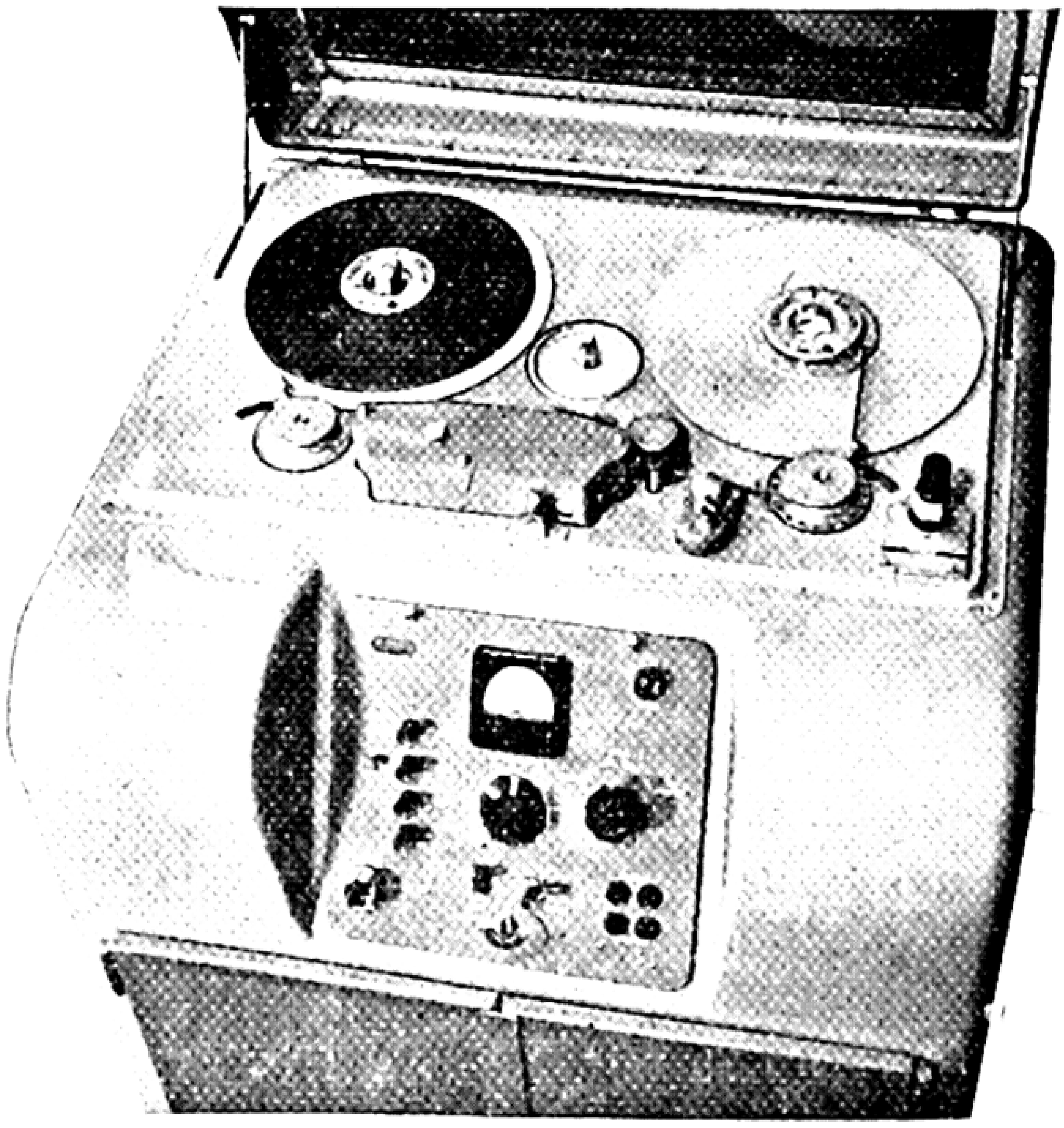
MAGNETIC TAPE RECORDING AND REPRODUCTION

Recording in a magnetisable object began with the invention of the *telegraphone* by Poulsen in Copenhagen in 1899, when he found that he could record sound by causing a vibrating diaphragm to induce varying magnetic effects in a moving steel tape. In one form of his apparatus, he used steel tape which pressed against the poles of an electromagnet as it moved from one rotating drum to another, while in another type he used a steel wire. He further experimented with a sheet of material such as paper covered with a magnetisable magnetic powder, and also with a disc of magnetisable material over which the electromagnet might be conducted spirally. Also, anticipating future developments, he explained that the recording could be " wiped off " the magnetisable object by passing a steady current through the electromagnet, so that the same tape, wire or disc could be used over and over again almost indefinitely. The process of recording the sound is reversed for reproducing it.

In 1918, in America, Fuller used a high-frequency current instead of a steady current as the eraser, and in 1921, Carlson and Carpenter, also in America, used a high-frequency current together with the recording impulses to set the molecules of the magnetisable material vibrating and so make it more easily influenced by the phonic currents.

Development in this form of recording was rather slow until the Second World War, although the British Broadcasting Corporation used apparatus of this type some years before the War,

comprising steel tape, wound on steel drums of 2 ft. diameter, weighing 11–12 lb. and able to run for 30 minutes. During the War, machines of very small size and weight, using wire instead of tape, were produced in America for recording in aircraft; one type of machine used wire of only 0·004 in. in diameter and was capable of running for 33 or 66 minutes, depending on the speed. Since the War, the same principles have been applied as an auxiliary fitting on a radiogram, and for long-playing records of music. There is no doubt that recent development with modern technique based on Poulsen's ideas have produced methods and apparatus in this field of recording which compare well with the photographic film and the gramophone disc, and which are of particular use for radio broadcasting, public address recording and background music for films.



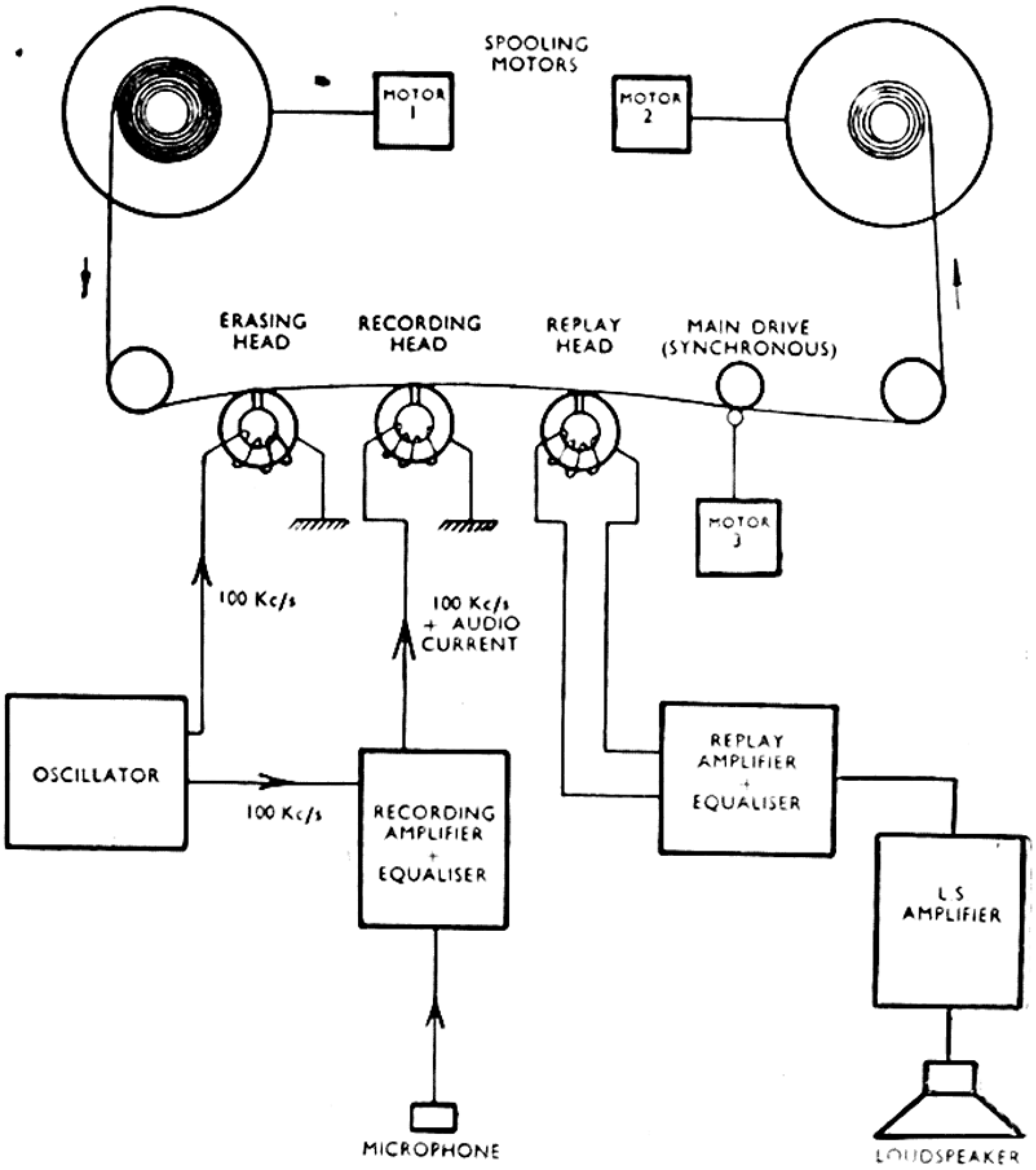
By courtesy of E.M.I. Factories, Ltd.

In the E.M.I. magnetic tape recorder illustrated above, which was designed and made in England for use in broadcasting stations and film studios, the tape consists of a base of cellulose acetate two-thousandths of an inch thick and $\frac{1}{4}$ in. wide coated on one side with finely divided ferric oxide in a particular physical form. The spool of tape weighs one pound, has a diameter of $11\frac{1}{2}$ in. when wound up, and plays for twenty minutes. The tape passes over three magnetic heads. The first head erases the recording

if and when desired and prepares the tape for fresh use. When in action this " wiping " head is energised by a current of 100 kilocycles per second. The recording head carries the phonic currents and a magnetic conditioning current at a frequency of 100 kc. per second, while the third head is for reproducing, and into it phonic currents are induced by the magnetised tape during replaying.

The recording and reproducing characteristics of this equipment are well adapted for faithful reproduction of musical sounds within range of the average musical ear, covering a frequency from 30 to 10,000 cycles per second.

It has been found that the above type of tape becomes brittle with age and tends to break when the machines are started. This difficulty has been overcome by the introduction of a homogeneous tape composed of polyvinyl chloride and magnetite (ferric oxide), the tape being heat-treated and stretched after rolling. A still later type, which gives greater mechanical



By courtesy of E.M.I. Factories Ltd.

Scheme for magnetic recording and reproduction.

strength, consists of a base layer of polyvinyl chloride coated with a mixture of the chloride and magnetite.

Having dealt with the various methods of sound recording and reproduction, it is only necessary to say that much research and development are proceeding in order to secure new advances. It may be that in the future gramophones of a new type will be designed which can play either disc or tape (or wire). At present the disc has the advantages of being easily mass-produced, especially for popular selections and musical subjects which have a duration of a few minutes and can be selected as desired. On the other hand, tape (or wire) records are useful when it is desired to reproduce a long musical item, say, an opera, without having to change the record.

CHAPTER XII

SOUND SIGNALLING, DIRECTION FINDING AND RANGING

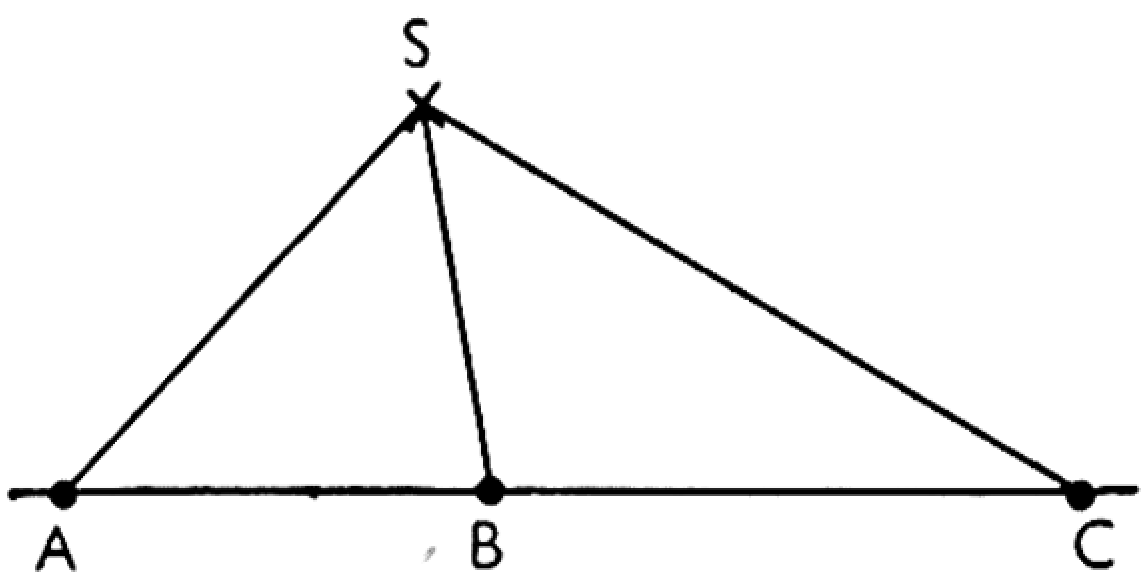
In this chapter, consideration will be given to applications of the principles of the transmission of sound in both air and water. So far as air is concerned, it must be said at once that the methods used for signalling, direction finding and ranging prior to the radar era have been largely superseded by this short-wave electrical method, which is of course outside the scope of a book on acoustics. But a brief reference should be made here to the older acoustic methods if only to show how man, confronted with a problem, sets himself to solve it with the knowledge and technique available to him at the time.

AIR AS THE MEDIUM

The problem of sound ranging and direction finding in air is the detection by an observer of the true direction and range of the sound from an unseen source such as an aeroplane or a gun, and clearly in any experimental method used corrections will have to be applied to allow for variations in the velocity of sound and for effects due to refraction, etc., bearing in mind that the air must be regarded as a stratified medium.

Detection of ground objects. The method of locating the position of an unseen source such as a gun was developed for use in the First World War and is a very good example of direction finding by measuring time intervals. In principle, the method consisted of arranging a set of microphones along a front opposite the concealed gun to receive the sound, and these were connected with a central base in the rear. Each microphone would pick up the sound at a time depending on its distance from the source, and by measuring the time intervals between the sounds recorded at the base the position of the source could be located. It should be noted that man's ears were replaced by microphones, and if the base along which the microphones were set is long, the time intervals would be long and more easily measurable.

The use of two microphones would of course give a direction only, but three would give two directions and an intersection, and therefore a location ; hence three microphones are the minimum for the purpose. In practice, six instruments were actually used to give five directions, and a mean location was obtained, thus improving accuracy. The microphones employed were of the Tucker hot-wire type (see p. 23) mounted in front of a resonator suitable for responding to the low frequency of the particular sound of the gun (the *onde-de-bouche*). The wire of the microphone was connected into one of the arms of a Wheatstone bridge which was balanced in the usual way. When the sound reached the microphone the wire was cooled by the moving air and the bridge became unbalanced, the lack of balance being recorded electrically at the base. If V is the velocity of sound under the prevailing conditions and t_a, t_b, t_c the times for the signals to reach the stations A, B and C, we have



$$\frac{SA}{V}-\frac{SB}{V}=t_a-t_b \quad \text{and} \quad \frac{SB}{V}-\frac{SC}{V}=t_b-t_c.$$

Now the locus of a point the difference of whose distances from two fixed points is constant is a hyperbola with the two fixed points as foci. Hence S must lie on the hyperbola with A and B as foci and also on that with B and C as foci ; therefore, it must lie on the intersection of the two hyperbolae, which fixes its position. These hyperbolae were constructed by a specially devised curve tracer and the position found.

In a later method the cables from the microphones were connected to a " string " galvanometer ; this consisted of six wires, one for each microphone, stretched at right angles to a strong magnetic field. When the sound reached one microphone an extra surge of current passed through that particular circuit and the wire of the galvanometer was caused to vibrate momentarily. Shadows of the wires produced by a fine slit and a lamp and system of lenses were thrown on to a moving photographic film which was automatically developed and fixed, thus saving time. Time intervals were marked on the film by vertical lines, and by noting the positions of the " kicks " in the otherwise straight

wires the intervals between could be measured. From these and the velocity of sound the position of the gun could be fixed.

Detection of aerial objects. The sound from an aircraft in flight is extremely complex, consisting of elements of high frequency and low frequency and of musical and non-musical sounds, and it certainly provides ample scope for acoustic investigations.

Most of the sound locators that have been devised are based on **binaural effects,** which were mentioned in Chapter I and which will be briefly considered again here. It is well known that the ears can localise the direction of a source of sound to a fairly large extent. If a source is on the right of a plane bisecting and normal to the line joining the ears, the listener recognises that the sound is on his right. The ears owe their directive capacity to the fact that they are separated, and the distance between them is a determining factor. If a sound is directly in front of the listener, the energy arrives at the two ears simultaneously ; but if it is to one side, one ear receives the sound a very short time earlier than the other, the measure of the time being determined by the angle that the direction of the sound makes with the line at right angles to that joining the ears.

It should be noted that there are *two* principles to be considered when dealing with binaural effects, namely the *intensity* effect and the *phase* effect. It certainly seems reasonable to suppose that the source of sound is on that side where there is a greater intensity at the ear, and this explanation was the one accepted for many years. However, if there is a *path-difference* in the sound arriving at each ear, there will also be a *phase-difference* ; and the difference in phase at the two ears has been shown to be the more effective in localising the origin of the sound, although there *is* an effect caused by an intensity difference at the ears, the phase being kept the same. Dr. G. W. Stewart, of the University of Iowa, studied both effects quantitatively and came to the conclusion that the intensity effect cannot explain the ability to locate sound under all conditions, while the phase effect can, at any rate, up to frequencies of the order of 1500 c.p.s. Lord Rayleigh held a similar opinion (see Chapter I).

The first British sound locator used the principle of widening the effective distance between the ears and employing horns to collect the sound and magnify it. Four horns were used, two in a horizontal plane and two in a vertical plane, the first pair taking the place of hypothetical ears, and the whole assembly was rotated until the sound was loudest, that is, when the mouths of the horns faced the direction of the sound. In one type of

locator, an American type, the horns were exponential, which gave very impressive magnification but which were extremely resonant, while in a later British type parabaloid reflectors were used to collect the sound, these locators being highly directional, free from resonance and fairly efficient in noisy conditions. In all these types of instruments the human ear was the ultimate criterion, and for later types microphones were produced for all-round listening which could be matched as accurately as man's ears are matched.

Difficulties arose here, for the sensitivity of the microphones themselves was inadequate and amplifiers had to be used, and these had to be matched or they would of course introduce time errors. Then again, the power of picking out the wanted from the unwanted sounds, used automatically by the ear, was lacking, and electric filters were used to cut out the undesired sounds. It was eventually recognised that the cathode ray oscillograph probably offered the best alternative to the ear. The pattern given by this instrument is a straight line only when all the constituents of the sound are in phase, namely when the microphones are equidistant from the source.

Apart from the difficulties so far enumerated in this acoustic method of direction finding, variable atmospheric conditions have to be allowed for. Both wind and temperature may affect the direction of listening due to their effect on the velocity of sound and also to refraction (see p. 74). For example, if the velocity of the wind increases with height in the direction of listening, the sound energy comes down more steeply, but if opposite to the direction of listening it comes in at a smaller angle, while gusty winds make direction finding very difficult since the time difference becomes variable. So far as temperature is concerned, it generally happens that in daytime an aircraft would be heard at a lower angle of elevation than it actually is, but at night-time when the air temperature is generally higher than that of the ground the reverse effect frequently occurs. When the aircraft is directly overhead the errors involved become negligible. The small velocity of sound relative to that of light is responsible for another error which may be called the lag of sound, and it can be defined as the angle between the line of sight and the direction of listening. Correction for this can be applied with considerable precision if the air speed of the machine and the line of travel are known.

The above brief discussion of direction finding by acoustic methods of an aircraft in flight applies mainly to the detection of

perhaps a single machine, and the problem would become much more complicated when it is desired to detect a number of machines. This problem becomes of course more acute in war-time, when both speed of detection and accuracy concerning number of aircraft, height and speed are of great importance. It will be seen that acoustic methods do not compare in these respects with the modern method using radar.

WATER AS THE MEDIUM

Several methods might suggest themselves for use in signalling work in water, namely, optical, electrical and acoustic, but of these the last method is the only effectively practical one at present. Optical methods can be dismissed when it is realised that water is highly opaque to infra-red and ultra-violet radiations and not particularly transparent even for visible light ; for example, it has been estimated that the visibility distance for light in the Mediterranean is of the order of 60 metres, while in the English Channel the distance is only about one-tenth as much. So far as electrical methods are concerned, since sea water is a good conductor of electricity, electromagnetic waves are rapidly absorbed on passing through it ; hence the possibility of using an electrical method in this medium is greatly diminished. Against these two methods the acoustic method has a big advantage, since, as has been seen in earlier chapters, the sea is a relatively good medium for the propagation of sound waves.

Sound ranging, based on an accurate knowledge of the velocity of sound in sea water, has also been used to enable a ship to locate its position in cloudy or foggy weather. The ship emits simultaneously two signals, one a sound signal from perhaps a depth charge and the other a wireless signal, and these are picked up by two land stations at an accurately known distance apart. If T_1 is the time interval between the reception of the two signals at one land station, we have $T_1 = D_1/v - D_1/V$, where D_1 is the distance of the ship from the station and v and V are the velocities of sound waves (in the water) and of wireless waves respectively. Now, V is so great compared with v that D_1/V can be disregarded ; hence $D_1 = T_1 v$, which shows that the position of the ship is on a circle of radius D_1 and centre at the station. Similarly, the position will lie on a circle of radius equal to the distance (D_2) of the ship from the second station, and we have $D_2 = T_2 v$, where T_2 is the appropriate time interval. Hence by drawing two circles the position of the ship can be determined. The two land stations

send their calculated distances by wireless to the ship, which can then obtain its position.

The ways in which sound energy behaves in its transmission through water, and what happens when the energy passes from one medium to another have already been dealt with, and the rest of this chapter will be devoted to considering how acoustic methods are used in practice for subaqueous signalling, echo-sounding, etc. Such methods, which are based on the reflection of sound and the production of an echo, are applicable both in times of war and peace : but before dealing with these two aspects it would be as well to consider how the two types of sound energy, low-frequency and high-frequency, can be employed for the purpose.

LOW-FREQUENCY OSCILLATORS AND RECEIVERS

When this type of energy is used, the frequencies involved are those within the range of audibility, and of course a suitable transmitter and receiver are required.

Transmitter. Several types of instruments have been devised for generating sound under water. One of the early types was the submarine siren, a purely mechanical device built on the same principle as the ordinary air siren, but with jets of water passing through holes in a vibrating plate or series of plates. This has now been superseded by the electromagnetic sound generator, although in the Second World War a more or less mechanical instrument was used to combat the acoustic mine (see p. 269).

The electromagnetic type of generator may be of two kinds, one using an electromagnet and a diaphragm as in the telephone receiver, and the other a moving coil to which the diaphragm is attached. In the first, alternating current is supplied to the coils of a large electromagnet arranged to attract a heavy diaphragm clamped along its periphery ; the diaphragm is thus caused to vibrate with the frequency of the supply. In the second, the alternating current is supplied to a coil surrounded by a magnetic field and directly connected to the diaphragm, and a modification of this, known as the **Fessenden oscillator,** is one of the most satisfactory of low-frequency generators.

In this, an electromagnet is energised by passing direct current through the coils, and the pole pieces are rigidly connected to a thick steel diaphragm. In the gap between the poles there is an iron core surrounded by a coil wound in one direction round one half of the length of the core and in the reverse direction round

the other half; alternating current of the desired frequency is supplied to this coil. Between this coil and the pole pieces is a copper cylinder fastened at each end to discs which are secured to a shaft in the middle along the axis of the cylinder; the front disc is rigidly attached to the diaphragm. The alternating current passing through the central coil sets up eddy currents in the copper cylinder which cut the lines of force of the magnet and are opposite in direction over the two halves of the cylinder. Hence, during one alternation of the current the cylinder is pushed forward and during the following alternation it is pushed backwards, and the movements are communicated to the diaphragm which is caused to vibrate with the same frequency as the current.

Detectors. Low-frequency detectors are divided into two classes: the purely acoustic type and the acoustic-electric variety. The earliest type of acoustic receiver was the so-called **Broca tube**, which is a pressure detector and consists of a sphere of rubber or sheet metal attached to the end of a listening tube. When a sound-wave is incident on the outside of the sphere, variations in pressure and volume of the air in the chamber are caused, which are communicated to the air in the tube; it is assumed that the air in the chamber acts like an incompressible fluid.

The theory of this type of receiver has been worked out by H. A. Wilson, and it has been found that with such a detector there is a resonance frequency for which the power transmission is a maximum, and also there is an optimum cross-sectional area of the tube. Further, for a fixed value of the cross-sectional area of the tube, the power transmission falls off as the frequency increases, while the intensity, other things being equal, varies inversely with the volume of the receiving chamber. These conclusions have been confirmed qualitatively by extensive experiments.

Of acoustic-electric sound receiving devices, the hydrophone has so far proved the best for sub-aqueous reception. One requisite of this instrument for work under water particularly is that its sensitivity should remain constant. This, however, is not possible with the ordinary carbon granule microphone, which is a pressure type of detector, for the slightest change in the static pressure due perhaps to a change of depth or to currents will certainly alter the sensitivity. The button type of instrument described in Chapter IX, and which is a displacement detector, is the type most suitable. Even with this it is found that the maximum sensitivity is not attained immediately after closing

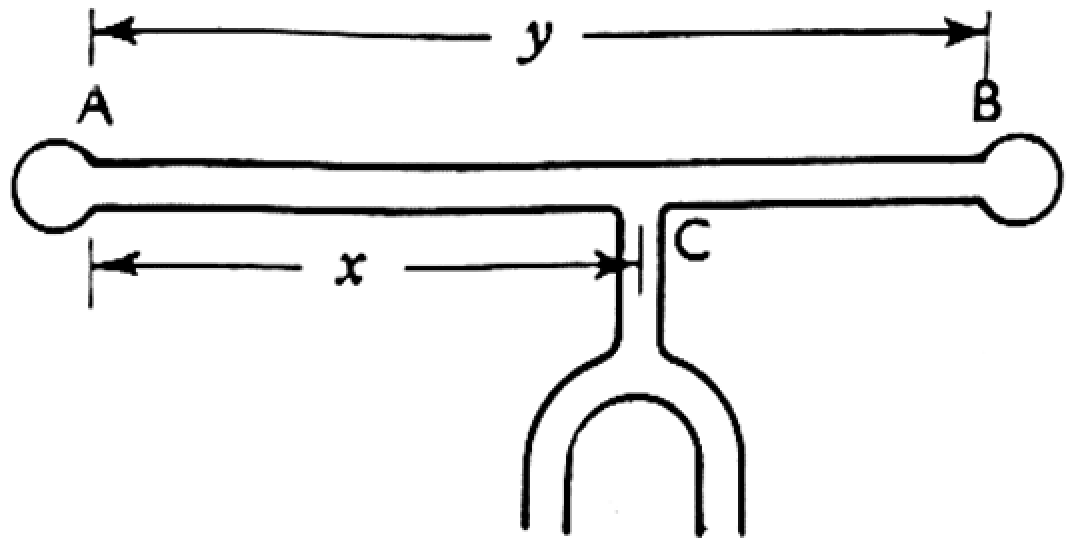
the circuit. A possible explanation of this is that the air in the instrument has first to come to some kind of equilibrium during the process of expansion of the air; but this difficulty can, however, be largely overcome.

The construction of the **hydrophone** is well known and a brief description was given in Chapter IX. If several microphones are used together, they must of course be matched for phase. Hydrophones are sometimes used outside the hull of a vessel, and sometimes just inside the hull in a tank of water; they are not used in air inside the hull on account of the loss of energy involved.

Most microphone diaphragms have definite resonance frequencies, but it is of interest to note that in 1921 L. V. King developed a receiver with a resonating frequency alterable over a considerable range; briefly, the method was to alter the air pressure in the microphone chamber and so alter the tension on the diaphragm.

Multiple receiver systems. Quite early in the study of acoustic receivers, it was recognised that if several receivers were used with their individual tubes leading into a common tube, more sound might be collected; and a great variety of multiple acoustic receivers have been made. Although the purely acoustic receivers are now almost entirely superseded, it is worth while considering one type. In this, two receivers, A and B, are joined by a tube of air tapped by another tube at a point C, and the sound led to the ears. If the sound approaches from the left, the receiver A will be excited first and the disturbance will travel down the tube towards C. At the same time the sound will travel direct through the water from A to B; thus B will be excited and the resulting disturbance will go through C to the ears. If two disturbances reaching C are in the same phase, the sound will be of maximum intensity, and if this is desired, C must be in such a position to provide for phase agreement. Let the distance $AC = x$ and $AB = y$. Then for the two sounds to arrive in the same phase, we have:

$$\frac{x}{\lambda_a} = \frac{y}{\lambda_w} + \frac{y-x}{\lambda_a},$$



where λ_a and λ_w are the wave-lengths in air and water respectively.

Hence

$$x = \frac{y}{2V_w}(V_a + V_w),$$

where V_a and V_w are the velocity of sound in the air and water, and the position of C is fixed.

HIGH-FREQUENCY OSCILLATORS AND DETECTORS

The two effects utilised for high-frequency work are the piezo-electric and the magnetostriction effects which were mentioned in Chapter I. In both cases the emitted waves of a frequency varying between 15,000 and 60,000 c.p.s., thus coming into the category of ultrasonic frequencies, are of a comparatively sustained and regular nature, and their use for sound signalling is based on the effect of resonance. The period during which the waves are emitted must be short enough to ensure that the transmission has died away entirely before the echo returns.

The methods involving high frequency have certain advantages over the low-frequency methods and in modern practice they are generally used. In the first place, high frequencies avoid interference by audible frequencies, for example, noises produced by the propeller and those caused by the movement of the ship in the water, and it is also found that with a vibrating diaphragm there is a greater efficiency of radiation at high frequencies. Finally, with high frequencies it is possible to obtain a comparatively narrow beam for the transmission of the energy. This is important, for clearly it is wasteful to distribute the emitted radiation over a wide area when only a fraction of the energy is likely to return to the receiver. The directivity of the beam depends on the ratio

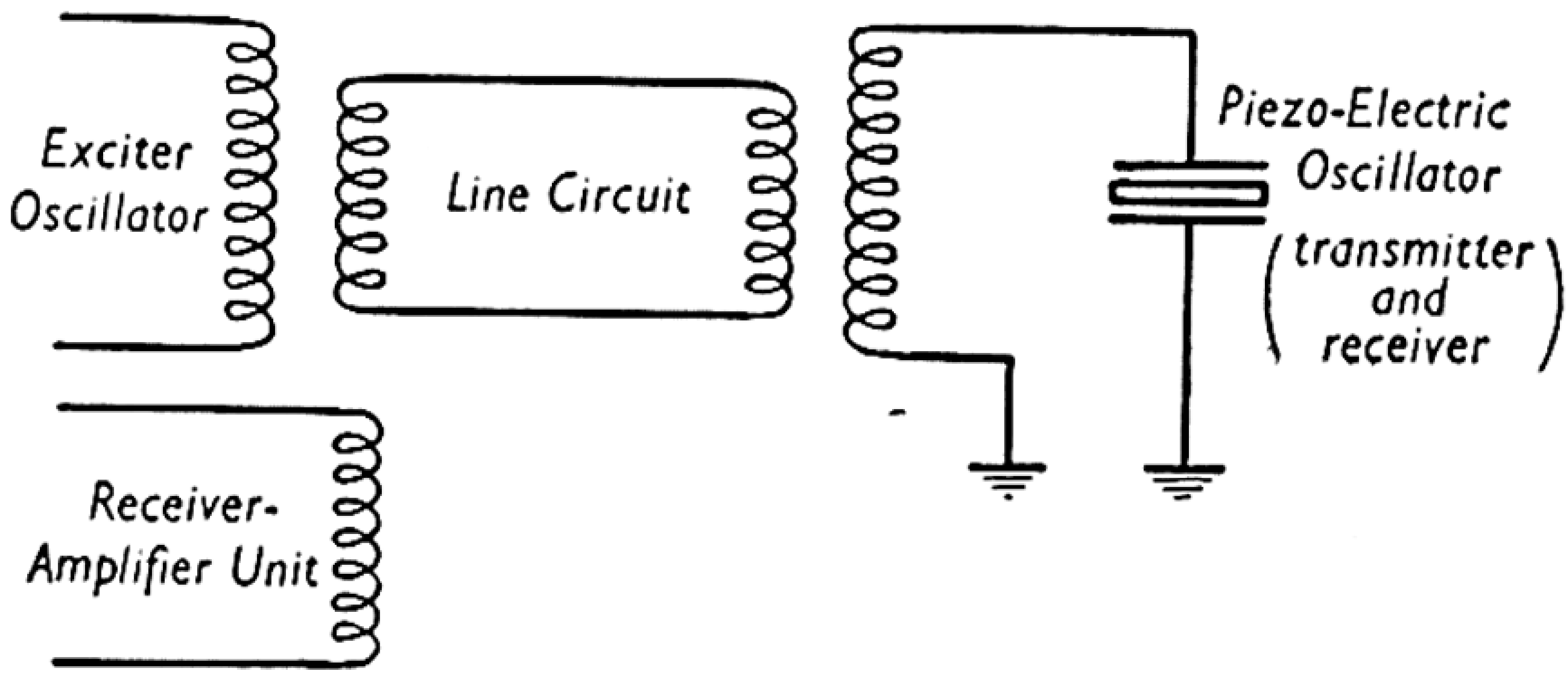
$$\frac{\text{diameter of emitter}}{\lambda \text{ of emitted radiation}};$$

hence λ must be short if the transmitter is to be of reasonably small dimensions. The magnetostriction oscillators usually give wider cones of energy than the piezo-electric type, probably on account of their smaller dimensions. Both effects are reversible, hence, in either case, the transmitter can be used reversibly as the receiver. This is the usual procedure (but not always—see detection of flaws, p. 64) in the piezo-electric apparatus, but a separate receiver is used in the case of the magnetostrictive equipment.

Piezo-electric oscillator. The earliest kind of oscillator of this type was made of mica, but nowadays quartz is generally used. A typical form of such an oscillator consists of a thin layer of pieces of quartz fitted together with a special cement, and on

either side of this mosaic is a thin steel disc, to each of which the quartz layer is fastened with the same cement. The thickness of the whole " sandwich " is half a wave-length at the resonant frequency of the sandwich, which, of course, is different from that of the quartz alone. The sandwich is excited by means of a quenched spark or other oscillator tuned to the resonant frequency, and the diagram gives an indication of the method used. The receiver-amplifier unit is cut out while signals are being sent, but it can be inserted in the circuit when the quartz oscillator is used for reception.

When the piezo-electric projector is used for echo-sounding, it is preferable to have the outer plate of the sandwich in contact with



the water. Hence to fit a ship with such a projector necessitates dry-docking her and cutting a hole in her skin. The projector can be worked through the hull plating, but considerably more power is required.

The transmitted signals may be received and recorded in several ways. In the early days the frequency of the transmission was modulated by a beat method so as to render the incoming signals audible ; but generally nowadays the indicator gives a visible record by either an oscillograph method or by electrolytic means. In the former system, as used for echo-sounding, a beam of light is reflected, first from an " oscillograph " mirror, and then from a " swinging " mirror which rotates at a uniform speed proportional to the velocity of sound in the water. The beam of light is reflected on to a scale, and if the oscillograph mirror is still, the light spot moves uniformly across the scale. When the echo returns, the electrical impulse is amplified, and by means of an electromagnet causes the oscillograph mirror to tilt upwards. Thus a peak is formed in the line of light on the scale ; in practice, two peaks will occur, one corresponding to the transmission

and one to the echo, if the receiver-amplifier unit is not cut out during transmission.

In the case of the electrolytic recorder, the light spot is replaced by a swinging arm carrying a stylus, the arm travelling across the recording paper with a speed proportional to the velocity of sound in the water. The paper is impregnated with starch and iodine and moves at constant speed over metal rollers. When signals are received, a small current from the amplifiers is caused to flow between the stylus and the rollers via the sensitised paper, and a brown mark is made on the paper. If the apparatus is being used to ascertain depths of sea water and the sea bed is level, a series of marks in a straight line is made on the paper, but if the depth varies, corresponding variations appear in the line.

Magnetostriction oscillators. These oscillators are of two forms: in one, a nickel cylinder or tube is caused to vibrate longitudinally by means of alternating currents of frequency equal to the fundamental frequency of longitudinal vibration of the tube; in the other, a nickel ring or annulus is employed. When used for under-water work, the efficiency of both forms is considerably improved by having a nearly closed magnetic circuit, and steps are taken to achieve this object.

In the first type, the nickel cylinder is supported at its centre, and small air gaps are left between the nickel and the non-magnetostrictive, but highly permeable, material which closes the magnetic circuit. A high-tension machine charges a bank of condensers which are then discharged through a solenoid surrounding the nickel. The nickel cylinder is thus set into vibration and produces a high-frequency sound pulse of short duration. In practice, this projector is mounted in a steel tank which is filled with water after being pressed down by jacks on a rubber pad in contact with the hull plating of the vessel; less power is required, of course, if a hole is cut in the hull and the end of the nickel core mounted in direct contact with the surrounding water. To prevent the emitted wave spreading through too wide an angle, two conical reflectors are fitted round the projector.

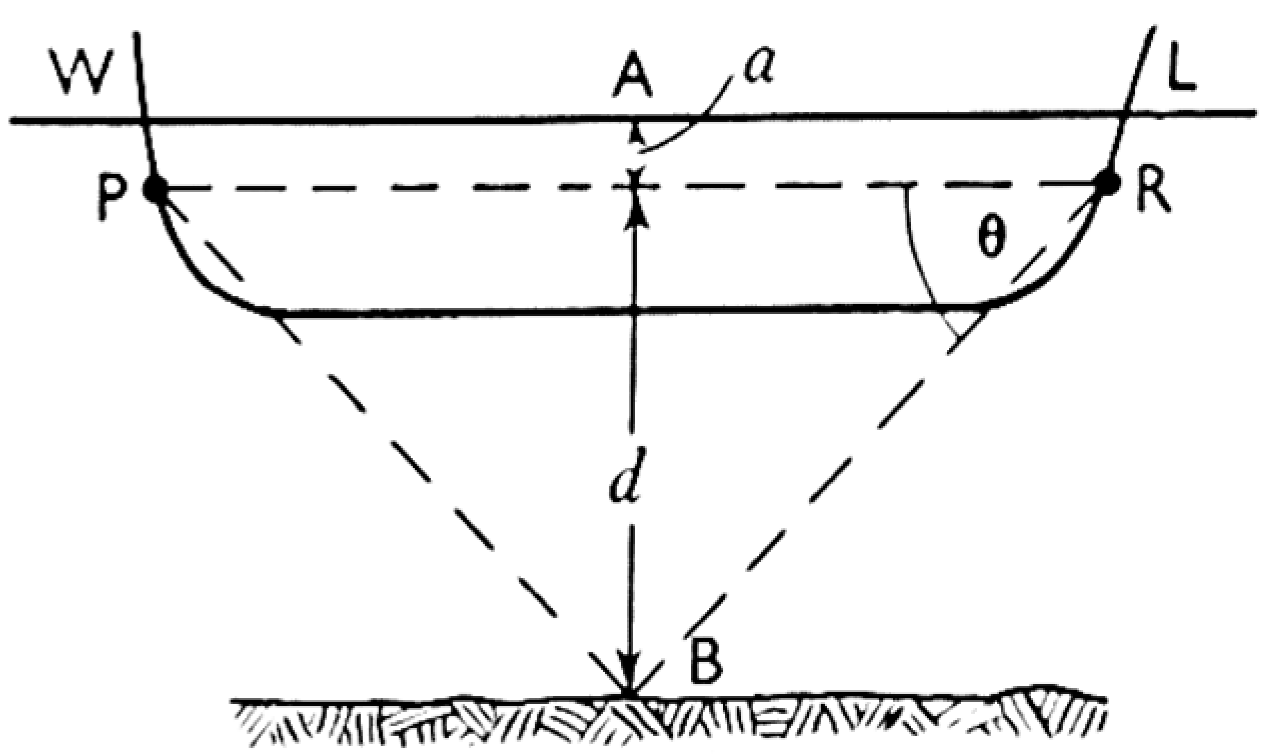
For the reception of the echo a similar oscillator is used. The nickel core is magnetised by direct current, and the state of strain set up in the nickel by the echo slightly changes the magnetic flux through it and sets up a small potential in the winding surrounding it. Thus a small current, alternating in a manner corresponding to the vibrations received, is produced and this is amplified to operate a recording machine.

In the second type, made by Messrs. Henry Hughes and Son

Ltd., a completely closed magnetic circuit is obtained by using a nickel ring ; this system will be dealt with in greater detail in the next section.

PEACE-TIME APPLICATIONS

The uses to which acoustic energy can be put in peace-time so far as signalling, etc., are concerned, are based on the technique of echo-sounding, the principles of which have already been dealt with. In the early days of echo-sounding, the method involved the use of audible frequencies, as employed in the **fathometer** designed by the Submarine Signal Company Ltd.



But in view of the advantages of high-frequency transmission over low-frequency given on p. 257 the ultrasonic system is generally used nowadays.

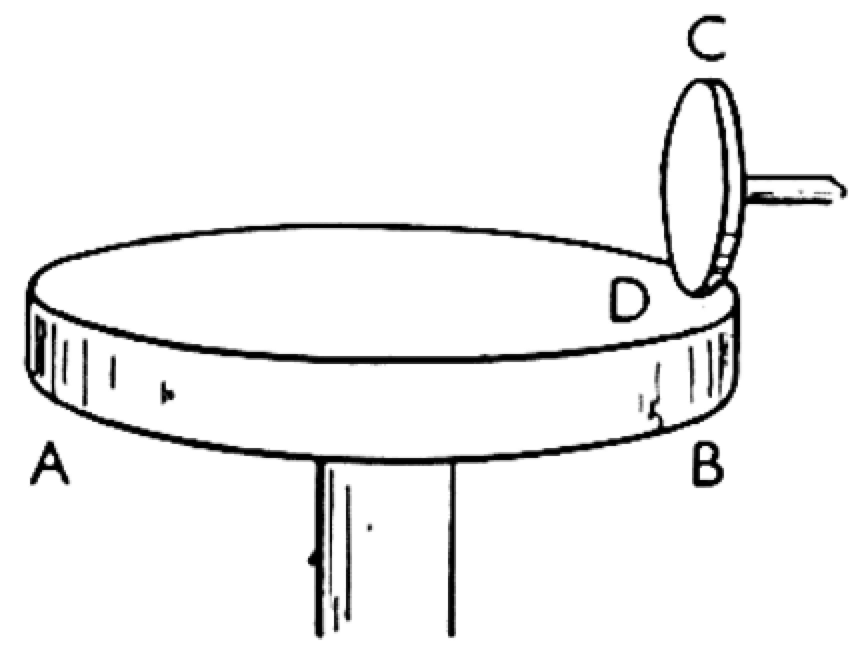
An early method of depth-finding for depths up to about 100 fathoms by using low-frequency sounds consisted in using the ordinary aural method to get a bearing on the reflected sound and then to calculate the depth. Let P in the diagram represent the position of the propeller, which is the source of sound, and R the receiver, and let the water line, represented by WL, be at a distance a from PR. The path of the sound is PBR, where B is on the sea-bed. If θ is the bearing of the reflected sound at R, we have $d = PR/2 \,.\, \tan\theta$, and the full depth of B is therefore

$$a + PR/2 \,.\, \tan\theta.$$

The student might enquire whether the sound taking the path PAR, being reflected at the surface of the water, will affect the result. It has been seen earlier, however, that when sound is reflected at a water-air surface when it is travelling through water there is a change in phase. Hence the reflected sound taking the

path PAR will interfere destructively with the sound travelling direct along the path PR, and the bearing at R will be on the sound reflected at the bottom only.

For deeper soundings the above method is not very accurate, and for this purpose the United States Navy has developed another method which is capable of a high degree of accuracy. The principle is as follows. AB is a uniformly rotating disc with a rather rough upper surface, and CD is a smaller vertical wheel which is caused to rotate by the friction between its edge and the surface of AB. The velocity of rotation of CD depends on its distance from the centre of AB, and this can be varied by means of a screw attachment on the shaft of CD. At each rotation of CD a sharp sound is emitted by a Fessenden oscillator, and this is heard direct by the listener since one earpiece of a set of head phones is connected to the oscillator. The other earpiece is connected to the receiver, which picks up the signal reflected from the bottom. Now, if CD rotates a whole number of times while the sound from a given signal travels to the bottom and back, it is clear that both sounds will reach the ears simultaneously. Suppose this happens when the time between the outgoing signals is t. If T is the time of rotation of AB, we have $t=T \, . \, r/R$, where r is the radius of CD and R is the distance of CD from the centre of AB. If the required depth is l, then $2l=kVrT/R$, where V is the velocity of sound in the water and k is some integer. To eliminate k another sounding is obtained by reducing R until coincidence is again established. We then have



$$2l=\frac{(k-1)VrT}{R_1};$$

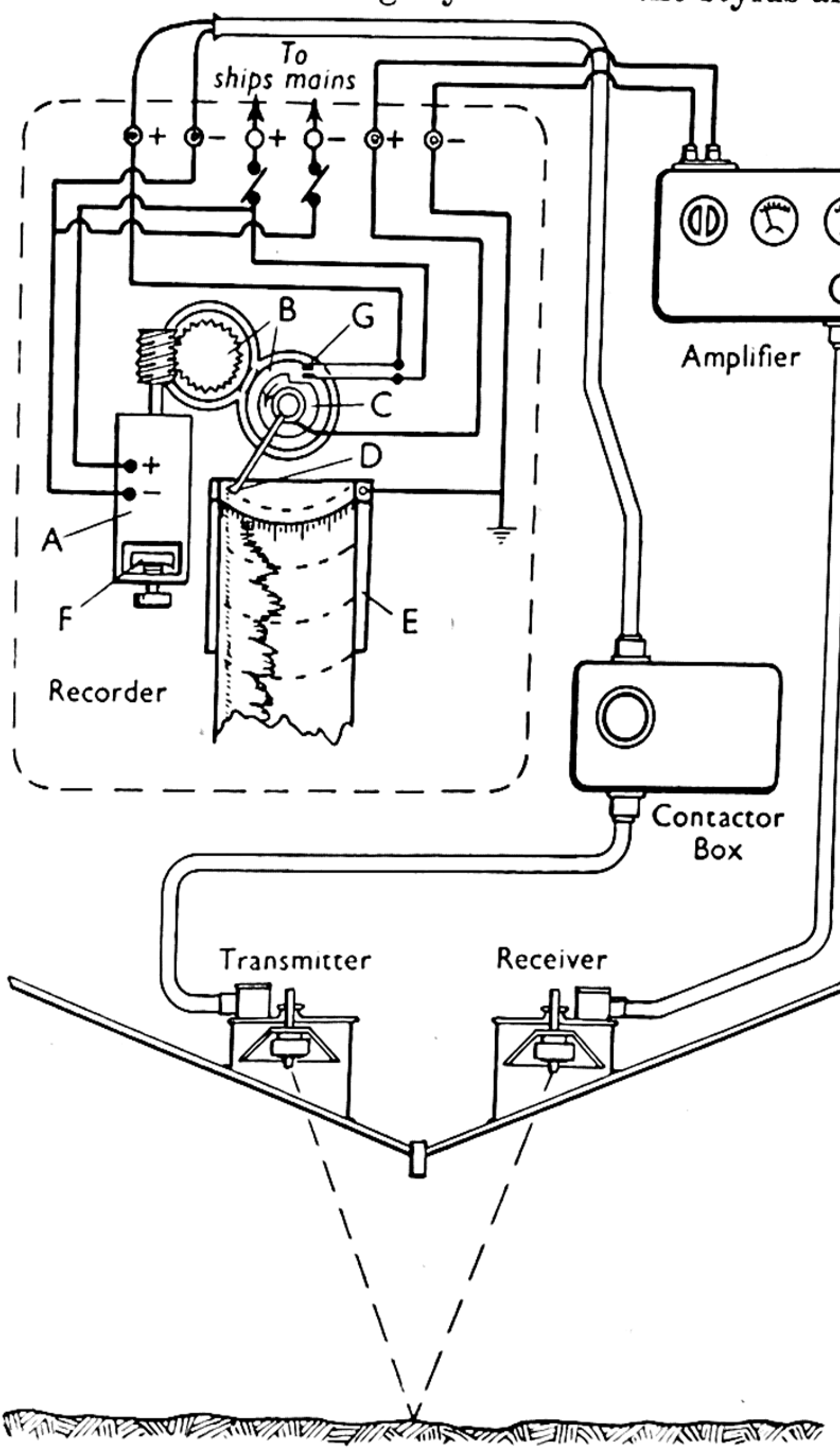
whence
$$k=\frac{R}{R-R_1}$$

and
$$l=\frac{VrT}{2(R-R_1)}.$$

Before dealing with the specific applications of echo-sounding, it might be useful to consider a typical high-frequency installation to show the general layout and indicate the method. Mechanical layouts vary considerably with the different types of recorder,

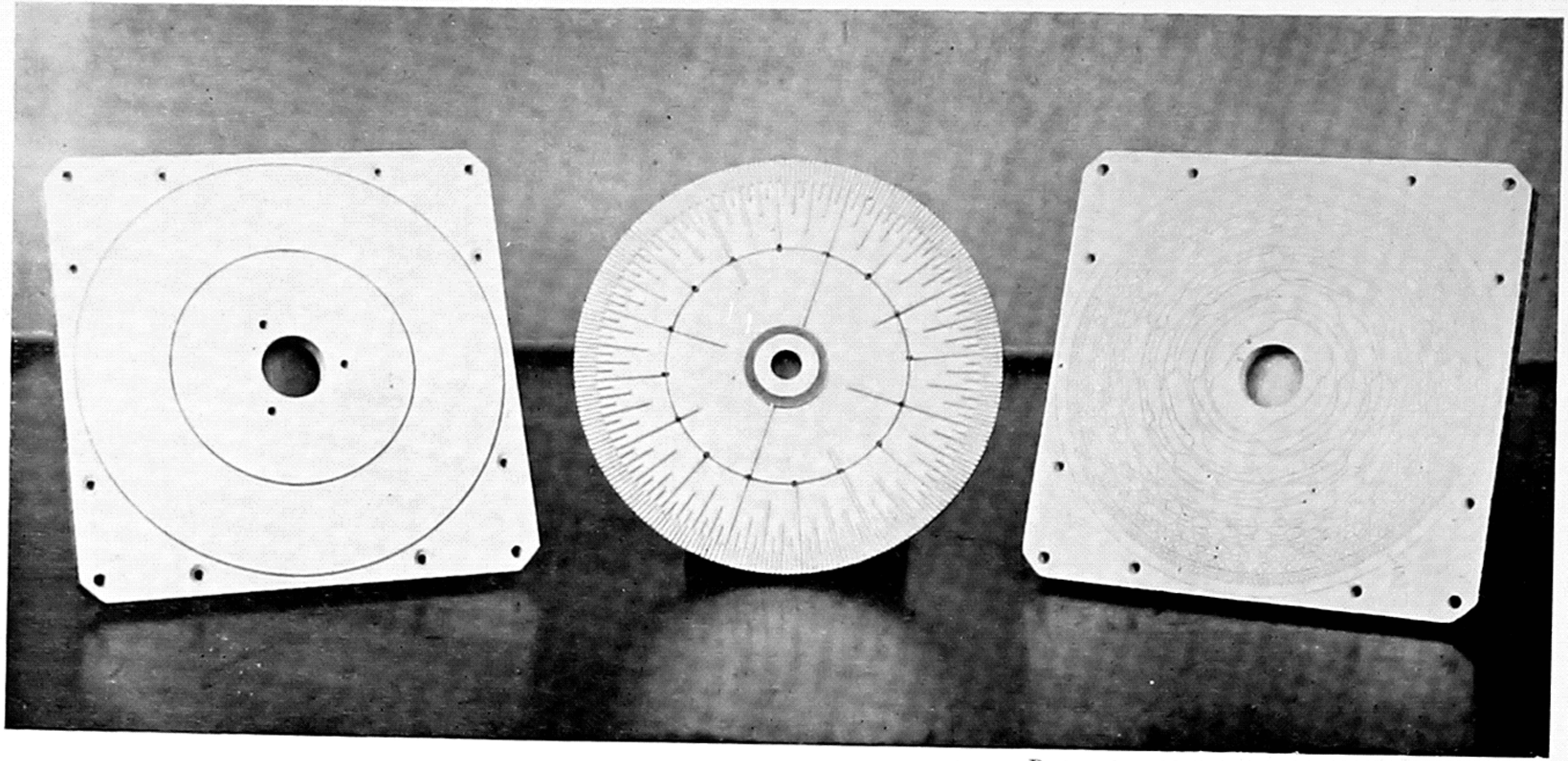
and the diagram indicates the schematic arrangement of the installation evolved and made by Messrs. Henry Hughes and Son Ltd. As will be seen from the diagram, the units in the installation are the recorder, the oscillators, the contactor box and the amplifier.

Recorder. Through the gear train B the motor A drives the switch cam C, to which is rigidly attached the stylus arm D. As



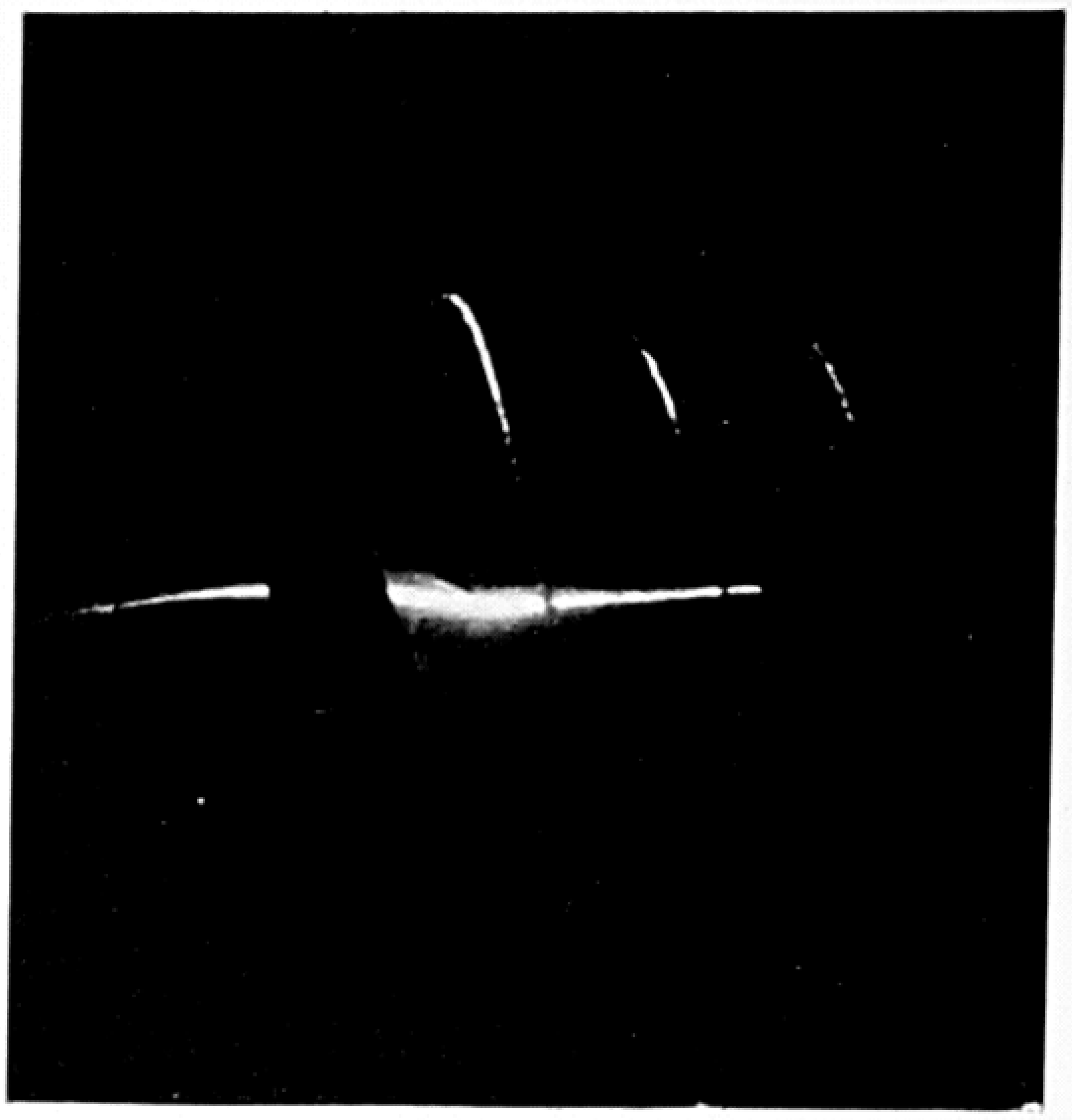
By courtesy of Messrs. Henry Hughes & Son Ltd

High-frequency sounding equipment.

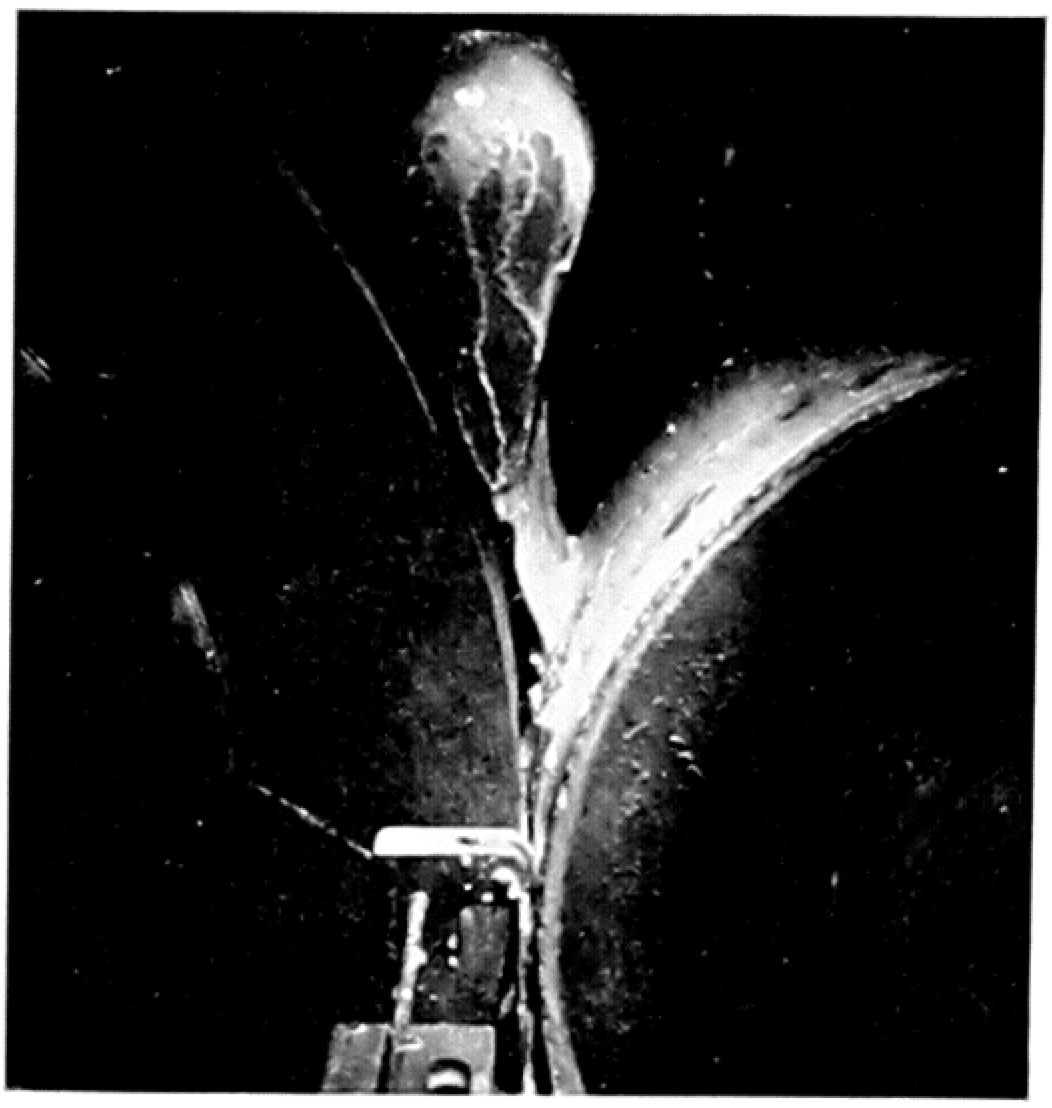


By courtesy of the John Compton Organ Co. Ltd.

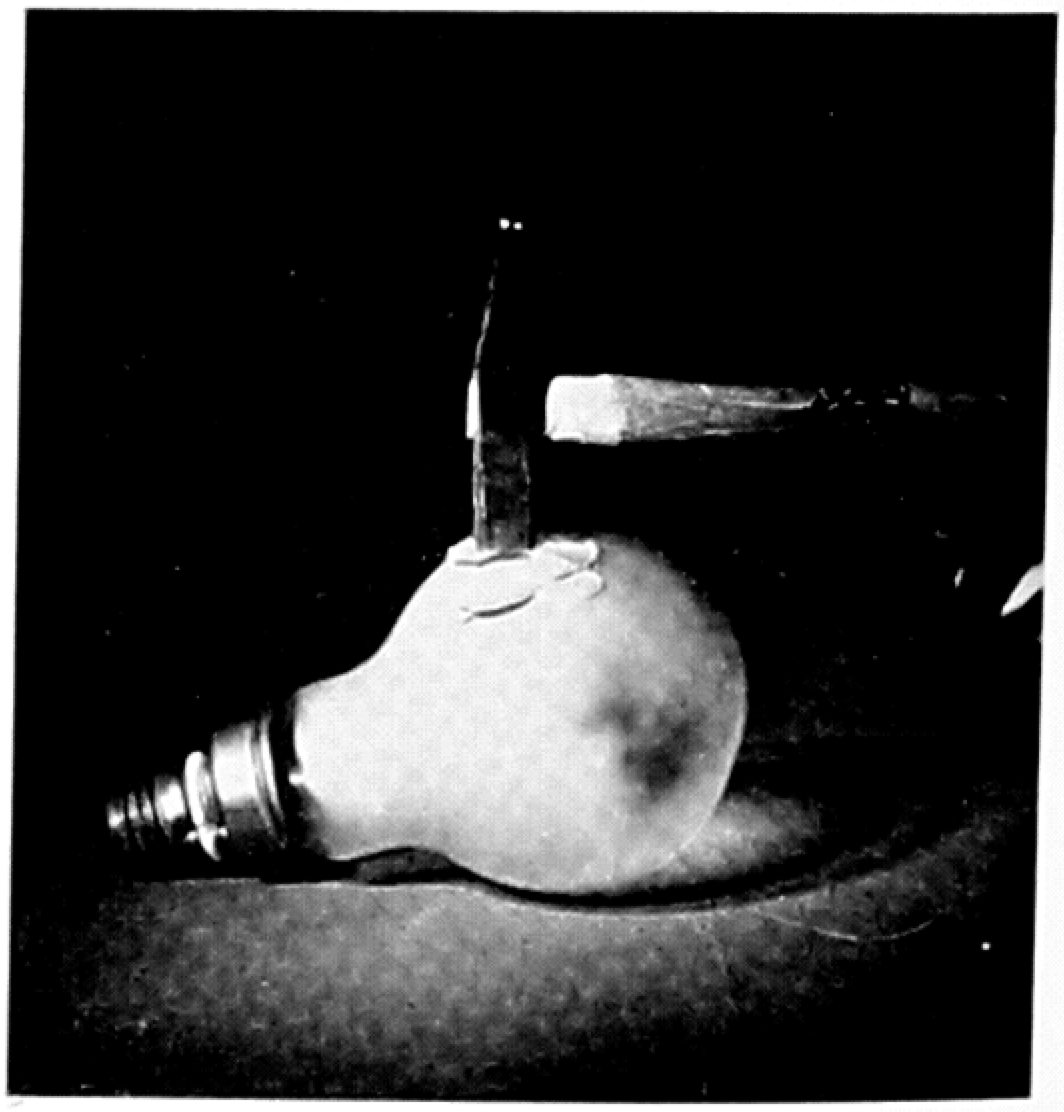
PLATE 5. Rotor (centre) and stator (right) plates of an electronic organ.

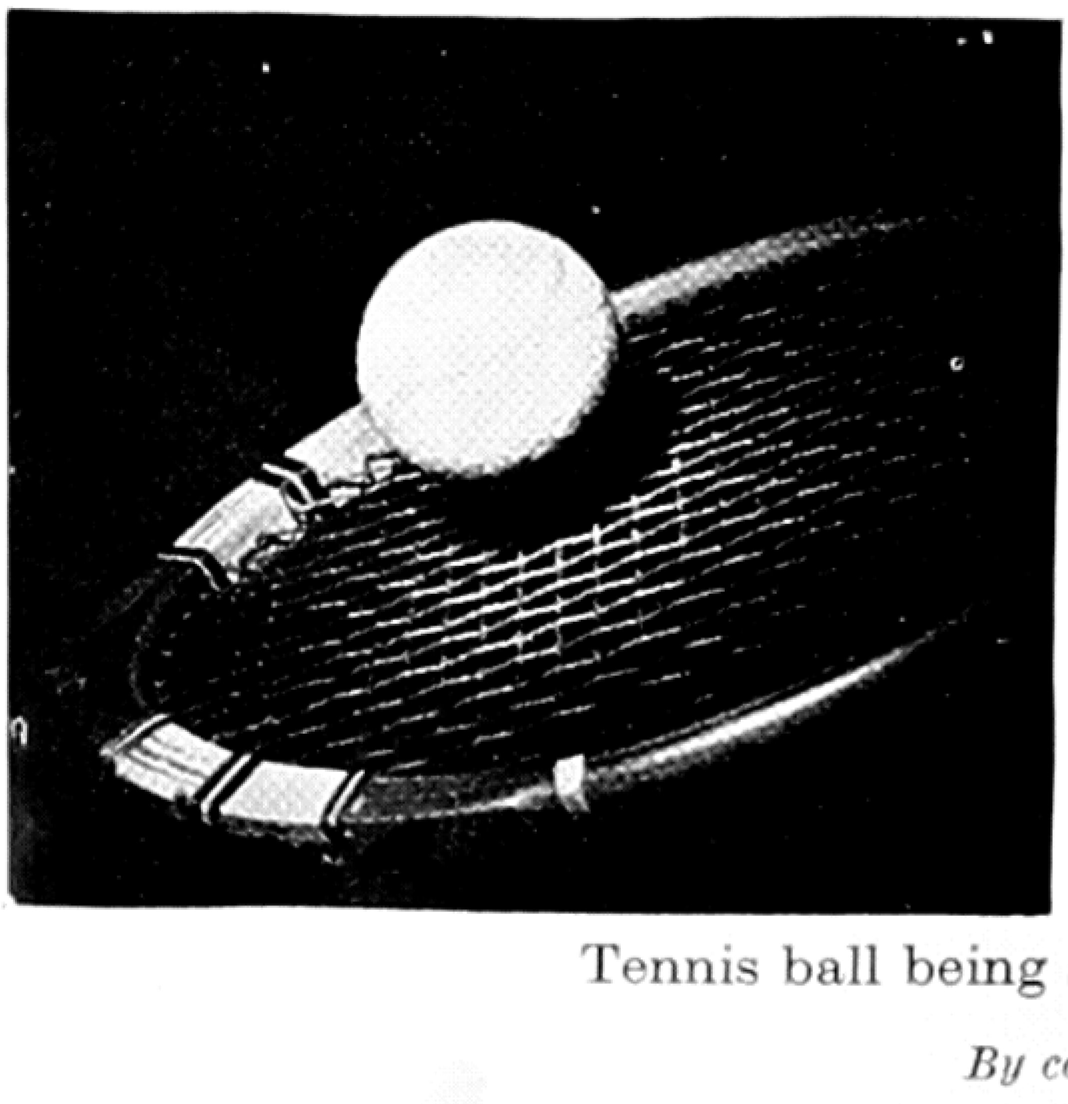
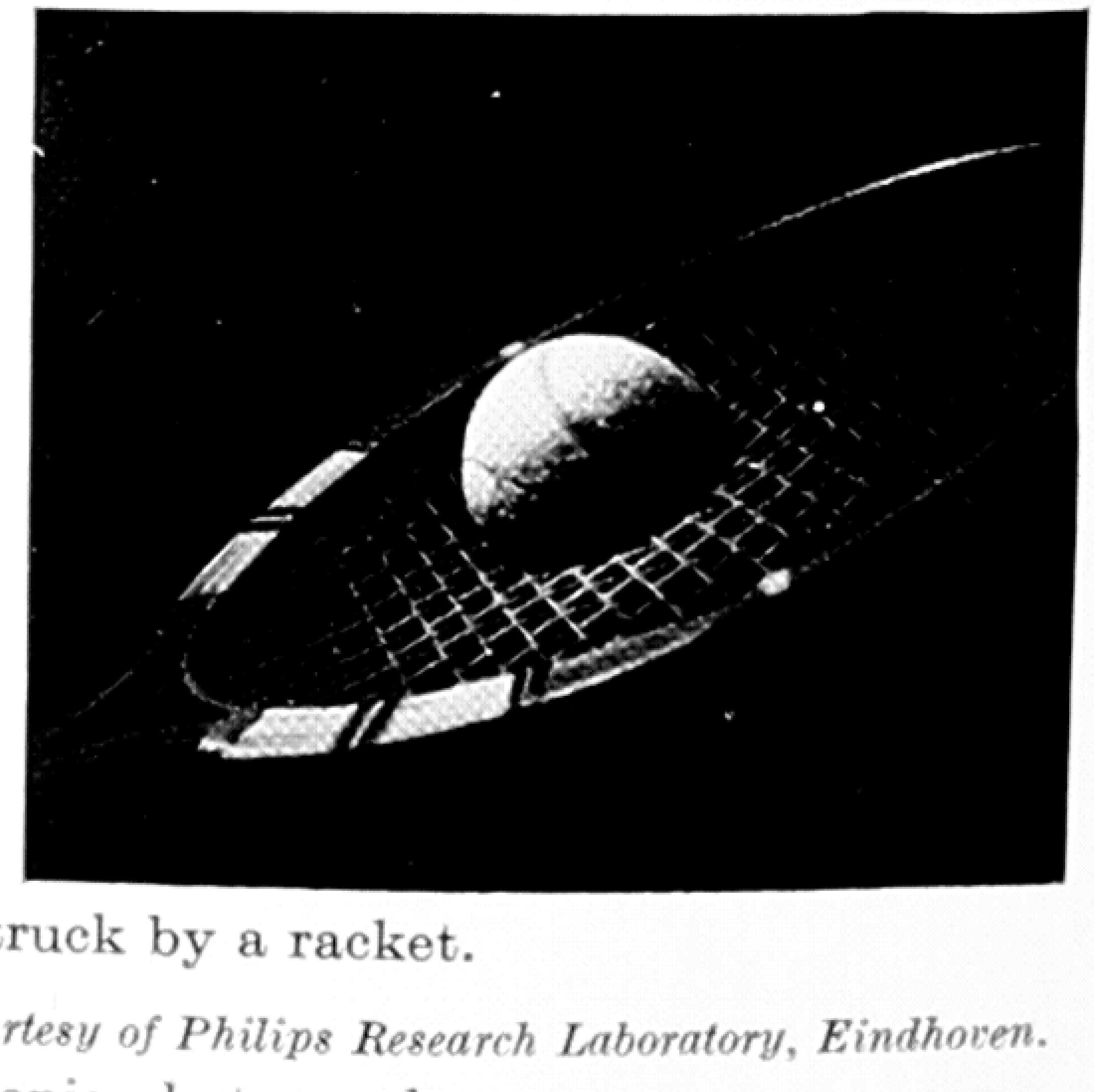
Cavitation (left), and improvement produced by modifying screw (right).



Lubricating liquid being blown away from a centre-less grindstone.



Hammer smashing an electric light bulb.

Tennis ball being struck by a racket.

By courtesy of Philips Research Laboratory, Eindhoven.

PLATE 6. Stroboscopic photographs.

the stylus arm revolves, the point is made to pass over the surface of the recording paper. Once each revolution, the switch cam, by operating the transmitting contacts *G*, causes a pulse of sound to be sent out from the transmitter in the bottom of the ship. At the same instant the stylus passes the zero of the scale, and immediately afterwards marks the paper at a point proportional to the time interval between transmission and reception of the echo. This being directly porportional to the depth, the scale may be graduated in fathoms or any other depth unit.

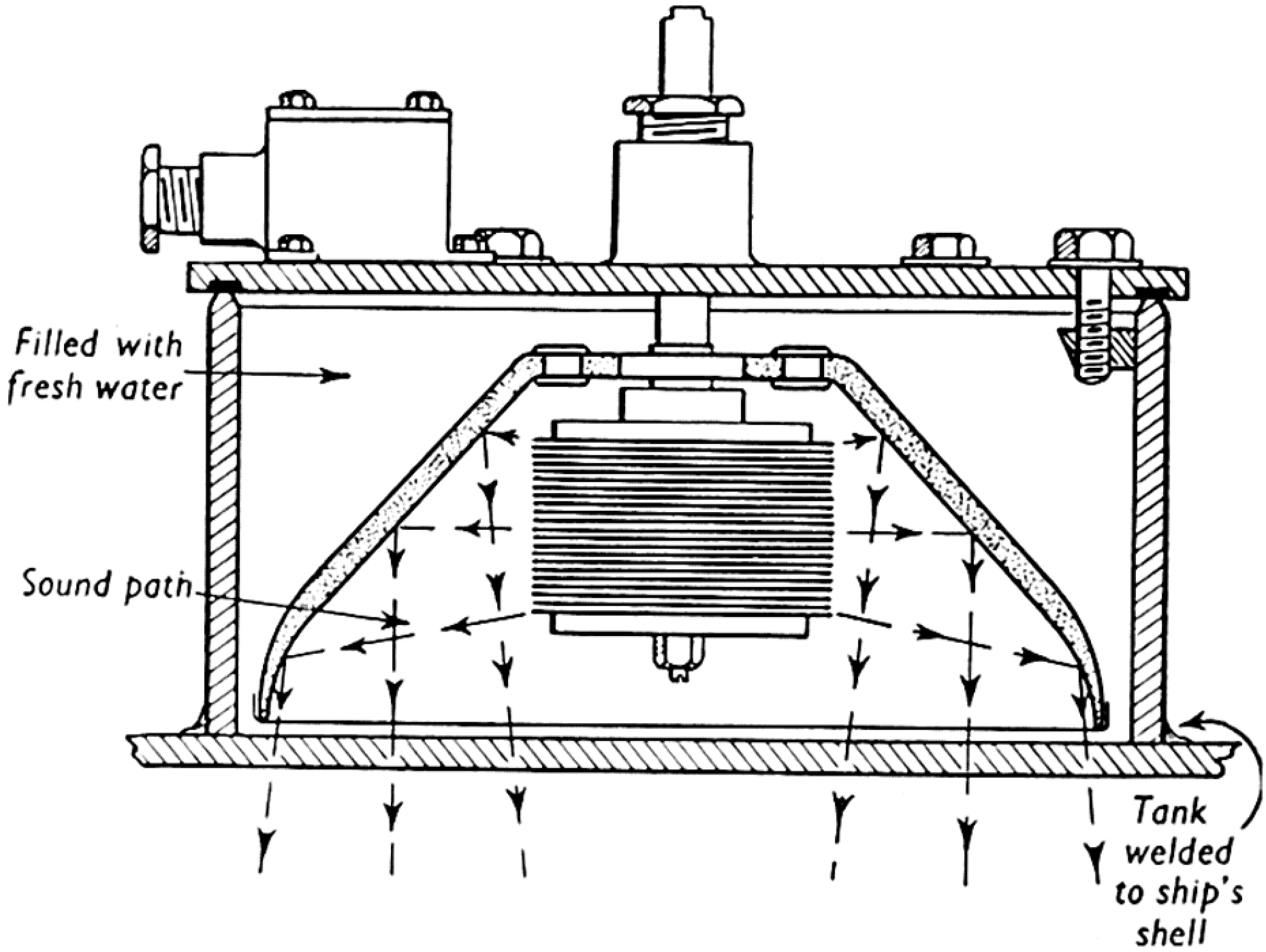
The recording paper is chemically treated so that a current passing through the paper from the stylus point to the metal tank front *E* causes a brown mark on it. The returning echo is suitably amplified to supply a short pulse of current to the moving stylus at the moment of its arrival. On going into deeper water the stylus moves farther across the paper before the echo is received, and the recorded line moves a corresponding distance across the paper. If the paper is made to move a short distance at right angles to the movement of the stylus for each passage of the stylus, the successive echoes will form a contour of the sea bed.

For calibrating the scale, the velocity of sound in sea water may be taken as 800 fathoms per second, so that it requires 1 sec. to go and return in a depth of 400 fathoms. Hence, if the scale is divided into 400 divisions, and the stylus moves through these 400 divisions in 1 sec., then for every fathom of depth the stylus will move through one division. If the switch cam is adjusted so that the transmission occurs at the instant the stylus passes the zero division, then an echo from a 100-fathom depth will return as the stylus passes the 100th division. Such a scale may be marked directly in fathoms. Recorders possessing different scales may be produced by simply varying the rate of travel of the stylus. It is clear that for any given recorder it is essential to keep the speed constant at the rated value, and for this purpose an automatic centrifugal governor *F* is fitted. The function of this governor is based on the fact that variations in the field strength of the motor will affect its speed ; within certain limits a reduction of current will increase the speed, and an increase in current will bring about a reduction in speed.

The stylus, which can be quickly detached from the arm, is a piece of steel wire with the ends bent over at right angles, the one marking contact with the paper being tipped with iridium. The other end passes over ramps which serve to lower the stylus point gently on to the paper at the start of the traverse and to lift it off again just before it reaches the edge, so that the stylus does not

make contact with the metal surface of the tank. The recording paper is treated with the necessary electrochemical material and is moist, being packed in air-tight tins.

Oscillators. Both the transmitter and the receiver are of the magneto-striction type, and consist of a pile of thin nickel rings with a winding in toroidal form. In the transmitter the oscillatory discharge (see contactor box) sets up an oscillatory magnetic field in the nickel stampings, and this causes the mean diameter of the stampings to undergo periodic changes and so vibrate



By courtesy of Messrs. Henry Hughes & Son Ltd.

High-frequency transmitter in tank.

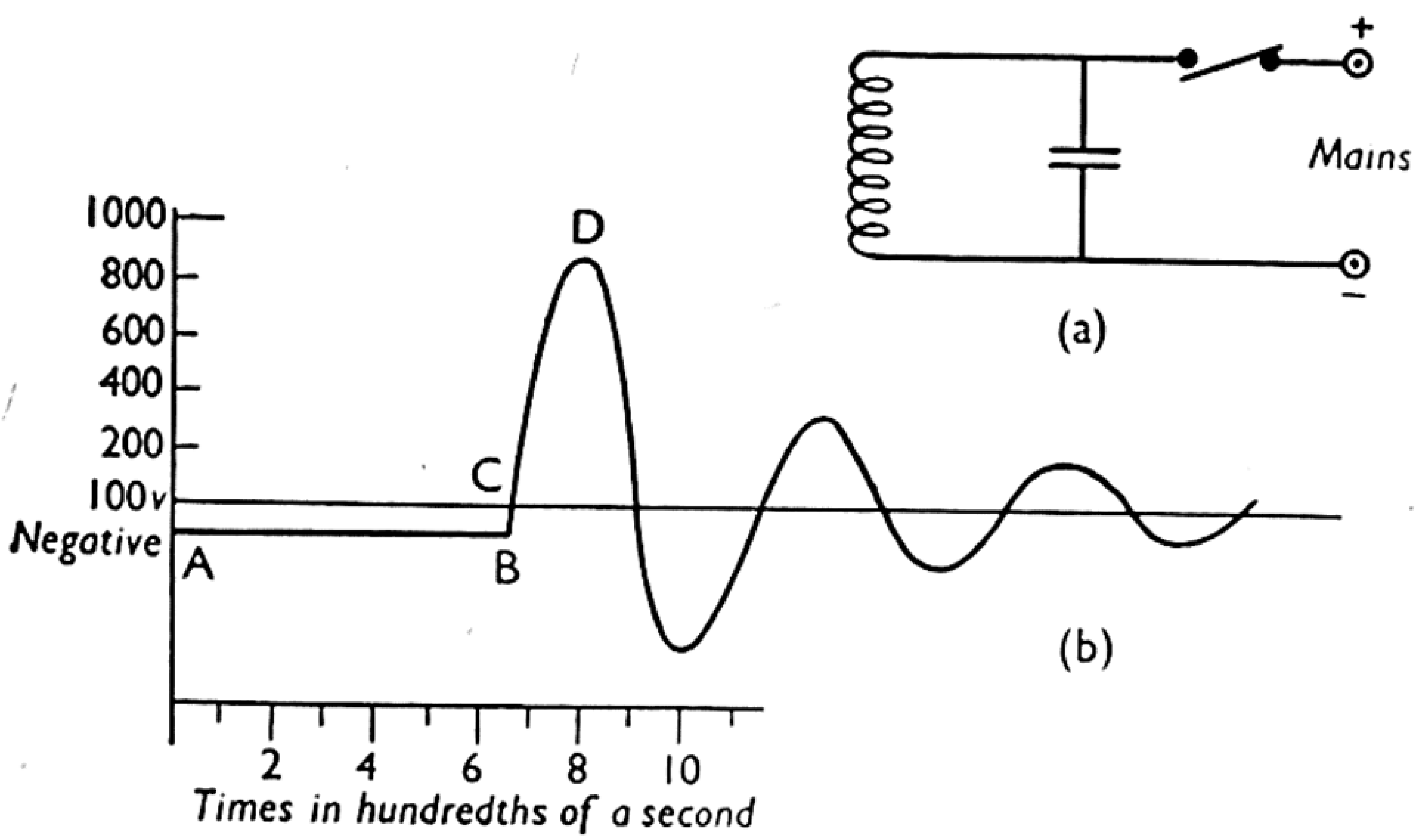
radially. The frequency of the discharge is adjusted so that the pile resonates at its fundamental mechanical frequency. The radial vibrations are transmitted to the water in the tank enclosure in a horizontal direction and are reflected downwards through the bottom of the ship by a conical reflector. The reflector concentrates the sound energy into a conical beam of about 40°, which is considered wide enough to prevent the loss of echo through rolling, and narrow enough to give an appreciable concentration of energy.

The process which occurs in transmission is reversible, so that when the sound energy sets the receiver in vibration the magnetisation of the nickel is altered periodically and sets up an oscillatory electric current in the windings. This current requires

amplification in order to operate the recorder, and in most respects the amplifier used is similar to the standard wireless instrument.

Where very low temperatures are likely to be encountered, the tanks housing the oscillators may be filled with a mixture of four parts of *fresh* water and one part glycerine ; this mixture does not freeze until approximately $-4°$ C.

Contactor box. With this arrangement, the transmitting condenser is charged by the inductive surge from an iron-cored choke coil, the underlying principle being the well-known one that with a circuit as shown in diagram (*a*) breaking of the circuit by the



switch brings about an oscillatory exchange of energy between the inductance and the condenser. By a suitable proportioning of the capacitance, inductance and resistance of the circuit, the condenser may be made to attain voltages very much higher than the mains voltage, as is shown in diagram (*b*). Here the line *AB* represents conditions with the switch closed, current flowing through the inductance, and the condenser being charged to the mains voltage. At *B* the circuit breaks and from *B* to *C* the condenser discharges through the inductance. From *C* to *D* the inductance transfers the energy of its magnetic field to the condenser, voltage being a maximum at *D*. If not interfered with, this cycle would be repeated until the resistance of the circuit absorbed all the energy. For echo-sounding purposes the interval *CD* represents a time of about one-hundredth of a second.

If matters are arranged so that the contactor closes at *D* and discharges the condenser through the transmitter, practically all

the energy will be transferred to the transmitter. The setting of the timing of the contactor is the only critical feature of the arrangement, and is the only part likely to require adjustment.

APPLICATIONS OF ECHO-SOUNDING

Hydrographic surveying. On account of their portability and compactness, magnetostriction oscillators are particularly useful for water surveying as they enable very shallow soundings to be taken, and at the same time have sufficient power to penetrate soft strata and indicate hard rocks lying underneath. Much information, quantitative as well as qualitative, is required in the modern survey work of docks and harbours concerning the volume of siltage and also dredging quantities, and this information must be obtained rapidly, continuously and with the greatest accuracy. The ultrasonic equipment used for the purpose takes and plots on a graph as many as 300 soundings per minute, enabling an extremely accurate exploration of any particular profile to be made. A typical record is shown in Plate 7, facing p. 278. By sounding along parallel lines spaced reasonably close together and crossing these obliquely from time to time, it is possible to obtain what is in effect a relief map of the bottom, accurate to a few inches. Furthermore, from the record obtained, it is also possible to interpret the nature of the bottom, the presence of hard rock, or rock on sand, or rock under mud, being clearly shown. Wrecks and other submerged obstructions are of course automatically located.

In the summer of 1937, a survey was made of Lake Windermere by echo-sounder. The object was to determine the original conformation resulting from glacial erosion of the bed of the lake, and how it has been modified by silt from the various streams which run into it and by deposits from animals and plants in the lake itself. The survey is being repeated at intervals to analyse the changes which take place in the bed. The record obtained showed the depth of water and the depth of mud, and in some places a triple echo was received from mud, glacial clay and rock. For this type of survey work the recorder has a basic scale of 0–40 ft., but this scale can be extended up to 480 ft.

It is often desirable to put the oscillators outside the hull of the boat, either for portability or when soundings are required in awkward or inaccessible situations. Also, when working in motor-boats with inboard oscillators, after reaching a certain speed there is often a tendency for the soundings to be blanketed by

"aeration", that is, the formation of air bubbles in the path of the sound waves. In such cases it has been found that outboard oscillators eliminate this effect and allow much higher speeds to be reached.

The outboard "fish" consists of a matched pair of oscillators, mounted in reflectors and housed in a *D*-section stream-lined form divided into five compartments. The ends and middle contain air, and between them lie the two oscillator compartments, which are filled with fresh water. Thus there is an air space between the oscillators; this acts as an insulator to any interference which might otherwise occur with so small a separation between them, and also gives a considerable degree of buoyancy to the whole unit, making it only slightly non-buoyant when submerged.

Navigation. The use of the echo-sounder together with Admiralty charts constitute the most valuable aid to navigation. For navigation purposes, soundings are taken at the rate of about 250 per minute, and are presented as a continuous profile of the sea bed by the recorder on the ship's bridge. This enables the navigating officer to fix the position of the vessel on the chart at any time, during dense fogs or when sights cannot be taken; risk of grounding is practically eliminated, and, of great importance, the captain is able to make the shortest distance between ports. It also allows the ship to proceed with caution towards the pilot so as to make port without having to lie at anchor until the weather clears.

Trawling. Echo-sounding machines have proved their worth on trawlers for many years, but only since the Second World War has their value been fully realised. For this type of work the equipment must have special features though, of course, the basis of operation is the same as in other magnetostriction oscillators. It must be of robust construction with the utmost simplicity of operation, and must be reliable, working for indefinite periods without breakdowns. Further, it must record from 1 to 500 fathoms or more with absolute accuracy, and it must have small dimensions on account of the limited space available in such vessels. Such a machine is the multi-phase sounder which has been evolved by Messrs. Henry Hughes & Son Ltd., and by its means a time reading of the ocean bed can be obtained to within half a fathom in accuracy. The skipper can now hold his trawler along his own favourite ledges, ridges and banks, where in his past experience the fish are likely to be at the time of year that he visits any particular ground. Drift while hauling in the catch

can be easily checked, and the risk of losing valuable gear through shooting on an unknown ground is much reduced. The echo-sounder also records the presence of shoals of fish. On some occasions the record of the soundings shows that the fish are too deep for the nets, and by successive soundings it can be deduced whether the shoals are rising towards the surface or going still deeper.

WAR-TIME APPLICATIONS

The uses to which acoustic energy may be put in war-time by any country depend largely on the situation and needs of the particular country. For example, Great Britain has an immense merchant fleet and is largely dependent on that fleet for its survival in times of war. Any nation attacking a country like Great Britain would therefore direct its scientific research along lines which would cause as much damage as possible to the merchant ships ; on the other hand, the country to be attacked would concentrate its energy on the development of protective devices. It is true that a strong navy can in various ways offer protection to the merchant fleet, but in war-time vessels of all types are liable to be attacked, and the most offensive weapons for under-water attack are mines and submarines.

The acoustic mine. Various types of mines, of both the buoyant and the submerged type, have been invented for attacking ships. First of all came the magnetic mine with its many technical devices, then came the acoustic mine. and lastly the type based on the slight decrease in pressure which occurs in the water below a moving ship. The only one that need be dealt with here is the acoustic mine. This was first invented for the British Admiralty during the First World War and was developed and used in the Second World War chiefly against the Allies. The essential feature of this type of mine was a vibrating reed tuned to a definite frequency (a usual one was 240 c.p.s.), and when this was excited by the noise from a ship it indirectly controlled the detonation. Later forms of this mine contained various ingenious devices, one type containing counters which operated like a telephone exchange. The mine would not explode until it was " called up " for, say, the seventh time—the first six ships would pass over it safely, but the seventh would cause it to go off. Another type contained a clock which would keep the mine disarmed for many days, until a fixed date.

To combat these mines it was necessary to investigate as

thoroughly as possible the sounds of ships, not only in the audible region, but also in the frequencies above and below. Nine hydrophones were set up in the Clyde for listening to passing ships. It was found that the sound output came chiefly from the propellers, and the complete spectrum of sounds extended from 1 c.p.s. to about 100,000 c.p.s.; at low frequencies the sound consisted mainly of harmonics of the frequency of the shaft. Modified quartz piezo-electric hydrophones were used to measure vibrations as low as 1 c.p.s., and the higher frequencies were measured with tourmaline hydrophones.

For the purpose of exploding mines working on the audible frequency of 240 c.p.s., a type of road drill was used. A noise-producing box was evolved containing a metal diaphragm about $\frac{3}{8}$ in. thick, and this was beaten by a type of penumatic hammer. The whole assembly was towed in the water by a minesweeper, the conical box being held away from the ship by paravanes. For exploding mines working on very low frequencies of the order of 10 c.p.s., a piston-displacing mechanism was used, in which rhythmic pushes were supplied by an electric motor by means of a cam.

Detection of submarines. In time of war, submarines spend as much of the daytime as possible under water where they are immune from detection by visual methods and by radar, and use the darkness to surface and travel about at high speed. The introduction of a long pipe, known as the Schnorkel or snout, for discharging the gases from a submerged submarine to the surface enables these vessels to remain under water for a longer time. The exhaust pipe sticking above the surface is too small to be detected easily by radar; hence acoustic methods involving the use of ultrasonics become more and more important, and a great deal of knowledge of the behaviour of high-frequency sounds in fresh water and the sea is required before completely reliable detection is obtained (see Plate 7, facing p. 278).

The detection of submerged submarines has been particularly highly developed by British scientists, the method being evolved from a suggestion made in 1912 after the sinking of the liner *Titanic* by collision with an iceberg, and in 1918 scientists at the Admiralty Experimental Station succeeded in using quartz oscillators to obtain echoes from a British submarine at a range of a few hundred yards. The method has now been perfected to such a degree that objects as small as mines and midget submarines can be detected at a distance of one mile. Quartz-steel sandwich oscillators are used, operating at a frequency which

research has found to be most useful, namely, in the neighbourhood of 20,000 c.p.s. By using a signal-strength meter it was found that with a frequency of 100,000 c.p.s. the waves are attenuated as much after passing through 50 yards of water as waves of frequency 10,000 c.p.s. after passing through 1,000 yards of water. The sound-power used is about 50 watts, which gives echoes from a submarine at 5,000 yards.

The development of the most useful form of transmitter is, however, only a part of the task of the detection of a submarine, for the transmitter has to be carried in the water below the keel of the ship it protects, and has to be slowly rotated so that all-round acoustic " vision " may be maintained ; the steady rate of rotation is controlled by the ship's gyro-compass. The transmitter must also be enclosed in some compartment, for without some protective covering it would set up turbulence in the water which would refract and scatter the sound. Therefore it is put into a " dome " which is filled with water and is fixed in relation to the hull of the ship. The covering of the dome, its shape and its position on the ship's bottom are all important points to be considered. The covering must be as transparent as possible to the sound waves and yet be strong enough to withstand the forces of the water acting on it, and the position of the dome must be such that the effects due to turbulence and suction are minimised. As a result of much experimenting and observation, it was found that the best position was as far forward as possible on the centre line of the bottom, and modern designs of domes can be used with safety and efficiency at speeds more than 20 knots.

The early detecting devices depended on the hearing of the echo through ear-phones, but later a chemical recorder was used incorporating starch-potassium iodide paper which of course is discoloured when any current passes through it. By this means the track of the submarine in relation to the ship is automatically drawn and the range accurately determined.

The British detecting device above described is called *Asdic*, and there are now many forms of this for special purposes, including the assistance of submarines in offensive as well as defensive work, and, of course, peace-time operations.

CHAPTER XIII

ACOUSTICS OF BUILDINGS

THE need for good acoustics in connection with the satisfactory hearing and appreciation of the performances of players and speakers has been recognised for centuries, for as early as about 55 B.C. Vitruvius stated clearly the problem confronting the actors in the drama. He said: " at some places the sound will strike against solid bodies and check in its return the rise of the succeeding sound so that the word endings are mingled into indistinct noises " ; and he suggested that the actor might stand in such a position that his voice is not so reflected as to cause confusion to the listener.

It must be remembered that the theatre of the ancient Greeks and Romans had no roof, and although there was probably slight reflection from the walls the problem of echoes and excessive reverberation could not have been troublesome. Indeed, the chief defect in the design of this and all open-air theatres is the lack of what Vitruvius called the " strengthening of the voice by reflections from well-placed surfaces ". To remedy this defect, Vitruvius suggested setting bronze vases in the auditorium so that the voice would be strengthened by the resonance of the air in the vessels; although no such vases have survived, cavities have been found in the chancels of some of our English churches and it is now believed that these were for the purpose of amplifying the sound.

The Elizabethan theatre seemed to be modelled on the classical prototype in that, although it was smaller in volume, it had a partially covered stage but was open to the sky above the pit. From time to time since those days much thought has been given to the question of design for good acoustics, and an appreciable amount of useful information was accumulated. It was left, however, to Professor W. C. Sabine, of Harvard University, to initiate a series of experiments on the problem and to put the investigations on a truly scientific basis.

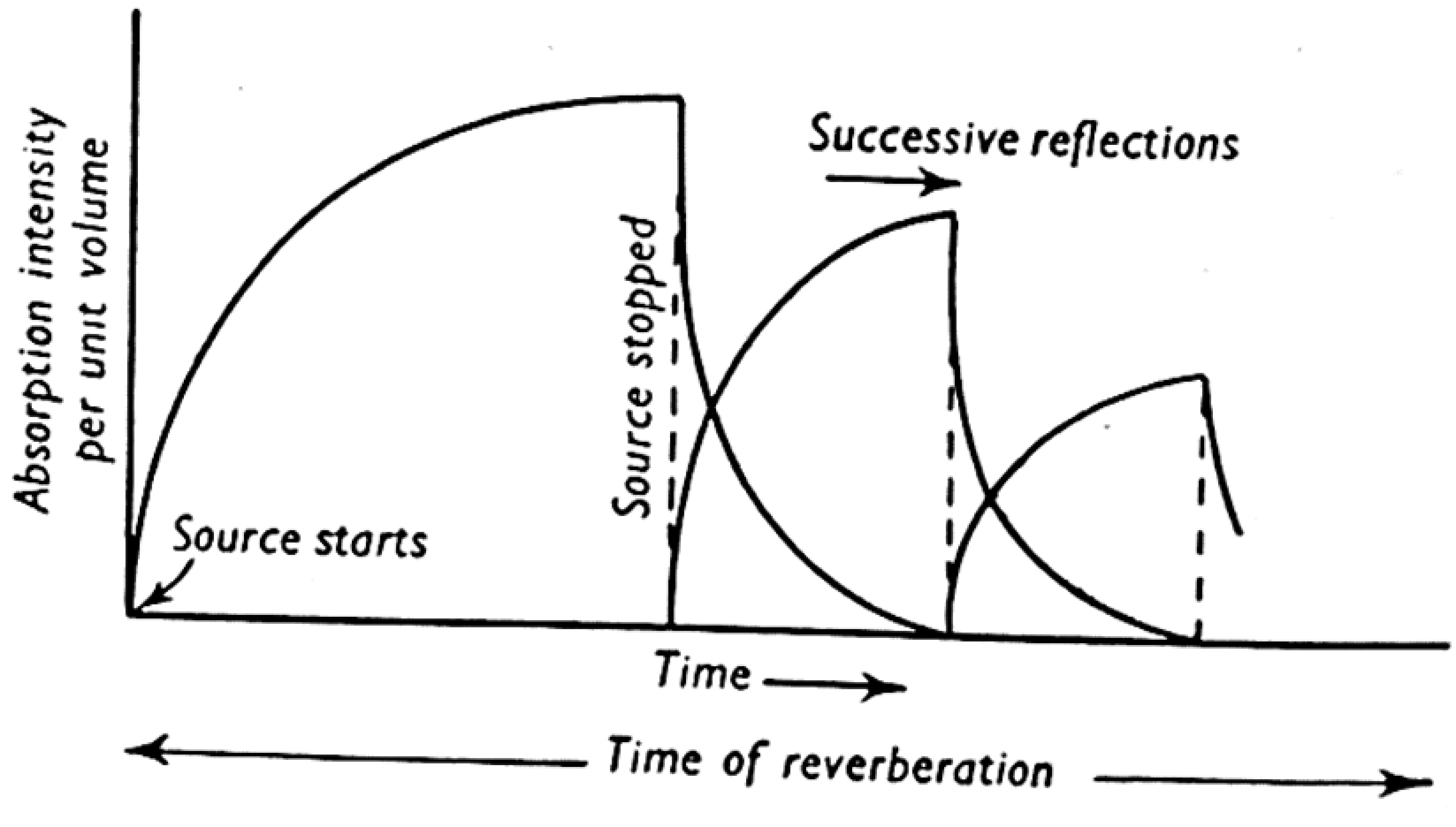
Requirements for good acoustics. For an auditorium to give satisfactory listening, it is necessary that every syllable or musical

note should reach an adequate level of intensity at every point and then die away sufficiently quickly before the next syllable or note is emitted. It is doubtful whether any auditorium can be designed to give perfect hearing of words and music to suit everyone's taste, but certainly the combined efforts of acoustic and structural engineers and physicists can now design a hall which will give very satisfactory results. It is also true that it is far better and less costly to plan the hall from the point of view of the acoustical requirements before it is built, rather than build it first and then have to modify it to make it acoustically satisfactory.

In designing an auditorium, the first main objective should be to ensure a good direct path for the sound, for it is important to hear the original sound loudly and clearly in order to avoid confusion from subsequent reflections. Secondly, none of the subsequent reflections should compete in strength with the direct sound, and thirdly, the appropriate degree of reverberation should be provided, and in particular the high frequencies must be preserved.

SABINE'S WORK

Reverberation and absorption. Sabine's early investigation soon led him to appreciate the main problem involved in the quest for satisfactory acoustics, which is the relation of reverberation to the amount of sound-absorbing material present. By reverberation is meant the prolongation of the original sound by the successive reflections which occur too quickly to produce distinct echoes ; the diagram illustrates this graphically. An outstanding example in Nature on a grand scale is afforded during a thunderstorm. If a strong reflection is heard at more than 1/15 to 1/20



second after the original sound, a distinct echo is apparent. In that time, sound will travel approximately 50 to 70 ft., so that for an echo to occur in a hall this latter must be large enough for the path of the reflected energy to exceed that of the direct sound by such a distance.

When a sound is emitted in a closed room, the energy striking the walls, etc., can be reflected, absorbed or transmitted. If the walls are of brick covered with plaster, most of the energy is reflected and very little is absorbed or transmitted ; a heavy curtain, however, probably transmits more than half the energy, absorbs perhaps one-third and reflects very little.

If a continuous source of sound operates in a closed space, say a hall, it is clear that the intensity is prevented from becoming indefinitely great in the hall mainly by the amount of absorption by the surrounding surfaces ; hence the magnitude of the reverberation must depend largely on the absorbing power of the surfaces. It is found that the shape of the hall is in general not important so far as reverberation is concerned ; but clearly if there are deep recesses in the room, both the time of reverberation and the distribution of sound are likely to be affected.

Sabine started his experiments by timing the reverberation of an empty hall. His source was in the form of a small organ pipe which was arranged to give an intensity of sound 10^6 times that of the minimum audible intensity, this being about the average intensity used by a speaker addressing an audience in a small hall. Thus he defined the **time of reverberation** as the time in which the intensity of sound decays from one limit to the other, that is, a difference in intensity of 60 decibels.

He then tried the effect of putting cushions on the chairs, etc., and he found this caused a steady decrease in the time of reverberation. This result was expressed by the relationship

$$(A+a)t=k,$$

where A is the absorbing power of the walls, etc., measured in square metres of cushions, a is the area of the cushions in square metres, t is the time of reverberation and k is a constant. From the two experiments it is possible to calculate how many square metres of cushions are necessary to reduce the time from that of the empty hall to any desired value.

Sabine next carried out experiments with various materials and was able to make comparisons of A for these materials with a standard absorber. This was an open window, which, so far as

reverberation is concerned, can be considered as a perfect absorber, since, as the sound waves simply pass through, there is no reflection. The results showed that the absorption is proportional to the area of the material, and the amount of absorption per square metre of surface Sabine called the **coefficient of absorption** (s), which may be defined as the ratio of sound-energy absorbed to the incident energy. The range of values of absorption coefficients is not large, varying from about 0·01 to unity, and the accompanying table indicates the values for a few materials.

Absorption Coefficients

Absorbent	Absorption coefficient at frequency of		
	125 c.p.s.	500 c.p.s.	4,000 c.p.s.
Open window	1·0	1·0	1·0
Plaster on brick	0·013	0·025	0·045 (est.)
Concrete	0·01	0·01	0·015 (est.)
Wooden floor	0·05	0·06	0·20
Linoleum on concrete	0·02	0·03	0·05 (est.)
Carpet	0·09	0·21	0·37
Curtains, velour	0·05	0·35	0·35
Hairfelt, 1 in. thick	0·1	0·52	0·44
Fibreboard tiles, perforated, 1¼ in. on solid	0·05	0·54	0·60
Plywood, 3 mm. thick, 5 cm. air space	0·25	0·20	0·10
Ceiling, plaster on lath	0·03	0·03	0·045
Acoustic plaster	0·18	0·32	0·58
Slagbestos	0·30	0·65	0·30
Audience per sq. ft. of floor	0·5	0·82	1·0 (est.)
Air	—	—	0·01
	Absorption per object (sq. ft. units)		
Audience, per person, seated	1·0–2·0	3·0–4·3	4·0–6·5
Seat, plywood	0·15	0·17	0·38 (est.)
Seat, heavily upholstered	2·8(est.)	3·0	3·6

The three frequencies given in the table represent the bass, the middle of the scale and high treble, 125 c.p.s. corresponding

roughly to the octave below middle C on the piano, 500 to the octave above and 4,000 to the note three octaves above middle C.

It should be noted that at high frequencies (above about 2,000 c.p.s.) a proportion of the sound energy is absorbed in the air itself.

In connection with this table, it must be remembered that the absorption coefficient is not a specific property of a material but depends on the thickness and on the method of mounting and it will be noticed it also varies with the frequency. For example, a thin layer of felt backed by a rigid wall absorbs mainly at high frequencies and may be inefficient in the low-frequency range. Improvement at the low frequencies is obtained by increasing the thickness and by spacing the material away from the backing, for example by mounting it on battens.

To find the total absorbing power of the room it is only necessary to multiply the area of the surface of each material present, including walls, etc., by the appropriate coefficient and add the results; thus

$$A = a_1 s_1 + a_2 s_2 + a_3 s_3 + \dots .$$

Finally, Sabine carried out experiments in rooms of different sizes and found that for a given amount of absorbing power the time of reverberation was proportional to the volume of the room irrespective of its shape; in fact, he showed that for all rooms $k/v = 0{\cdot}164$ if all measurements are in metres. Hence, the equation on p. 273, $At = k$ (where A represents total absorption) becomes $At/v = 0{\cdot}164$.

$$\therefore \ t = \frac{0{\cdot}164v}{A} .$$

This is known as **Sabine's equation.**

If all measurements are expressed in terms of feet, $t = 0{\cdot}05v/A$.

Since v and A can be calculated from plans and specifications, it is possible to build a structure of any desired time of reverberation. To test his work, Sabine planned a new hall, and his theoretical calculations were found to give the correct time of reverberation when the hall was built.

Modification of Sabine's formula. Sabine obtained his formula as a result of his experiments, but in 1903 W. S. Franklin dealt with the problem theoretically. On the assumption that the distribution of sound is completely random, he showed that the average energy-density I at a time t after the source is stopped is given by $I = I_{\max} \exp\,(-VAt/4v)$, where $I_{\max}$ is the average

energy-density at the instant the source is stopped, V is the velocity of sound, A the total absorption and v the volume. From this, the time of reverberation is given by

$$10^6 = \exp(-VAt/4v), \quad \text{or} \quad t = 0{\cdot}161v/A,$$

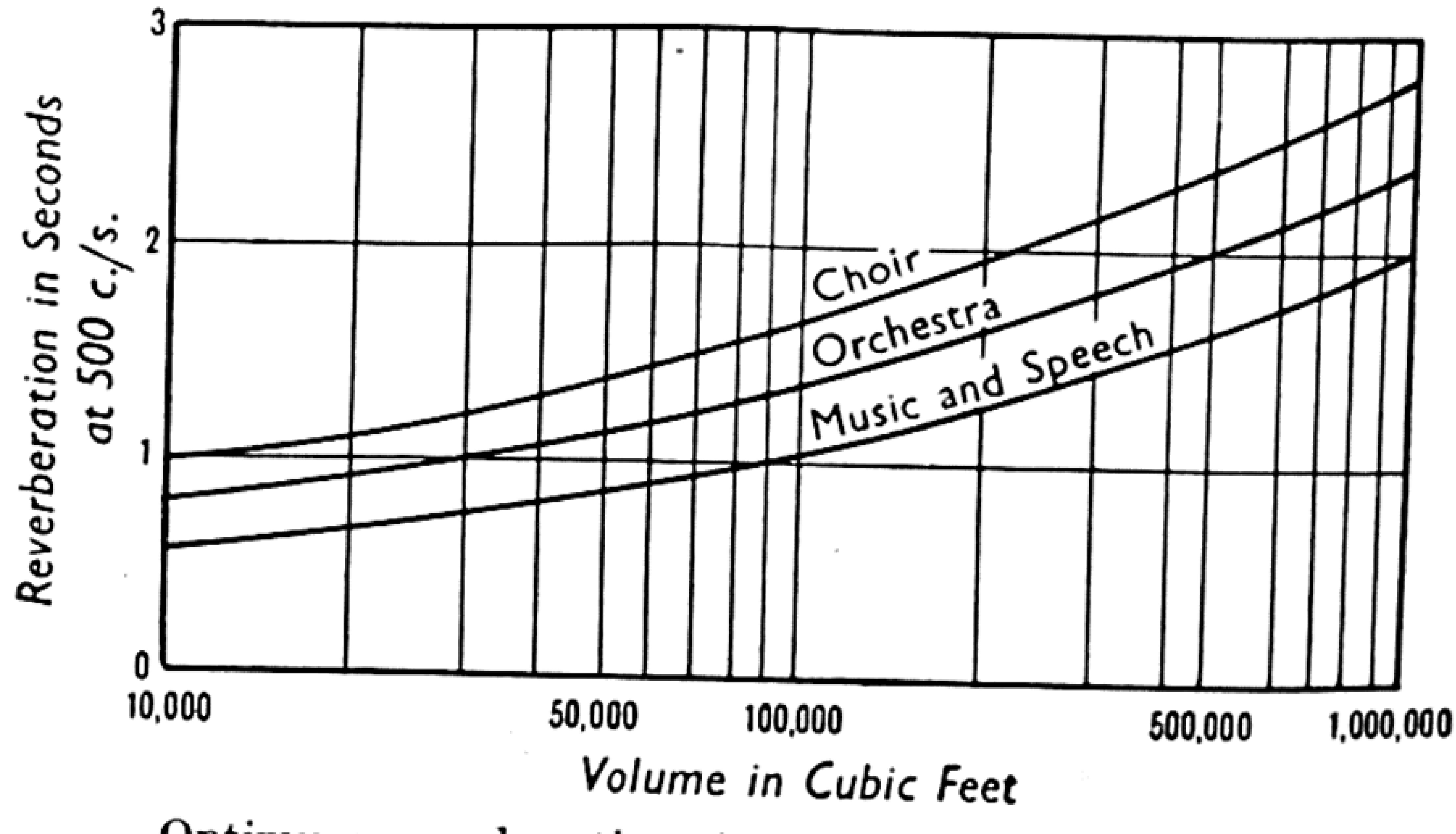
if all linear dimensions are in metres and V is taken as 344 metres per sec. at room temperature (20° C.). It will be seen that this is in very good agreement with Sabine's formula.

But more recent work by Eyring and others has shown that Sabine's formula applies essentially to "live" rooms in which there is small absorption and large reverberation, but is unsatisfactory for rooms of very large absorption when the sound suffers few reflections during the course of decay. Eyring derived a new equation by considering the reflection at the walls as due to a sequence of "image" sources which all come into action at the instant the source starts. According to his theory, the formula for the time of reverberation is

$$t = \frac{0{\cdot}05v}{-S\log_e(1-a)},$$

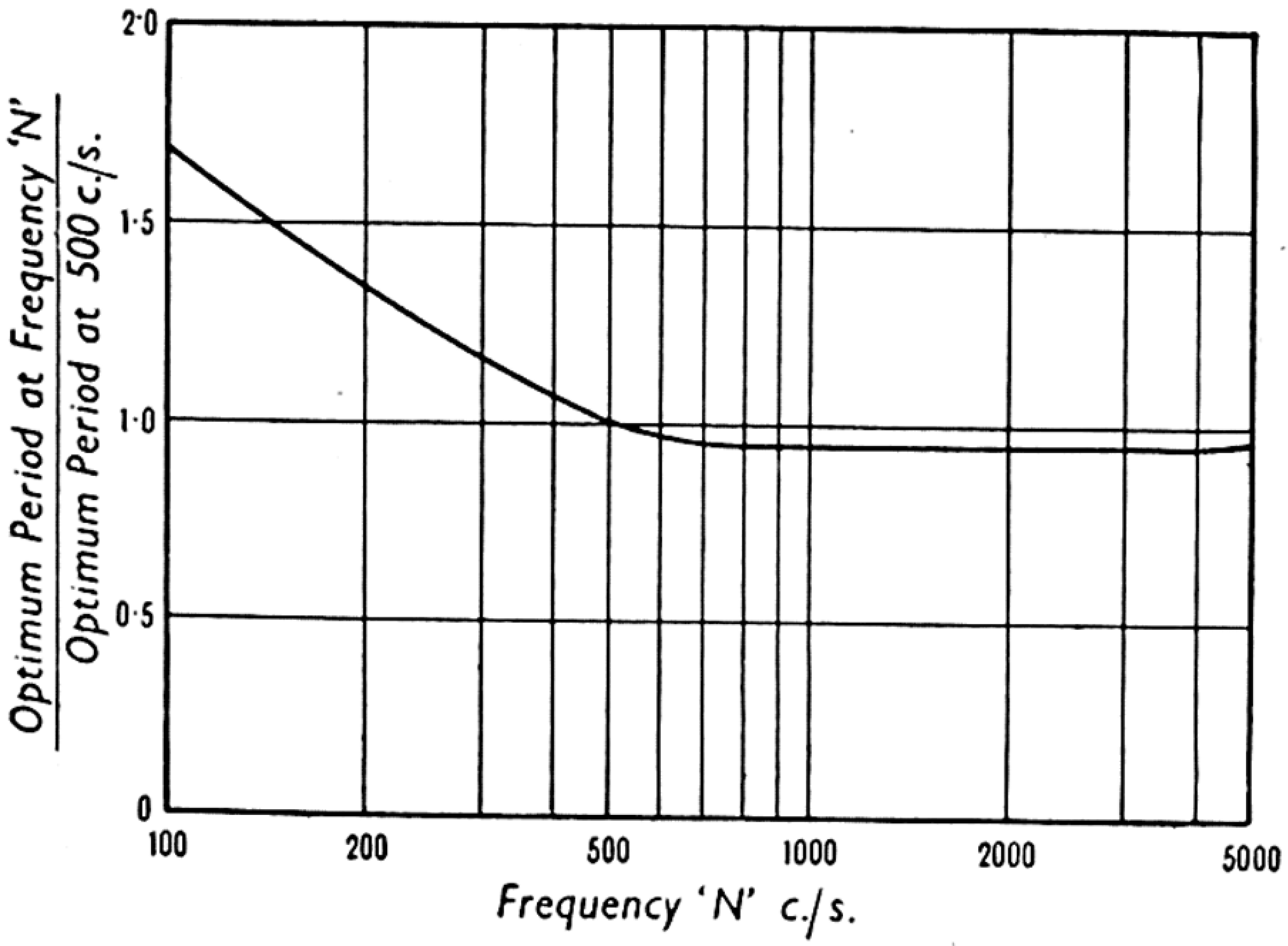
where S denotes the total area (in sq. ft.) of the exposed surfaces in the room and a is the average absorption coefficient. It is, however, only when the average absorption coefficient is greater than about 0·2 that there is any appreciable difference between the results given by the two formulae.

Optimum time of reverberation. One of Sabine's major contributions to the proper understanding of auditorium acoustics was



Optimum reverberation times (Bagenal and Wood).

to establish that there is an optimum time of reverberation which varies with the size of the auditorium, being longer for larger halls, and with the type of performance, being longer for music than for speech. So far as volume is concerned, the accompanying curves, due to Bagenal and Wood, show the variation for halls intended primarily for musical purposes and for halls in which both speech and music may equally have to be catered for, the



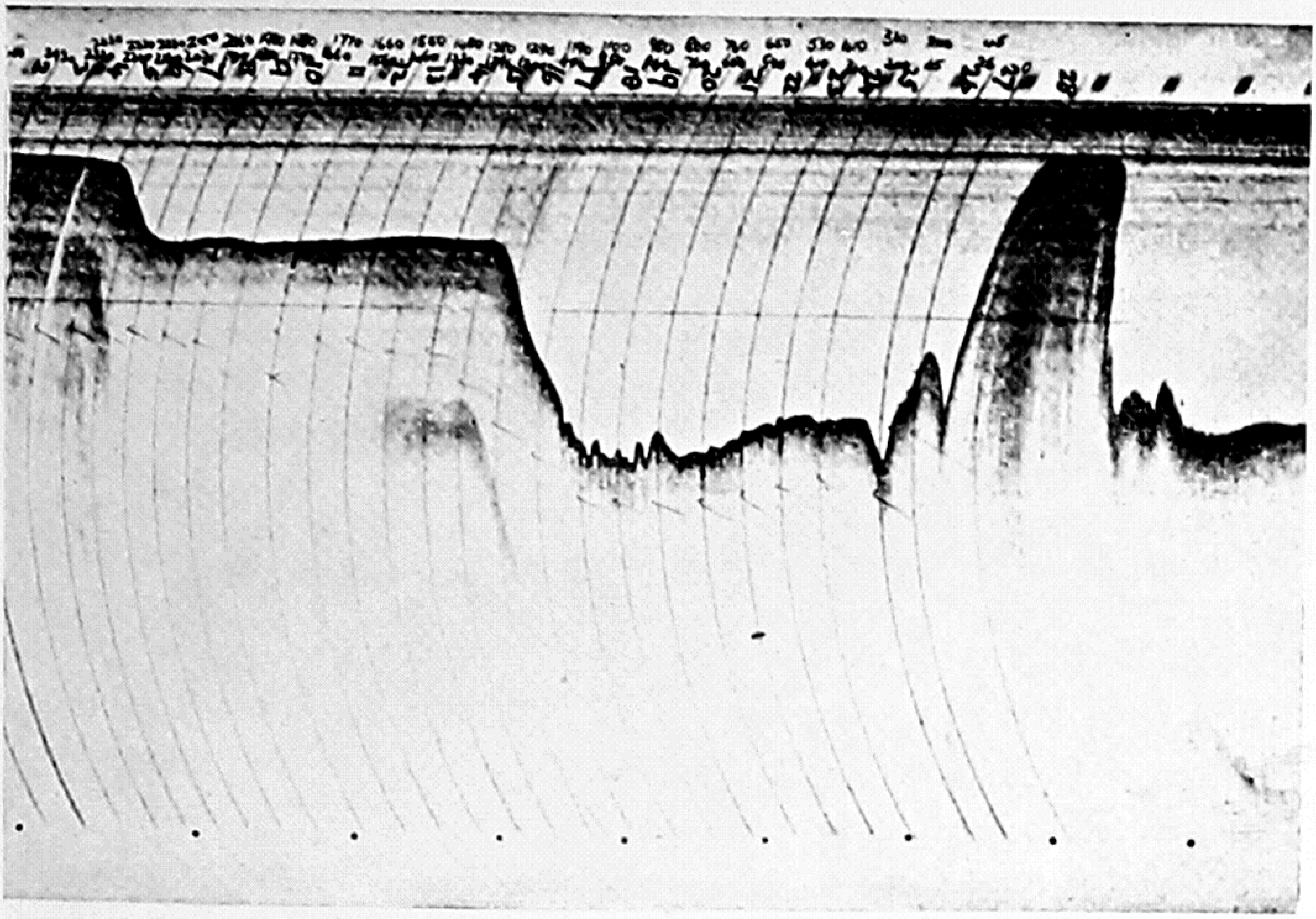
Variation of optimum period of reverberation with frequency (MacNair).

chosen frequency being 500 c.p.s. The values of the reverberation periods were based upon calculations of the periods of a number of halls which were generally accepted as satisfactory; it must be noted, however, that in the light of further experience Mr. Bagenal appears to think the curves may be high. At lower frequencies, and to a lesser extent at very high frequencies, a rather longer period is preferred; it is particularly important that the reverberation should not be too low at high frequencies, because upon these mainly depend intelligibility of speech and much of the character of other sounds. The variation of time of reverberation with frequency does not appear to be very critical, but MacNair has produced a very helpful graph, as shown, based upon the assumption that the loudness of all frequency components should decay at the same rate. By using the two sets of curves, it is possible to make a suitable choice of absorbent

surfaces in the auditorium to provide the appropriate degree of absorption at the low, medium and high frequencies.

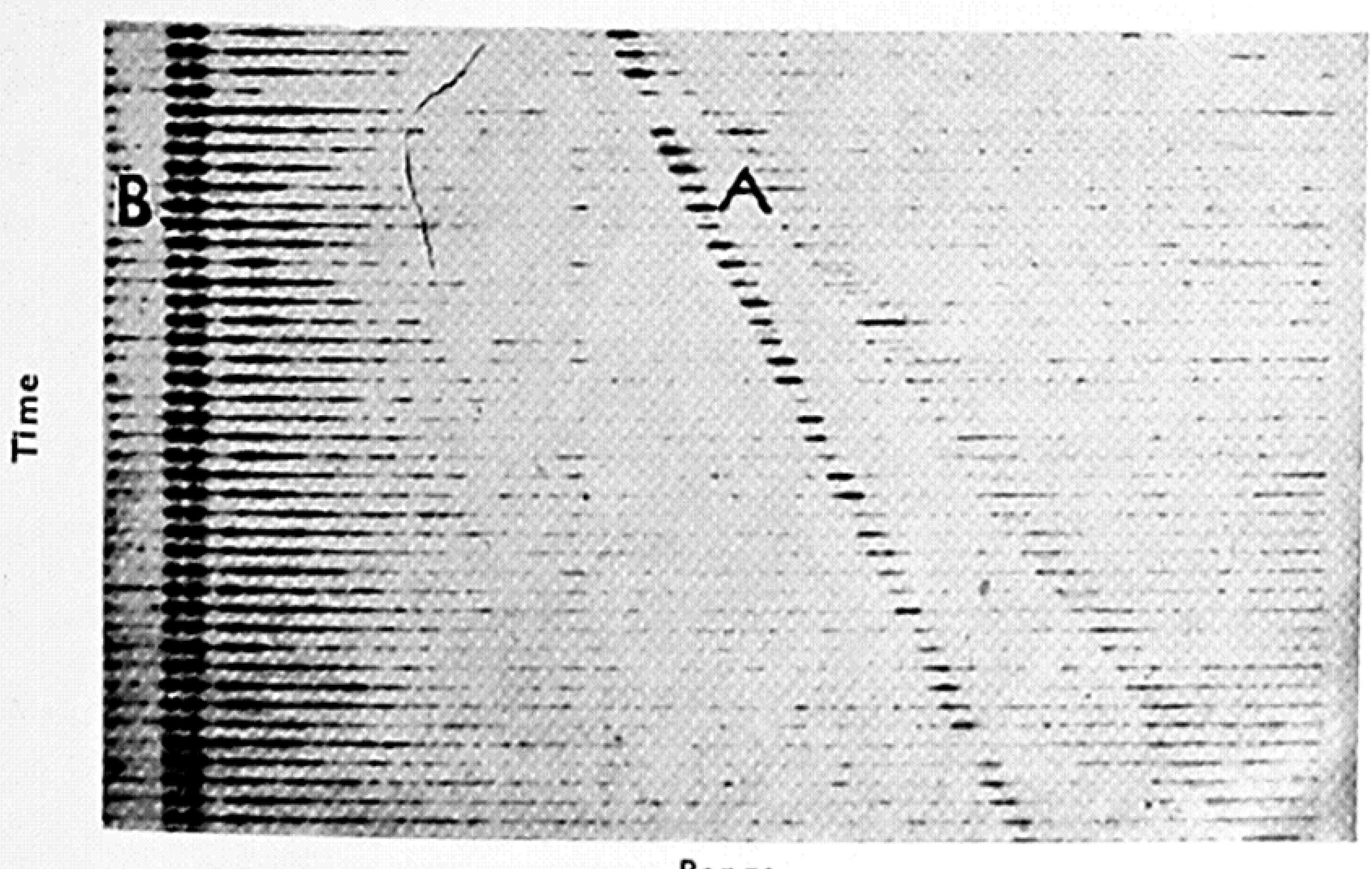
Control of reverberation. It is seen from the above formulae that the time of reverberation depends upon both volume and absorbing power. If v is increased and A remains constant, then t is increased; hence to obtain small values of t the auditorium certainly should not be too lofty. It has been suggested by Fleming and Allen that volumes of 150–200 cu. ft. per person are appropriate targets for moderate-sized halls, with slightly more for larger places. If v is fixed, then the value of A will determine the value of t.

In the design of a hall, assuming that v is settled and the value of t at a certain frequency has been decided upon, the value of A can be obtained from the formula, this value representing the *total* absorbing power in the hall, including walls, fittings, furnishings, etc., and of course the audience. In calculating the absorbing powers of the individual items it is convenient to work in *units* of absorption. To determine how far the correct value of A is met by the proposed design, the total absorption of the proposed surface finishes and furnishings is then calculated. For this purpose the area of each kind of material lining the various surfaces is multiplied by its absorption coefficient, and the whole is totalled, giving a certain value in units. So far as the audience is concerned, it might be assumed that a seat in the hall had an absorption of 2 units and an individual of perhaps 4 units; it can also be assumed that the size of the audience is two-thirds of the full seating capacity. The absorbing power of the audience is then calculated as follows. Suppose the seating capacity of the hall is 600. On the assumption made, 400 seats will be occupied and 200 vacant, each of the latter absorbing 2 units, making a total of 400 units. The occupied seats will not have a value equal to the individual plus the chair because a good part of the seat will not be exposed. It is usual to suppose that the individual and seat combine to have a total absorption equal to that of the individual alone, and on this assumption, the 400 seats will absorb 1,600 units. Thus the grand total of the absorbing powers of everything in the hall can be found, and if this differs greatly from the calculated total using the formula, steps must be taken to bring the two totals nearer together. Generally, the audience accounts for the largest absorption in the hall; therefore, to minimise variation of the period of reverberation with the size of audience present, the seats should be well upholstered so that they provide a high degree of absorption when empty.



By courtesy of Messrs. Kelvin & Hughes, Ltd.

Cross-section of river, 2,600 ft. wide from bank to bank, made by an echo sounder installation. Horizontal scale gives distances and vertical depths. The river has a steeply shelving bank on the right, shown by the peak on record due to the survey launch turning at this point to retrace its course across the river.



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Echoes from a submarine at 150 ft. depth. Range is measured by the distance of the records of the echoes, *A*, from the datum echoes, *B*.

PLATE 7.

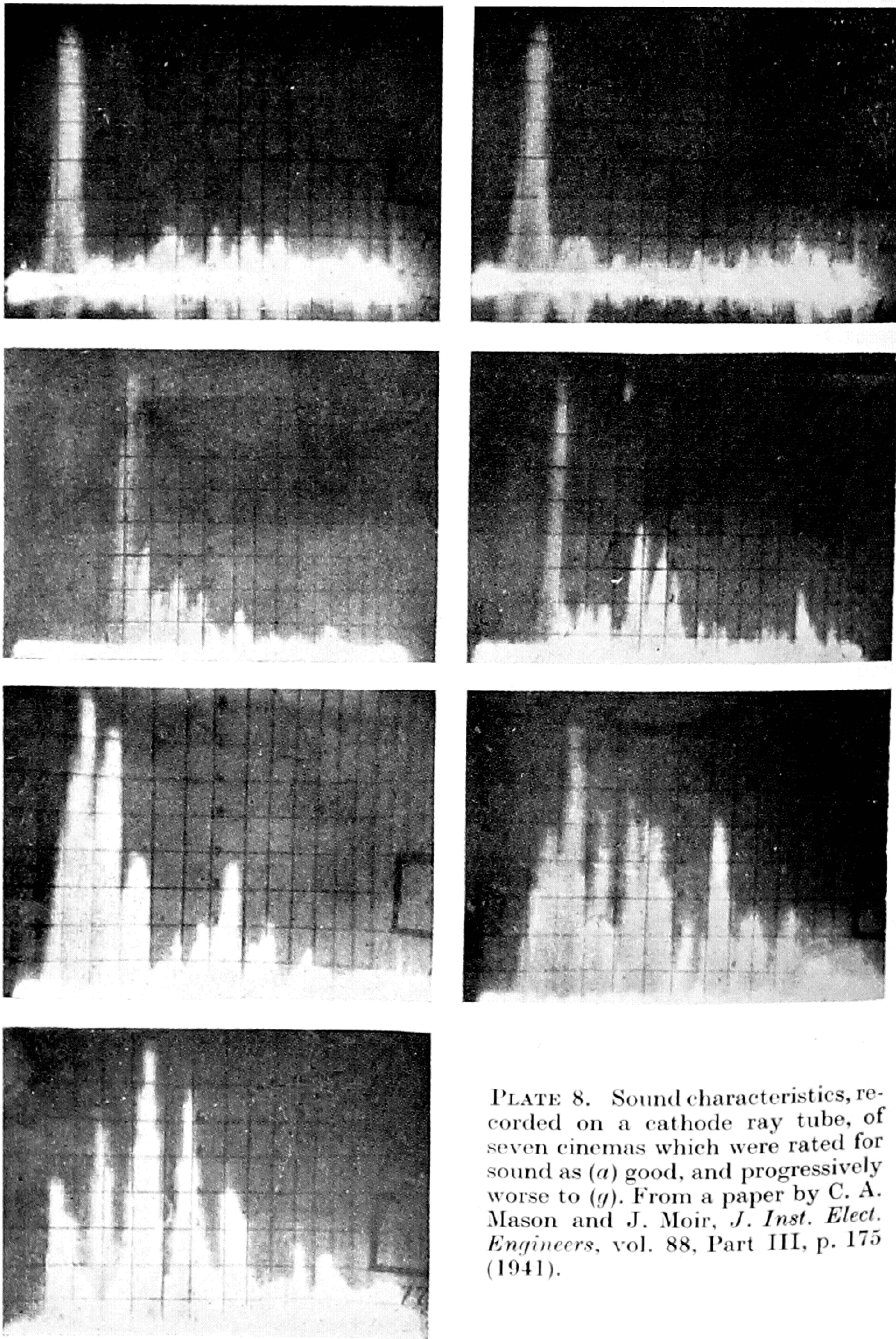


PLATE 8. Sound characteristics, recorded on a cathode ray tube, of seven cinemas which were rated for sound as (*a*) good, and progressively worse to (*g*). From a paper by C. A. Mason and J. Moir, *J. Inst. Elect. Engineers*, vol. 88, Part III, p. 175 (1941).

DISTRIBUTION OF SOUND

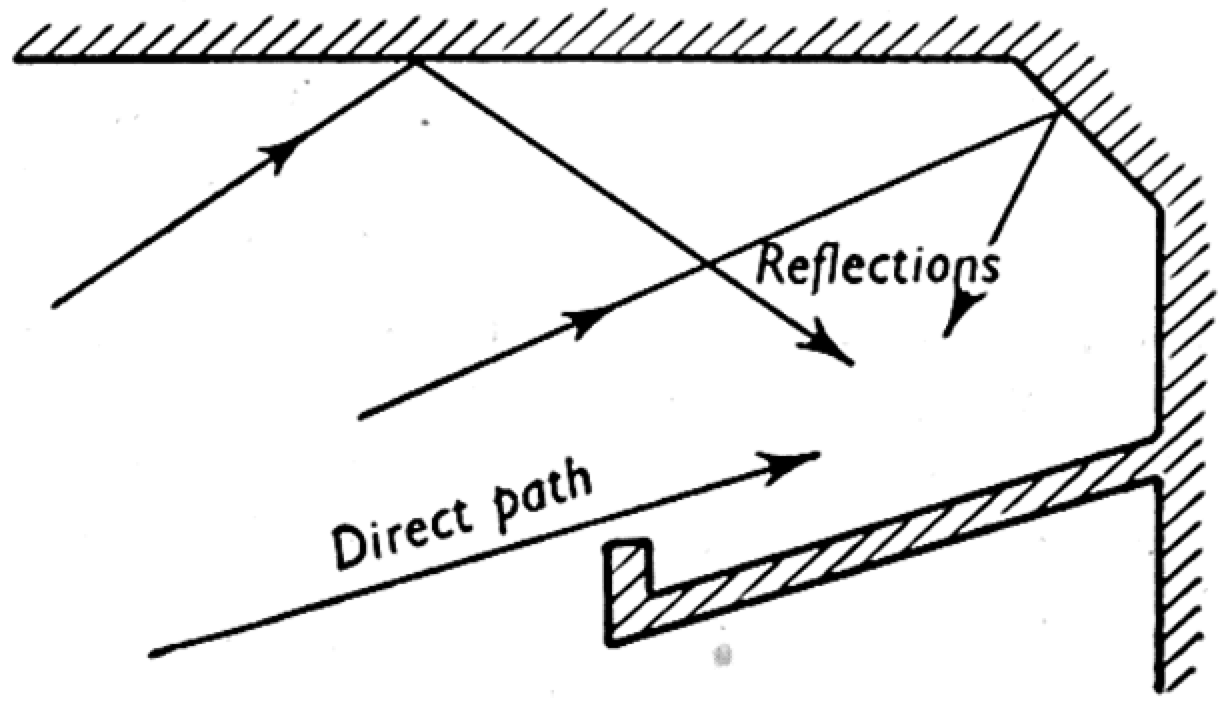
It does not always follow that because an auditorium has the correct reverberation period it is entirely satisfactory acoustically, for it is necessary to ensure that there is an adequate degree of loudness at every point, and an absence of echoes or dangerous concentrations of sound. These defects may arise through both direct and reflected sound.

Direct path. The loudness of the sound coming direct from the source seems to depend mainly upon the angle at which it reaches the audience ; if the angle is favourable, hearing will be good, but if the angle is very low the sound may be weak. This may possibly be due to two causes. First, as the audience is a good absorbent, the sound-waves travelling across it at grazing incidence may have their energy rapidly drained from their lower edges. Then again, if the air just above the audience is warmer than in the body of the hall, as it very well might be, the sound will be bent slightly upwards and pass over the audience.

Concerning the first point, if the floor of the auditorium is not tilted, the angle of incidence of the waves travelling from the stage to the audience is bound to be low, and the result is generally poor hearing for a large part of the audience. The remedy for this seems to be to provide ramped seating, temporary if necessary, in the auditorium, and also a ramped stage, though Dr. A. H. Davis has recently suggested that an alternative might be to have a splayed reflector over the performers to throw the sound down on to the audience. To help further with the satisfactory distribution of the sound, supplementary screens made of resonant panels with highly finished surfaces can be put behind the performers.

Reflected sound. If the ceiling of an auditorium is flat the reflected rays tend to spread out, and if the total path from source to listener is sufficiently long an echo will be produced, for some listeners at any rate. Hence either the ceiling must not be too high or the surface must be finished with some absorbent material to weaken the reflection ; flat walls should be similarly treated. If the ceiling is curved, however, the sound is concentrated in a particular region of the hall, and it is possible for the concentration to exceed the strength of the original sound. In certain circumstances a distinct echo may be formed, and even if this is not the case it must be remembered that undue intensity of reflection at some parts is accompanied by reduced intensity at others. It is evident then that the use of domes and curved surfaces in

auditoria requires the greatest caution. In halls where such features already exist and it is not found desirable to use absorbents, the curved surfaces acting as a sound-mirror can be " broken up " so that the sound is scattered rather than reflected. In this connection it must be borne in mind that on account of the long wave-lengths of sound, the irregularities must be fairly large ; reflection from a dome may be reduced by hanging a large object such as a chandelier at the centre of curvature.



There are certain places in a concert hall where a curved surface is advantageous. For example, if a paraboloid reflecting surface is put behind a speaker or soloist standing at the focus, a plane beam of sound is sent down the hall and good distribution is secured ; this is the function of the sound-board over a pulpit, but of course it must be large to be effective. This treatment is scarcely possible with a large orchestra, but a shell-shaped cavity or even a plane wall close behind the players will help considerably. In this case, the surface of the wall must be hard and nonabsorbent, so that good reflection is obtained. Another place where a hard surface is helpful is the roof over the gallery at the theatre, for the reflected sound, falling closely behind the direct sound, will improve hearing (see above diagram).

ABSORBENT MATERIALS

Sound incident on the surface of a material may be absorbed in several ways. In the case of porous materials, the energy is dissipated through friction between the material and the moving air within the pores. With soft, compressible materials, the energy may be absorbed by internal friction due to the compression, while with board-like materials capable of vibration it may be dissipated by internal friction arising from bending of the material. Most of the absorbent materials which have been used

in auditoria are soft and more or less porous, and it is not easy to give such materials a good appearance by surfacing them with a presentable covering. The coverings must be more or less pervious to sound, and they should not be painted or varnished as this will probably close up the pores and destroy the absorbent properties; coverings which can be used with care are brown paper, canvas and rep.

In the design of a hall, the architect must bear in mind that absorption coefficients of materials vary with frequency. For a hall designed for speaking only, probably an average value of coefficient would be sufficient; but if the hall is to be used for concert and orchestral work, the values should be ascertained over a fairly extensive range of frequencies, for the efficiency of one absorbent may be poor at low frequencies and of another poor at high frequencies. In general, porous materials absorb best at high frequencies, resilient ones at intermediate frequencies and materials like panelling at low frequencies, so for a concert hall a judicious combination of all three would probably be necessary.

In his experimental work Professor Sabine generally used *hair-felt* as the absorbent and this was covered with canvas which was lightly painted but not sufficient to close up the web. Since then many different types of absorbents have been evolved. After Sabine's death, his nephew, P. E. Sabine, carried on the work and succeeded in making an *acoustic plaster* from the slag of iron furnaces by dropping it while still hot into cold water. Lumps of this slag were then made into a plaster by using a paste made of magnesium oxide and a solution of magnesium chloride; when this mixture dries, the gas bubbles form pores. This plaster will adhere to a brick wall and can be coated with a thin distemper without interfering with its absorbent properties. Other well-known absorbents are "*Slagbestos*", *cabot quilt* and soft boards made of compressed pulp and known as *pulp boards*.

The absorption by certain porous materials, for example, canvas and wood panelling, can often be increased and the acoustics of a hall improved by mounting them a short distance away from the brick walls; sometimes two thin layers of panelling are used separated by an air space which can be filled with a material like cotton wool to damp the aerial vibrations in the space. In fact, the modern tendency seems to be to develop *cavity absorbents* in which the damping of air-borne sound is facilitated by resonance in an air vessel, while another development is to put absorbent coatings on to walls by high-pressure spraying plants.

BROADCASTING STUDIOS

For broadcasting purposes, the various studios must have definite acoustic characteristics in accordance with the purposes to which they are to be put; but in general the reverberation time should not depend very largely on frequency. Hence the acoustic materials used should have approximately the same absorbing power for all important frequencies within the audible range; of these, ordinary "building board" or "insulation board" cemented to a hard plaster beneath is very useful for wall treatment of studios.

A studio designed primarily for speech should be practically "dead", that is, it should be as free from reverberation as possible. Therefore the absorption should be a maximum, and to provide this the whole of the walls and ceiling can be covered with a material known as "mineral wool". A studio intended for music purposes generally has a reverberation time which decreases somewhat as the frequency increases, but not to a marked degree. The concert hall at Broadcasting House, London, in the condition in which it is normally used for broadcasting, has a maximum time of reverberation of about 1·84 sec. at low frequencies up to 300 c.p.s.; the time gradually decreases with increasing frequency, so that at a frequency of about 4,000 c.p.s. the time is just over 1 sec. The variability of reverberation time according to the size of audience can be overcome, as has already been stated, by the use of upholstery and carpets which will provide sufficient absorption in themselves.

When there are a number of studios in the same building, and used simultaneously, it is imperative that no sound from one studio should reach any other. To provent this happening, the best way is to arrange the separate studios as far apart as possible. In Broadcasting House, for example, no studios are located on adjacent floors; there is one floor of studios, the floor above being devoted to rooms such as libraries, etc., and on the floor above that there are further studios. If, however, space is limited and the studios have to be put close together, it is clear that a high degree of sound insulation will be necessary; methods of dealing with this problem will be considered in Chapter XIV.

Then again, in the construction of a good studio, it is important, in addition to the provision of an adequate amount of sound-absorbing material, that the building construction shall be such that, as far as possible, any resonance in the walls, floor or ceiling,

shall be avoided. Such resonance usually takes the form of the vibration of the partition concerned as a large semi-flexible diaphragm, and in practice can be readily observed as a tendency to " boom " on the part of the structure. One effect of partition resonance is often the apparent prolongation of the reverberation time at or near the natural frequency of vibration of the partition, while in bad cases a definite " coloration ", by the boom tone, of speech or music performed in the studio can easily be heard. It is difficult to make every part of the structure of a studio ideally rigid, and means have to be adopted to render resonance as innocuous as possible. One modern way of dealing with the problem which has proved very effective is to make the studio a more or less independent inner shell of the main room, with a " floating " floor and a " suspended " ceiling. This is part of the technique of what is called *discontinuity* in construction, and it will be considered more fully in the next chapter.

PUBLIC ADDRESS SYSTEMS

In a public address system, the speaker talks quietly into a microphone, and the sound, after being amplified, is fed into one or more loudspeakers conveniently placed in the auditorium. If there is more than one loudspeaker, care must be taken that the sound from one does not feed into another and so cause interference. Also a loudspeaker must not be put too close to the microphone, otherwise the system of microphone, amplifier and loudspeaker may start up oscillations at its natural frequency and give rise to a phenomenon known as the " howling telephone " ; even if the system does not howl, there may still be interference, the extent of which will depend both on amplification and frequency. Many loudspeakers are directional and send out the energy straight ahead ; but at low frequencies there is a certain amount of spreading, so that to prevent interference the distance between microphone and loudspeaker must be sufficiently large, or the low frequencies must be cut out. In other words, an injudicious disposition of loudspeakers in an auditorium is worse than useless.

The proper use of a loudspeaker is so that every person in the audience is ensured of the correct degree of loudness for satisfactory hearing without distortion. A public address system is not a remedy for excessive reverberation in a hall ; indeed, the use of loudspeakers might easily aggravate the condition, especially if the sound is over-amplified. The correct procedure is to

get the reverberation times right first and then introduce the loudspeakers.

Of course, there are some large and lofty buildings like churches and cathedrals where it is difficult and sometimes impossible to obtain the correct reverberation time. In such cases each section of the congregation can be treated as a separate reverberating chamber with its own loudspeaker put in a suitable place, and any excess sound energy would probably be absorbed by the congregation in that section. The loudspeaker should be arranged a few feet over the heads of the listeners and pointed slightly downwards; a good place is on a pillar, which will cast an acoustic shadow so that not much sound travels behind.

A good example of a very reverberant building is St. Paul's Cathedral, London, which has a reverberation time (at 500 c.p.s.) with a full congregation present of 6·5 sec. compared with 2·8 sec. in the Royal Albert Hall (full) and 1·7 sec. in the Royal Festival Hall (full). It is difficult for even an experienced speaker to make himself understood in the Cathedral, except perhaps in the dome area, and some form of speech reinforcement is necessary.

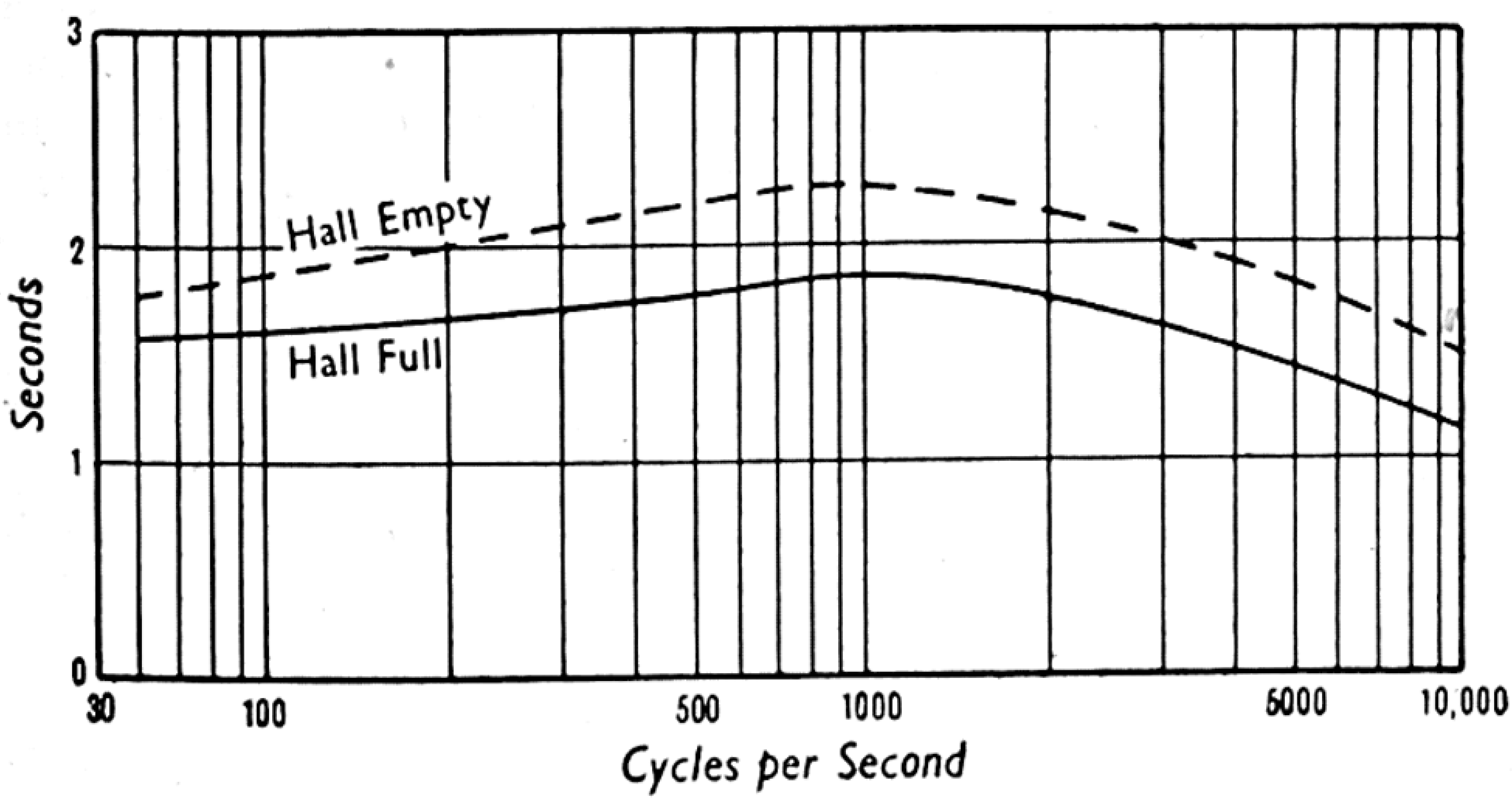
The following treatment has been recommended by the Building Research Board of the Department of Scientific and Industrial Research. The most suitable type of directional loudspeaker for use in St. Paul's—and in most reverberant auditoria—is a line source of several cone loudspeakers mounted one above the other in a common baffle and all operating in phase. Such a system mounted close to the pulpit would certainly cover the dome area, and would make speech intelligible even in the empty Cathedral. Adequately to cover the nave area, the suggestion is to use six similar but shorter line sources mounted on the pillars, and to cause these subsidiary loudspeakers to be delayed in time by electrical means so that, whatever his position in the Cathedral, the listener will hear any sound coming from the main source, the pulpit, at the same moment as the sound from the nearby loudspeakers.

In cinemas the usual practice is to mount one loudspeaker on each side of the screen so that the sound is synchronised with the movements on the screen, a condition known to the acoustic engineer as "intimacy". For persons a long way from the screen this synchronisation probably fails, for the sound reaches the persons later than the visible action, and to get over this difficulty it might be advisable to provide the more distant sections of the audience with a loudspeaker of their own. The absorption, however, in this section must be adequate so that no

sound gets back to the other loudspeakers and other parts of the audience.

In using a public address system, it is important that the speaker should not raise his voice above the ordinary conversational level. If he does so, apart from the " blasting " effect caused in the loudspeakers, the listeners in certain parts of the auditorium will hear the sound of every syllable twice, first from a nearby loudspeaker and afterwards direct from the speaker himself.

The most recent example in Great Britain of a hall in which the acoustic properties were considered before the building was erected is the Royal Festival Hall in London. The problem here was to exclude as much noise as possible from nearby road traffic and trains, and to assess and define the musical requirements of the proposed hall, such as definition, fullness of tone, balance, etc. As a result of many tests made on the site, a treatment of insulation was decided upon which renders the hall very satisfactory acoustically both for vocal and orchestral work.



Variation of reverberation time with frequency, Royal Festival Hall, London. From *Nature*, vol. 168, p. 264 (1951).

Absorbents of various kinds have been put on the rear walls, along the junction of the side walls with the ceiling, and on the side walls close to the orchestra. Such absorbing surfaces have been kept to the minimum necessary to prevent echoes. In addition, the ceiling is constructed with about twelve hundred holes, $2\frac{1}{2}$ in. in diameter, for use with resonators if the need should arise, and a canopy placed over the orchestra reflects the sound to the rear of the hall. To achieve fullness of tone the hall is

designed to have as long a reverberation time as possible, and the accompanying diagram shows the variation of reverberation time with frequency for the hall, empty and full.

RECENT INVESTIGATIONS

The work so ably begun by Professor Sabine has been continued and is continuing in many countries, especially in America, and also at the National Physical Laboratory and the Building Research Station at Garston, near Watford, as well as other acoustical laboratories in Great Britain. While accepting that the time of reverberation of a hall is an important factor, recent investigators are of the opinion that it is not the only factor.

Before the Second World War, the Acoustics Department of the British Broadcasting Corporation carried out some experiments in two studios which had more or less the same shape, but had different wall surfaces, one being irregular and the other regular. It had been found that, although the two studios, of identical acoustic treatment, had exactly the same reverberation time, there was a great difference between the sound that came to the ear from the two studios. Similar work was done in America and it was found that for studio purposes irregular walls are very important. Two types of irregular walls were used ; in one type the irregularities were a series of flat surfaces at different angles, whilst the other type had polycylindrical surfaces. In connection with the above work, H. L. Kirke believes that, apart from reverberation time, another important criterion in studio and auditorium design is what happens to the sound during the first few reflections or the first few milliseconds after the sound is started and stopped.

A similar conclusion was reached by C. A. Mason and J. Moir as a result of investigations carried out in the period 1936–39. Their method of investigating the acoustics of buildings consisted of emitting short pulses of sound from a loudspeaker placed at the point which would be used by an artist, and recording the successive reflections of the sound pulse as they struck a microphone placed at the point in the auditorium under investigation. Photographs of a cathode-ray tube, arranged to record microphone output as vertical deflections, the horizontal scale representing a total time-interval of about 300 milliseconds, were taken, and are shown in Plate 8 (facing p. 279). The photographs represent conditions in cinemas which were very good (illustration (*a*)) and getting progressively worse up to illustra-

tion (g). In the good hall the initial sound is a single strong pulse with all remaining reflections of much lower amplitude. Photograph (d) represents the conditions in an average hall and shows two secondary reflections delayed by approximately 80 and 150 milliseconds behind the main pulse. (g) represents conditions in a bad hall, and it seems that the initial pulse is completely swamped by a mass of sound that has been reflected from the side and rear walls, delayed by as much as 200–300 milliseconds behind the main pulse. In this case, speech intelligibility was very low, because each of the speaker's syllables was repeated half a dozen times by the wall reflection. The reverberation time was satisfactory in all the above auditoria, and other factors actually favoured the hall represented by (g).

The conclusion arrived at by Mason and Moir was that the acoustic excellence of a hall depended mainly upon the reflection pattern that existed during the first 100–250 milliseconds, and only to a second order upon the Sabine conception of reverberation time.

CHAPTER XIV

NOISE AND SOUND INSULATION

Introduction. It was stated in Chapter I that the distinguishable feature between a noise and a musical sound is that a noise is a sound of irregular wave-form and of a more or less unpitched character, whereas a musical sound has a definite pitch and a regular wave-form. But for the purposes of this chapter a noise will be defined very broadly as a sound which is undesired by the recipient, and in the modern world there are many sounds of this kind in factories, offices, out-of-doors and even in the home. The increase in mechanised processes has inevitably brought about an increase in noise, and even though the advantages to be gained by the mechanisation probably outweigh the disadvantages, yet for those people who do not immediately reap the advantages the noise constitutes at least a distinct annoyance. Although perhaps very little mechanical energy is wasted by noise, the energy dissipated may be more than a mere nuisance and may result in impairment both of working efficiency and of health. Hence from this point of view it is a pressing social problem to try to suppress unwanted noises whether they occur in factories, offices, houses or flats. It does not matter either whether the sounds are noises or musical. Most people would agree that a neighbour's radio can at times be very annoying even though quite tuneful music is being played; and again, the war-time innovation of "music while you work" while no doubt serving a very useful purpose, was certainly a source of irritation to unwilling listeners.

It is not likely, nor indeed would it be desirable, that all noise should be eliminated, for it is highly necessary for some people; for example, blind persons would be at a grave disadvantage if they could not hear the rumble of approaching traffic, and no doubt other examples can readily be found. But the general aim must be to eliminate or suppress as far as possible those sounds which can make no useful contribution to general happiness and welfare.

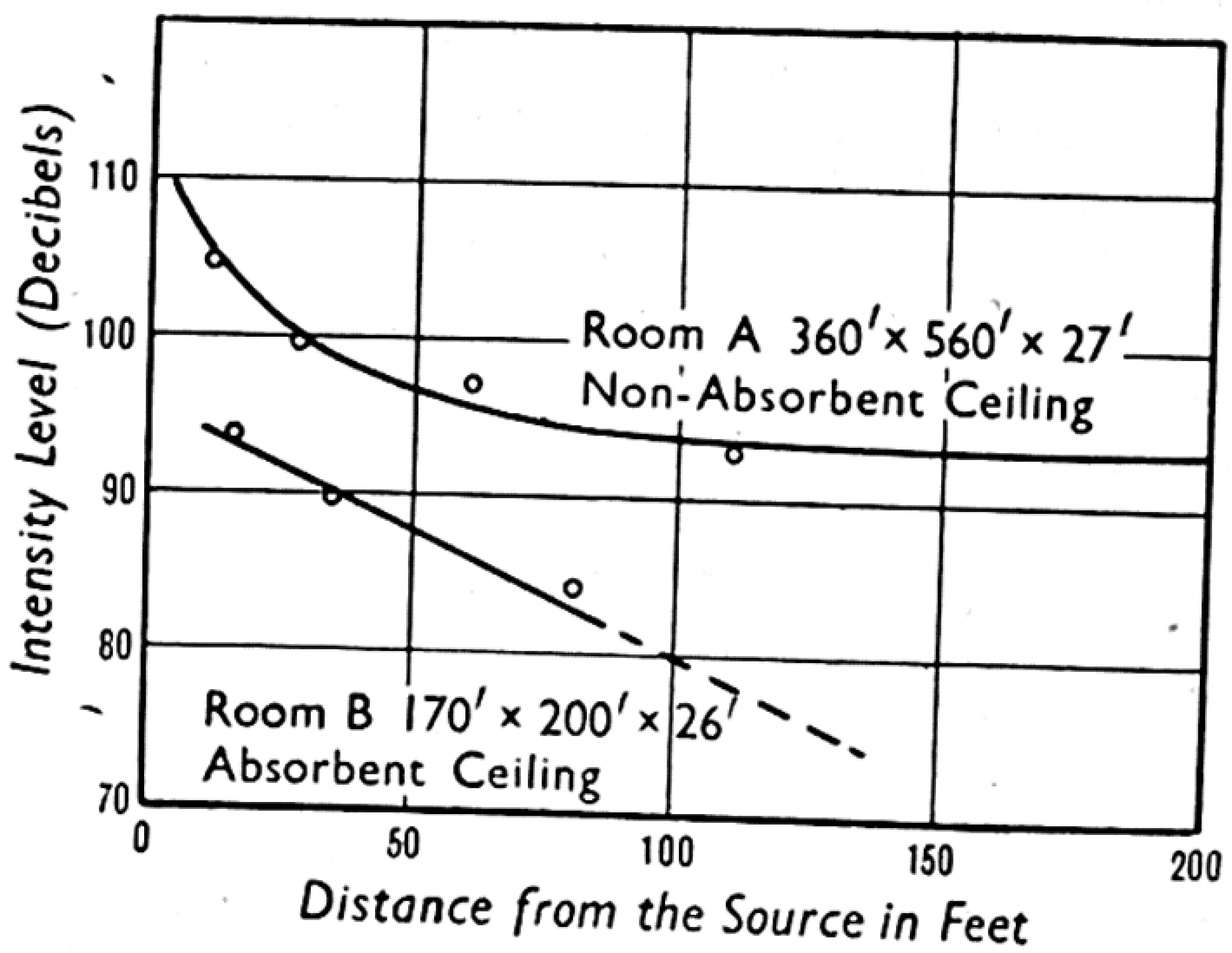
PHYSIOLOGICAL AND PSYCHOLOGICAL EFFECTS OF NOISE

Although the younger members of the community seem to delight in noise, on the whole, older people find it very unpleasant. Undoubtedly a sudden violent noise such as an explosion causes a change in the rate and regularity of heart-beats and is a great shock to the whole nervous system. It is fair to assume then that a persistent series of noises, even though less intense than an explosion, must have an accumulative effect; probably the first symptom of excessive and persistent noise is a violent headache. The most direct physiological effect of noise is upon the ear, and temporary or even permanent deafness may result; prolonged exposure to noise day after day, say, in a factory, certainly produces a steady deterioration of hearing. When temporary deafness does occur, the hearing loss is usually greater in the higher frequencies (2,000–4,000 c.p.s.) even though the predominant energy in the noise is in the lower frequency region.

A great deal of attention has been paid in recent years to the psychological aspect of the effect of noise, and although the factors involved in the human reaction to noise are difficult to assess, it has been shown in some cases that excessive noise adversely affects output of workers and increases the liability to error. This is probably the result of the distraction caused by the noise, which brings about lack of concentration, though a contributory cause of the decreased output and increase in error may be the effect of the noise on the health of the workers. In some cases relief has been obtained by giving an absorbent treatment to the ceiling of the noisy room, but in this connection mention must be made of some conclusions arrived at by H. J. Sabine and Wilson. After a series of investigations they reported in 1943 that relief has been obtained by absorbents in some workshops even where loudness reductions were negligible, and they made suggestions relating to the phenomenon of reverberation and to what they called the "spreading effect", which might be very helpful. They concluded that reverberation is regarded as an irritant because it prolongs an already disagreeable sound and also because it interferes with conversation. They suggest that reverberation becomes an important factor when the noises consist of distinctive impacts, and that the addition of reverberation to the steady noise of rotary machines is not so annoying.

The importance of the "spreading effect", which is described

as the tendency of sound in reverberant rooms to remain at the same level, or to decrease very slowly with increase in distance from the source, lies in the fact that it causes the noise of other machines to interfere with the operator hearing his own machine. Sabine and Wilson say that " as far as symptoms of strain and fatigue due to noise are concerned, a machine operator is affected much less by his own machine than by others . . . in spite of the fact that his own machine, being closest to him, sets up a higher



Variation of intensity of sound with distance from source in large rooms with absorbent and non-absorbent ceilings (Sabine and Wilson).

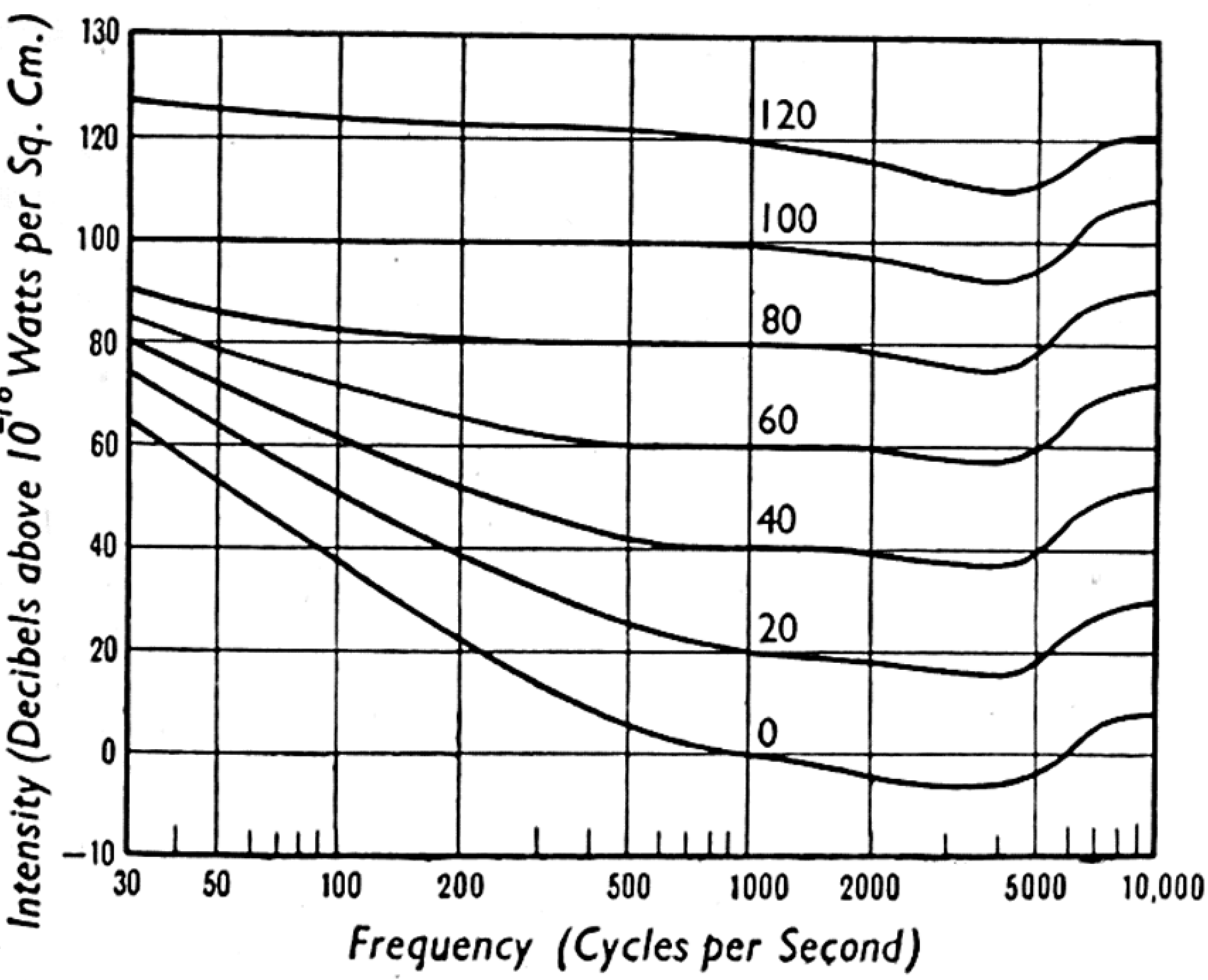
noise-level at his ear ". The result of putting absorbents in the room is to reduce the spreading effect, and thereby to limit the extent to which other noises can interfere with the sound the operator wants to hear. Sabine and Wilson measured the relation between absorbents and the spreading effect, and some of their results are shown in the graphs, indicating that the spreading effect is influenced considerably by the absorbent. The upper curve shows the decrease with distance from the source in a reverberant room, in which the levelling off is evident, and the lower curve shows the steady decrease in a room with absorbent ceiling.

MEASUREMENT OF NOISE

Before any methods can be prescribed for noise suppression, it is necessary to be able to measure the essential characteristics of a noise, for such measurements provide the basis for the ultimate

control of noise by specifying limits which must not be exceeded in particular cases. The chief characteristics concerned are the *intensity*, or physical power of the sound (dealt with in Chapter I), the *constitution*, or the distribution of energy with frequency, and the *loudness*.

Both intensity and constitution are physical attributes of the noise which are amenable to objective measurement; but loudness, which is the magnitude of the sensation a sound produces, is a subjective characteristic and must be assessed principally by subjective methods.



Equal loudness contours (Fletcher and Munson).

Loudness : the phon. There is no doubt that the loudness of a sound is related to the physical intensity, since increasing the intensity without changing any other characteristic increases the sensation of loudness. But it has been shown experimentally that loudness is a function of the character of a sound as well as its intensity, and that two sounds of equal intensity but of different character do not in general appear equally loud. Experiments have been carried out by several investigators on the equality of loudness of *pure tones*, and the results obtained by Fletcher and Munson are shown in the diagram, where each curve is a contour of equal loudness; that is, sounds of the intensities and frequencies represented by points along any one curve are

judged aurally to be equally loud. The lowest curve is the **threshold of audibility**, giving the minimum values of sound intensity in a plane progressive wave required to produce the sensation of sound, and the other curves are drawn for successive 20-decibel increments of intensity at 1,000 c.p.s. It will be noticed that the curves tend to become closer together at the lower frequencies. This means that a given change of intensity produces a greater change of loudness at low frequencies than at medium and high frequencies. Hence it is evident that even for pure tones, neither the absolute physical intensity nor its relation to the corresponding threshold value is sufficient to express loudness; when noises, which are complex sounds, are considered, the problem becomes still more complicated.

For the measurement of loudness a subjective method must primarily be employed. The principle adopted is to use a standard comparison sound of variable intensity to which a scale of loudness values is assigned, starting at zero at the threshold of hearing and increasing progressively with the intensity. The noise to be measured and the standard sound are then compared aurally under specified conditions of listening and the standard sound is adjusted until it is judged to be equally loud to the noise. On the international scale of loudness which was introduced in 1937, and in which the unit of loudness is called the **phon**, the standard sound is a pure tone of frequency 1,000 c.p.s. in the form of a plane progressive wave coming from directly in front of the observer. A scale of loudness values in phons is assigned, such that when the physical intensity of the standard tone is n decibels above a zero of 10^{-16} watts per sq. cm. (corresponding approximately to the normal threshold value) the loudness is said to be n phons. The loudness of any sound, in phons, is then defined as equal numerically to the intensity-level, in decibels, above the specified zero, of the standard sound which, under certain prescribed listening conditions, is judged on the average to be equally loud. The table opposite gives the loudness-levels in phons of some common noises.

The steps on the phon scale have been chosen quite arbitrarily as equal to decibel steps of intensity of the standard tone, and other scales have been proposed to avoid some of the disadvantages of the phon scale.

Subjective methods of measurement are not very reliable since they depend on the judgment and opinion of the listener, and probably no two listeners would agree as to what constitutes equality of loudness, though of course more reliable results can be

Loudness (phons)	Noise
130	In aero-engine test-house
110	Loud motor-horn at 20 feet
100	Pneumatic road drill at 20 feet
90	In tube train
70	In main line express. Conversational speech
50	Quiet office
30	Quiet whisper
0	Threshold of audibility

obtained by employing a team of listeners. Much work has been done in an effort to assess the loudness of any sound by objective measurement and to design instruments to measure loudness directly, thereby eliminating the human element. Such instruments are called **noise-meters**, and although several types have been produced, no form of such instruments has so far been considered to be sufficiently reliable over a wide range of noises to warrant its adoption as a standard.

Loudness is not the only subjective characteristic of a noise, for the annoyance a noise causes is another, and this is not entirely dependent on loudness. For example, noises of predominantly high pitch tend to be more annoying than equally loud noises in which the lower frequencies are more prominent; with noises containing a wide range of frequencies considerable relief may be obtained by suppressing the high frequencies only, although the resultant decrease in the sensation of loudness may be slight. Reduction of the high frequencies also contributes markedly to improvement in the intelligibility of speech in the presence of a high level of background noise. Again, noises of an irregular and intermittent nature tend to be more annoying than steady sustained noises, though the latter can be specially irritating in some circumstances.

SOUND INSULATION

General considerations. The two chief problems to be dealt with in connection with sound insulation are first, to prevent any noise from *outside* reaching an enclosure where it is not wanted, and secondly, to reduce any noise made by machinery, typewriters, etc., *inside* an enclosure to such a level that it will not

unduly annoy people working in the enclosure. The first problem is of course very important from a domestic point of view in connection with houses and flats, when the noise may be of the *sustained* type, or *impact* noises. In its entry into an enclosure the noise may come via the atmosphere through open windows, doors and the ventilating system (in which case it is termed *airborne noise*, the effect of which depends upon the pressure of the sound-wave), or via the walls, ceiling and floor which are set into vibration and act as new sources of sound. In the latter case, of course, the sound ultimately reaching the hearer is airborne, though some of it may be conveyed to his inner consciousness from the floor through his bones.

If a source of sound inside an enclosure emits energy at a constant rate, the average sound-energy density in the enclosure builds up until a level is reached at which the rate of absorption (including transmission through the boundaries of the enclosure) is equal to the rate of emission of energy. For a random distribution of sounds, the final level is given by $I = 4E/VA$, where I represents the energy density, E the rate of emission, V the velocity of sound and A the total absorption within the enclosure.

If there were no true absorption within the enclosure, the intensity would build up until the rate of transmission through the boundaries was equal to the rate of production by the source. Thus the noise outside the enclosure would be independent of any transmission-loss through the walls, and nothing would have been gained by enclosing the source. Also, if a space is enclosed to exclude outside noise and sound enters through the boundaries at a constant rate, the average sound-energy density inside the enclosure will build up until in the final state the above equation will again apply. With no true absorption within the enclosure the sound would build up until the rate at which it was being transmitted outwards through the walls was equal to the rate at which it was entering. In such a case the intensity level inside would be the same as that outside, and of course no relief from the noise would have been obtained.

Hence, it is clear that in insulating against noise, consideration must be given to the question of absorption as well as transmission. Both the **absorption-coefficient** (see p. 274) and the **transmission-coefficient** (which is defined as the ratio of the sound energy transmitted to the incident sound energy) vary with frequency and with the direction of the incident sound. The range of absorption-coefficients is not large, but transmission-coefficients may vary from nearly unity for a light curtain at low

frequencies to the order of 10^{-8} for a heavy wall at high frequencies. Consequently, in dealing with transmission problems it is convenient to employ a logarithmic equivalent instead of the direct energy-ratio. For this purpose the sound-reduction factor, or transmission-loss, in decibels, is used, and this is defined as

$$10 \log_{10} (1/t) \quad \text{(see p. 14)}$$

where t is the transmission-coefficient.

If sound enters a room wholly through a partition of area S separating it from an adjacent noisy room, the actual sound-reduction (R') between the two rooms is given very approximately by

$$R' = \left(R - 10 \log_{10} \frac{S}{A}\right) \text{ decibels,}$$

where R is the sound-reduction of the partition and A the total absorption in the receiving room. Thus, if the total absorption (which has the dimensions of an area) is equal to the area of the partition, the actual reduction is equal to the reduction factor of the partition. If A is greater than S, the actual sound-reduction is greater than the sound-reduction factor of the partition. In a normally furnished living room, A is probably less than S at low frequencies, and may approach equality at high frequencies, with the result that the actual sound-reduction from an adjacent room is a few decibels less than the reduction-factor of the partition at low frequencies, and about equal to it at high frequencies.

INSULATION AGAINST OUTSIDE NOISES

If it is desired to make a room in a building *sound-proof* against noises which occur outside the room, various elements in the structure of the building have to be considered, and these will now be briefly dealt with. It must be borne in mind that such noises may be produced outside the building altogether, in the street, etc., but they may also occur inside the building in rooms adjacent to the room where silence is required.

Walls, floors and ceilings. Except for thick walls at high frequencies, there is practically no wave-motion within the thickness of the material, and sound is transmitted mainly by diaphragm-like vibrations. In practice, a wall will have a large number of resonant frequencies corresponding to different modes of vibration, but if the effect of resonances can be neglected, the transmission-coefficient of an approximately homogeneous partition

should be proportional to the square of the frequency and to the mass per unit area of the partition, and independent of the nature of the material. This means that the transmission-loss should increase by about 6 decibels per octave increase of frequency and by 6 decibels each time the weight is doubled. A $4\frac{1}{2}$-inch brick wall has an average transmission-loss of about 50 decibels, and the corresponding reduction of loudness will depend upon the nature of the noise concerned, since the insulating value is better at high frequencies than at low, but it should be about 50 phons. A 9-inch brick wall, plastered, of about twice the weight of a $4\frac{1}{2}$-inch wall gives an increase in insulation of about 5 decibels only. This is insufficient for a high degree of insulation, for although ordinary conversation behind such a wall is inaudible, loud sounds from a radio set are certainly heard.

If the wall is composed of porous materials such as felt, or clinker concrete blocks, sound may be transmitted by the motion of the air within the pores as well as by the movement of the materials as a whole, and this will result in a much greater transmission of sound than with a non-porous material of the same superficial weight.

Increasing the thickness of a wall, then, will certainly increase the noise reduction, but of course there are practical limits to this increase in weight, and better results can be obtained by the use of a composite partition in the nature of a cavity wall. For example, two leaves of plastered clinker concrete, 2 in. thick, which separately give a transmission-loss of about 40 decibels each, may together give a reduction of 55 decibels, provided each is insulated around the edges with a layer of cork. This is equivalent to the insulation of a 9-inch brick wall of about three times the weight. The space between the two leaves can be air-filled, or some sound-absorbing material can be applied to the inner surfaces of the leaves, but not so as to bridge the gap entirely. It is important, however, that to obtain any advantage from a double partition in comparison with a single partition of the same total weight, throughout a wide range of frequencies, the spacing between the components should be made sufficiently large to push the frequency of minimum transmission down to a low value. This is equally important in the case of double windows (see p. 299).

So far, only the sound transmitted *directly* through a partition has been considered, but it must be remembered that a noise outside a room will cause vibrations, not only in the partition wall, but also in other walls, the floor and the ceiling. These vibrations

in the flanking walls, floor and ceiling may ultimately become the predominant factor in the transmission of the noise, and will undoubtedly reduce the insulation of the partition. Vibrational energy is much more effectively conveyed to a wall or floor by direct mechanical *impact* rather than by the small pressure variations of airborne sound, and the problem is of most importance in connection with floors. For example, a solid concrete floor is a reasonably good insulator against airborne sound, but a comparatively poor one where impact sounds are concerned. Of course, the surfaces of floors can be treated with resilient materials such as cork, rubber and linoleum to reduce impact noise. Very often these materials can prevent the generation of much of the higher-frequency sounds, but in cushioning the blow they secure only a limited effect upon the lower frequencies, and must be regarded merely as palliatives and not cures. Tests have been made on a number of these materials and only one of them has been found to give a reduction in loudness of more than 10 phons ; this was an elaborate finish in the form of a thick and springy sponge rubber with a smooth wearing face.

Discontinuous construction. In order to obtain the required degree of sound reduction in buildings, consideration has in recent years been given to techniques based, not upon continuity, but upon the interruption of continuity at certain points. The general principle of the method of discontinuous construction is to make the room to be isolated a kind of separate box insulated from the main structure, and this treatment will involve walls, floor and ceiling.

For isolating floors so that impacts upon them are not transmitted throughout the framework, the principle of the *floating floor* can be adopted. In essence, this type of floor may consist either of a concrete screed or a board-and-batten raft, carried on a resilient material. The simplest type of floating screed is constructed by laying a resilient quilt over the main supporting floor and pouring the screed directly on it. Precautions must be taken to prevent the concrete from leaking through the joints in the quilt and bridging the insulation, and a good way of doing this is to seal with bitumen paint, or to use waterproof paper. The best quilt for the purpose seems to be glass silk, though slag wool and eel grass are almost equally good. The junction between the walls and the floating floor must be closed to prevent the escape of airborne sound. The illustration (p. 298) shows such a floor being laid. In another type of floating floor, developed at the Building Research Station, Garston, Herts., the screed is lifted on to its

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Laying concrete on a glass silk quilt. Reproduced by permission of the Director of the Building Research Station, Garston, Watford.

resilient supports. The screed is laid upon waterproof paper to prevent adhesion to the base, and threaded sockets are incorporated in it at intervals of about two feet. When the screed has set, wood blocks are put into some of the sockets, and plugs screwed down upon them. By this means the screed can be lifted out of contact with the base, and when it is at a suitable height (about 1 inch) the wood blocks are replaced one by one with rubber cubes. The noise-reduction factor of this type of floor is of the order of 20 to 25 phons; that of the first type mentioned ranges from 15 to 20 phons for single quilts and of course increases with greater thicknesses.

The walls can be made of clinker concrete blocks with an air space between them and the main structure, and they can be built directly on to the screeded floor. It should be equally satisfactory to build them on a separate insulating layer if this is more convenient, and the floor can be placed later and poured in contact with the walls. Care has to be taken to see that no concrete leaks through the joints in the insulating quilt at this point or the insulation will be by-passed.

For ceilings, either an independent ceiling can be carried on the walls, or a *suspended* ceiling can be used. The latter type seems to be the more practicable, and experimental ceilings have been erected consisting of metal strap hangers holding a lattice of steel rods to which metal lath was fixed and plastered. Thus the whole room is like a box with a loose fitting lid. For broadcasting purposes, it is essential that structural vibrations should be reduced to a minimum, since one effect of partition resonance is often the apparent prolongation of the reverberation time at or near the natural frequency of vibration of the partition. For this reason, many broadcasting studios and listening and silence rooms are equipped with some form of suspended ceiling.

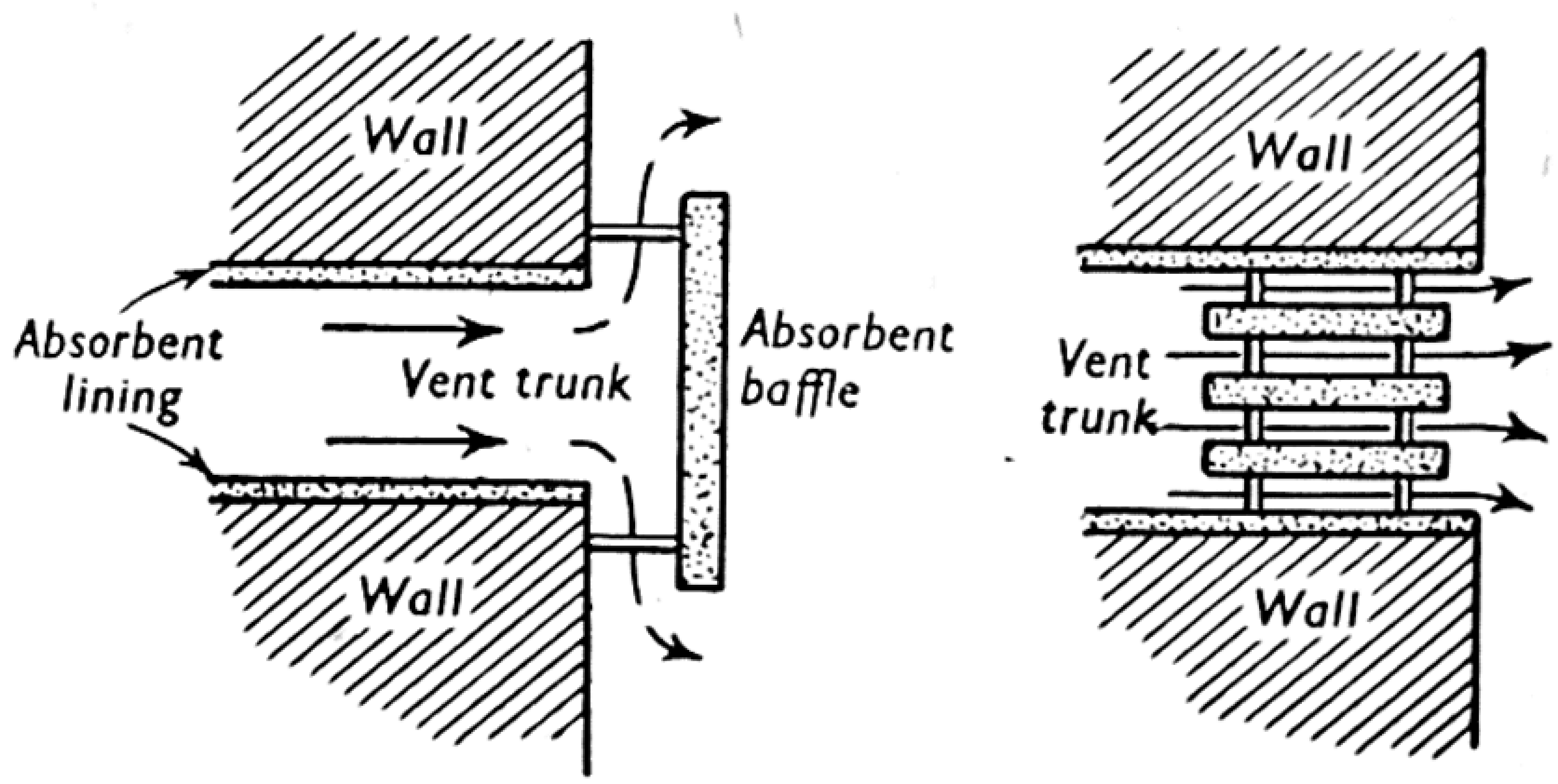
The above is only a very brief account of the discontinuous treatment of buildings. It must be remembered that the technique is still in its infancy and much research is going on in order to obtain the most efficient results. But it certainly seems that the question of introducing discontinuity in building construction is becoming more and more important.

Windows, doors, ventilating apertures. Often in a building, the windows and doors are the weakest links so far as sound insulation is concerned; everyone must have noticed that the slight opening of a window over a busy city street lets in a great deal of noise. In the design of a highly insulating enclosure, special attention must be paid to doors and windows and to any necessary ventilating apertures, or the high insulation of the walls, etc., may be completely vitiated. In the type of room described above in the section on discontinuous construction, double windows should be provided, one in the outer wall and one facing it in the inner shell. Such windows would have to be hermetically sealed and this would involve the provision of a forced ventilating system. The spacing between the two windows is important, because at a certain frequency, depending on the distance between the two components, selective transmission occurs and the reduction afforded may be less than that of a single component. Increasing the spacing lowers the frequency of selective transmission, and with adequate spacing it may be shifted outside the range of frequencies in the noise concerned. With two glazings of $\frac{1}{4}$-in. plate glass, spaced 6 to 8 in. apart, and the surround between lined with sound-absorbent material, there should be no serious loss of insulation compared with a 9-in. brick wall. In the case of most dwellings, it is (at present at any rate) inconvenient and not practicable to have double windows, and natural ventilation is relied upon. This, of course, means considerable

reduction in the sound insulation ; but some improvement can be obtained by the proper setting of window frames and by the use of absorbent baffles.

For high insulation, doors also should be double and of heavy construction, equivalent to at least 2 in. of solid wood. It is very important that they should effect a tight closure and the rebates should be lined with resilient material. The gaps at the bottom of the doors should be closed either by a sill or by an efficient form of draught excluder. One form of single door which apparently is an efficient sound-insulator consists of a heavy steel box packed, not with slag wool or other similar absorbent material, but with a large mass of dry bricks.

To prevent noise escaping through ventilating apertures, the noise should be made to pass through a duct or a system of parallel ducts lined with sound absorbents. This is particularly important where an artificial system of ventilation is in operation, and the noise, which may arise from the ventilating plant or from adjacent rooms, must be prevented from entering via the inlet and exhaust ducts. To suppress the noise from the fan, this and its motor should be mounted on anti-vibration supports and the ducts should be isolated from the fan by means of flexible sections ; in addition, the airborne noise should be absorbed by lining a length of the duct near the fan with absorbent material, such as slag wool. The attenuation of sound passing through a duct increases with the ratio of the perimeter to the cross-sectional area. Hence the duct should preferably be of narrow section, or should be divided into a number of narrow ducts by means of baffles of absorbent materials ; two suitable types of baffles are suggested in the diagram. Noise from an adjacent room may also find a path via the ventilating ducts, and to pre-



vent this, the portion of the duct between the branches to the two rooms concerned should be lined with absorbent.

Houses, flats, hospitals, etc. One of the big social problems to be solved in the future is to provide dwellings which will give to the occupiers a comfortable standard of sound insulation, and it is satisfactory to note that in addition to the extensive investigations on methods of sound-insulation, a recent step has been taken in the establishment of standards of airborne and impact insulation in respect of both houses and flats. The chief types of noises to be dealt with are those arising from neighbours' activities, including radio and television, impact sounds and noises produced outside the building altogether, and there is no doubt that occupants of flats are the biggest sufferers.

Quite apart from any specific treatment in sound-insulation that might be given to dwellings, much can be done to reduce noises in dwellings by careful siting and planning, and also by an attempt to suppress the noises, especially those coming from outside the building, at the source. Such noises as motor horns, motor cycle exhausts, pneumatic drills, etc., are a serious source of annoyance, and it is encouraging to note that tests are being carried out to deal with some of these. Traffic noises, too, could be minimised if it were possible for all vehicles to have pneumatic tyres and if the roads could be covered with some substance like rubber, but this perhaps is too much to hope for at present.

Houses, as a rule, must be built adjacent to the street or road, and unless special precautions are taken in the construction, the occupants must endure the outside noises from the street ; but certainly efforts can be made to design the houses so that noises from neighbours' activities are minimised. So far as blocks of flats are concerned, these are better set back as far as is practicable from the road, with possibly a screen of evergreen trees or tall shrubs along the verges of the road. There are other noises, of course, besides those emanating from neighbours in both houses and flats. Lifts often cause annoyance in flats and hotels, due to noise arising from the starting and stopping of motors, and from doors and gates, and the noise is accentuated because it wanders up and down the shaft and is audible on all the floors. Hence, lifts and stairs should not be placed next to living rooms and bedrooms, and noisy mechanical plant should not be set next to, or above or below, rooms where quietness is required.

Then again, chimneys might require to be treated as part of the problem in dwellings, and if possible the adjacent chimney stacks

should be separated. In blocks of flats where it is desired to insulate, say, the living rooms, the length of the chimney concerned can be isolated by interposing over the full area of one course in the brickwork at each floor-level a layer of soft asbestos quilt. This type of discontinuity is also useful in connection with hot-water systems in which pumps are used to assist the circulation of the water, and in ordinary water-supply systems. In the former, the vibration of the pumps may be conveyed along the pipes ; the resulting noise can be reduced by breaking the main supply pipe just beyond the pump and inserting a flexible connection of rubber. In the latter case, vibration may be caused by the turbulent flow of water at the taps, and although the vibration of the pipes alone does not produce much noise, much larger areas may be set in vibration since the pipes usually are rigidly connected to the building structure. Here again the remedy is to break the continuity of the pipes by inserting a flexible rubber section, and to insulate the pipes from the structure with materials such as felt or rubber.

So far as houses and flats of the future are concerned, it is reasonable to hope and expect that some form of discontinuity of structure such as is indicated in the previous pages will be used. Standards of sound-insulation have been arrived at, and for air-borne sound a reduction of not less than 55 decibels between the main rooms in adjacent dwellings has been recommended. If this is accepted, then there is no doubt some form of discontinuous construction is necessary.

INSULATION AGAINST INSIDE NOISE

When the source of noise is in the room where quiet is required, and the noise cannot easily be controlled, the only methods available for suppression are to try to isolate the source and to use sound absorbents. In the case of machinery in factories, etc., rigid connection with the building structure must be avoided so as to prevent the vibrations being communicated to the floor, walls, etc. Probably the best way of doing this is to mount the machinery on resilient supports such as pads of rubber or compressed cork or steel springs. For effective reduction in the vibrational force transmitted to the structure, care must be taken in the choice of the resilience of the supports so that the ratio of the frequency of the vibrational force to the natural frequency of the machine on its supports is as high as possible. Otherwise the transmitted force may be increased instead of reduced. In the

case of a light machine, it may be desirable to mount it on a heavy base plate in order to obtain a sufficiently low natural frequency.

When as much has been done as is possible in this direction, the only treatment left is absorption. Thus the ceilings should be covered with absorbents particularly efficient in the higher frequencies, and absorbent baffles suspended from the ceilings would also help ; the upper parts of the walls could also be similarly covered. Greater improvement can sometimes be obtained if it is possible partially to enclose the source of the noise by an absorbent screen, for then the absorbent, being in a position where the noise is greatest, is most effective. With some machines this treatment may not be possible, but it is certainly worth trying with machines like typewriters, etc.

Absorption has also been used effectively for reducing the noise on underground railways. The noise is normally confined by the reflecting walls of the tunnel, and the level is thus considerably increased ; hence when the walls are lined with suitable absorbents an improvement is obtained. In connection with the use of absorbents, it must be noted that noise reduction by absorbents alone can only be markedly effective if originally the room is very reverberant, so that the rate of absorption can be increased several fold. Doubling the rate of absorption in the room, for example, will give a reduction of about 3 phons in the noise level, and no amount of absorption at the boundaries of the room can reduce the noise to less than would be obtained in the open air.

The student who requires more information concerning noise and sound insulation should consult original papers on the subject. A list of these will be found in an excellent paper by N. Fleming and W. A. Allen called "Modern Theory and Practice in Building Acoustics" (*Journal of the Institution of Civil Engineers*, 1945), from which source most of the information contained in this chapter has been derived.

AIRCRAFT NOISES

Now that flying is a recognised and well-established mode of travelling both for business and pleasure purposes, it is necessary that the journeys should be made as comfortable as possible for the passengers and the crews, and certainly one problem which must be tackled is that of noise within the cabins. In war-time, the over-riding consideration must be of course speed and manoeuvrability, and although these are important in civil aircraft, the suppression of noise to a reasonable standard is also

important, even though any sound-insulation treatment given to the aircraft may interfere with the other two. The problem is not an easy one to solve, as new types of aircraft are continually being evolved. But a good start has been made by N. Fleming and others at the National Physical Laboratory to obtain data that will be useful to the aircraft designer ; much remains to be done, however, before a satisfactory solution of the problem is reached.

It has been stated earlier in this chapter that noises of a predominantly high pitch tend to be more annoying than equally loud noises in which the lower frequencies are more prominent, and that loud noises containing a wide range of frequencies are much less distressing when the high frequencies are suppressed, although the resultant decrease in loudness may be slight. Therefore, the first problem in dealing with aircraft noises, as with other noises, is to obtain measurements to show the distribution of sound-intensity with frequency. Methods are available for doing this, and as a result of tests made, certain provisional standards have been recommended as to the maximum levels of noise which should be permitted in aircraft of different types, military and civil. The problem of noise is essentially one which depends on the effects produced on human beings and not necessarily on the physical measurement of the magnitude of the noise, and even though it might be difficult to reduce the noise of aircraft to the requisite level, there might be a much greater chance of eliminating those characteristics which cause the annoyance.

The main sources of noise in aircraft are the engine, the airscrew and aerodynamic noise produced by the flow of air over the structure, and for an effective treatment an attempt must be made to reduce the noise at the source and to insulate the cabins by sound-proofing the walls, etc. The problem would be easier if the relative contributions of the various sources of the noises to the total noise intensity were known, but as these depend on various factors such as frequency, airspeed, tip-speed of the airscrew, etc., the matter becomes more complicated, and much more quantitative data will be required than are available at present.

Reduction at source. The predominant noise from a *piston-type engine* is that from the exhaust, and a considerable reduction of noise can be obtained by the use of exhaust manifolds instead of separate stubs, though this might be accompanied by loss of engine power and of exhaust thrust. When manifolds are used, the exhaust discharge should be located at a point screened from the passenger cabins by the wing or engine nacelle. In many

instances the exhaust is delivered downwards, and this generally causes the annoyance factor to people on the ground, especially in the case of low-flying aircraft. It has been found that the offensiveness is mainly due, not to the magnitude of the noise, but to the high-frequency component of wave-length about 1 ft. This sound could therefore be screened from the ground if the exhaust were discharged above rather than below the wing.

In a *jet engine* the main sources of noise are the whistle from the impellor, and the roar of the jet. No information is yet available on the noise of such engines in flight, and any comparison of the noise with that of piston engines must be based on measurements made on the test bed. It appears, however, that on the whole there are grounds for believing that, for the same performance, an aircraft powered with jet engines may be less noisy than one powered with piston engines with open stub exhausts. Against this, of course, the exhaust noise from the piston engine can be reduced by the use of manifolds, but the jet engine presents no such opportunity. To people on the ground, it is the high-pitched whistle, the frequency of which is determined by the number of blades on the impellor and its rotational speed, which is probably the most disturbing feature, though one consolation is that the noise does not last very long.

Various theories of the production of noise by a rotating *airscrew* have been given, and there are several factors which determine the noise. One portion is produced by the forces exerted on the air by each element of the blades, that is, by the torque and thrust of each element, and this consists of a series of harmonic components, the fundamental of which is equal to the rotational speed multiplied by the number of blades. At low tip-speeds the fundamental is the predominant component, but with increasing tip-speed the harmonics become more evident, and at high speeds they may become the most important. Another source of noise is that due to the shedding of vortices from the blades, and this may be the more important at high frequencies. When the tip-speed approaches the velocity of sound, shock waves are produced and these give a large increase in the high-frequency content of the noise. The reduction of airscrew noise requires primarily a reduction of tip-speed. In order to maintain performance, therefore, either the diameter of the airscrew must be increased or a larger number of blades used ; for example, it has been found that at 500 horse power and blade angles of about 12° the tip-speeds of a two-bladed airscrew and a five-bladed one were about 1,050 ft. per sec. and 650 ft. per sec. respectively, and

the noise of the five-bladed airscrew was more than 20 db. lower than that of the two-bladed airscrew. The measures suggested for noise reduction certainly have disadvantages, and it is for the designer to decide whether the disadvantages outweigh the need for smaller noise.

Very little information is yet available concerning the magnitude of *aerodynamic noise*, but it appears certain that it is due partly to the turbulent motion of the air over parts of the machine such as the wings, etc., and partly to factors such as small beaded parts around the windows and badly fitting components generally. At speeds in the region of 200 m.p.h. aerodynamic noises are not much below those of the engine and airscrew, especially at the higher frequencies; but at higher speeds the noise seems bound to increase and may be by far the most important source of the noise. Here again it is a problem for the designer to introduce more laminar flow sections and so reduce turbulence, but he must be assisted by a more thorough investigation into the nature of aerodynamic noise and the manner in which it varies with air-speed and other relevant factors.

Sound-insulation of cabins. The principles underlying the insulation of the cabins against noise are similar to those already discussed in connection with buildings. The noise entering the cabins may be airborne, coming direct from the engine, the airscrew and the source of aerodynamic noise, and it may also be caused by vibrations conveyed through the structure. Therefore, in order to bring about a reduction of the noise in the cabin, the latter would have to be treated as a sound-proofed room, and details of the necessary treatment have been given earlier in the chapter. Such treatment involves the walls of the cabin, the floor, and other parts such as windows, ventilation and heating ducts and gaps at imperfectly fitting doors or opening windows, through which the noise may enter. By using double walls for the cabin, fitting double windows and attending to the ventilating system and other openings, it has been found possible, by tests, to bring about a noise reduction between the outside and inside ranging from 10 db. at low frequencies up to more than 60 db. at high frequencies. Quite apart from any sound-proofing treatment, the designer may be able to assist in the problem by locating the passengers in the less noisy parts of the aircraft, when these can be definitely ascertained.

It will be recognised by the student that the above account of aircraft noise is very incomplete and does not pretend to indicate the great amount of research work which has already been done

on this rather intricate problem. But it is hoped that sufficient information has been given to arouse the enthusiasm of the young engineer, and to encourage him to realise that this is one of a number of problems in acoustics, to the solution of which he may one day be able to make his contribution.

APPENDIX

(1) **General differential equation for wave-motion, $\frac{d^2y}{dt^2}=V^2\frac{d^2y}{dx^2}$.** The equation for a simple harmonic wave-motion is

$$y=a \sin p(t-x/V) \text{ (see p. 5).}$$

If x is regarded as constant, that is, we are considering a particular particle the position of which is defined by x, we have by differentiation :

$$\frac{dy}{dt}=ap \cos p\left(t-\frac{x}{V}\right) \quad \text{and} \quad \frac{d^2y}{dt^2}=-ap^2 \sin p\left(t-\frac{x}{V}\right) \text{.......(1)}$$

Similarly, if t is regarded as constant, we have :

$$\frac{dy}{dx}=-\frac{ap}{V}\cdot \cos p\left(t-\frac{x}{V}\right) \quad \text{and} \quad \frac{d^2y}{dx^2}=-\frac{ap^2}{V^2}\cdot \sin p\left(t-\frac{x}{V}\right)\text{....(2)}$$

Hence, from (1) and (2),

$$\frac{d^2y}{dt^2}=V^2\cdot\frac{d^2y}{dx^2}, \text{..............................(3)}$$

which is the differential equation for a simple harmonic motion. Now consider the equation $y=f(Vt-x)$, or more generally

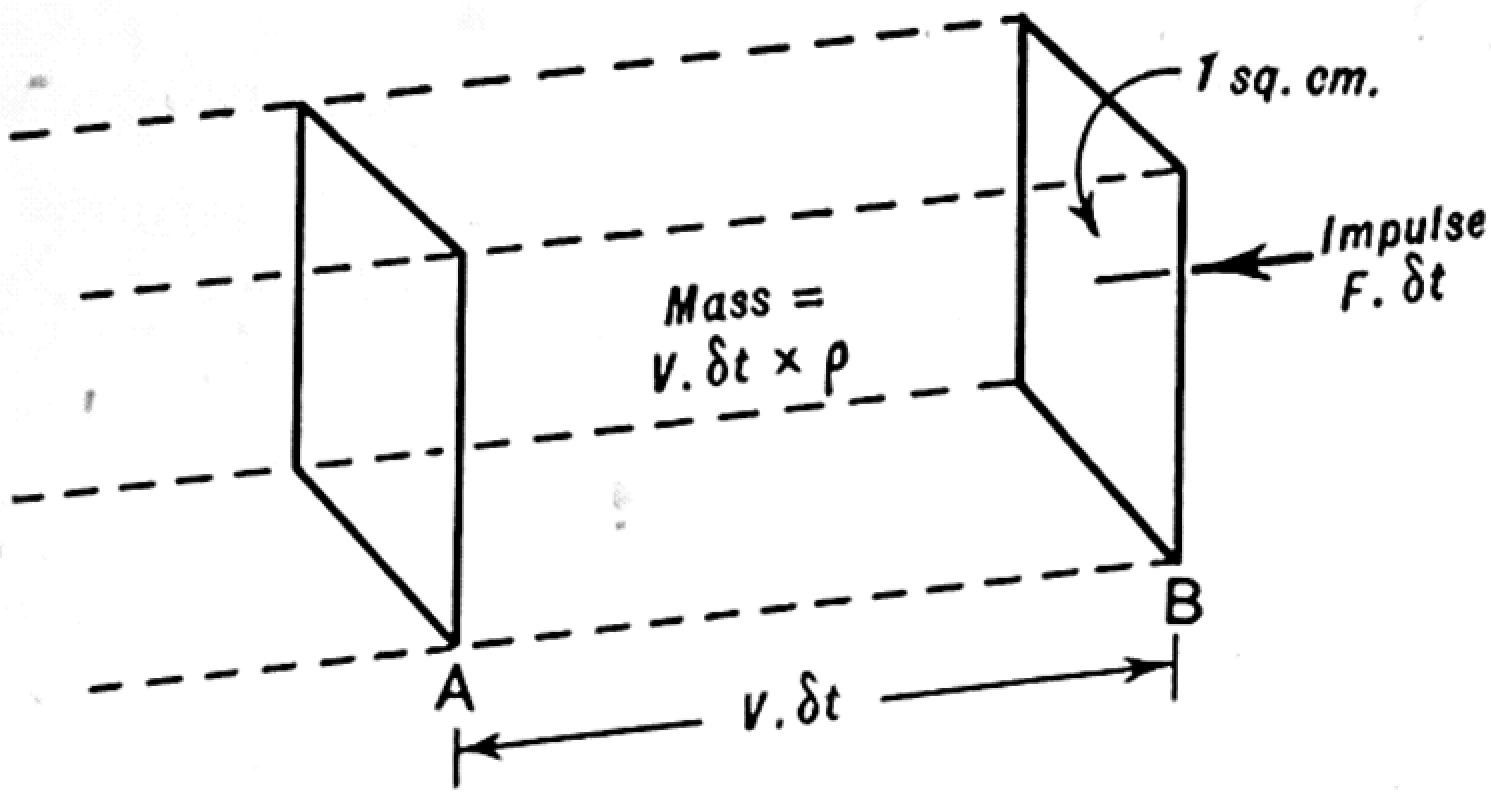
$$y=f(Vt-x)+f_1(Vt+x), \text{ ....(4)}$$

where f and f_1 represent *any* functions.

By differentiating equation (4) twice, it can be shown that this equation is the generalised solution of the differential equation (3). Hence equation (3) is the general differential equation for *any* form of wave-motion, not necessarily harmonic, and the equation $y=a \sin p(t-x/V)$ is a special case of equation (4), the case of a progressive simple harmonic wave.

(2) **Velocity of a longitudinal wave in a solid.** Consider a uniform rod of cross-sectional area 1 sq. cm. and density ρ gm. per c.c., and let an impulse of $F\,.\,\delta t$ be given to one end, say at B. This will produce a compression (δx) which will travel forward to A a distance $V\,.\,\delta t$ in the time δt. The momentum acquired by the length of rod $V\,.\,\delta t$ is given by mass × velocity

$$=V\,.\,\delta t\times\rho\times\delta x/\delta t=\rho V\,.\,\delta x\text{.....................(1)}$$



Now if E is Young's modulus for the material,

$$E = \frac{\text{stress}}{\text{strain}} = \frac{F}{\delta x / V \cdot \delta t} \text{ (where area is 1 sq. cm.)} = FV \cdot \frac{\delta t}{\delta x},$$

or the impulse $$F \cdot \delta t = \frac{E}{V} \cdot \delta x \quad \text{.........................(2)}$$

Since by Newton's laws, impulse = momentum ;

we have $$\frac{E}{V} \cdot \delta x = \rho V \cdot \delta x$$

whence $$V = \sqrt{E/\rho}\,.$$

(3) **Frequency of a tuning fork.** It was stated on p. 108 that the factors determining the frequency of a tuning fork are the length and thickness of the prongs (physical dimensions l) and the velocity of sound $\sqrt{E/\rho}$ in the material. If this may be assumed, an expression relating these quantities may be obtained by using the method of dimensions.

Let the frequency $n = k \cdot l^a E^b \rho^c$

Now dimensions of $n = [T]^{-1}$, of $l = [L]$, of $E = [M]\,[L]^{-1}\,[T]^{-2}$ and of $\rho = [M]\,[L]^{-3}$.

Hence $[T]^{-1} = [L]^a \cdot [M]^b [L]^{-b} [T]^{-2b} \cdot [M]^c [L]^{-3c}$.

Equating corresponding indices : for $[T]$, $-1 = -2b$; $[L]$, $a - b - 3c = 0$; $[M]$, $b + c = 0$.

Whence $b = \frac{1}{2}$, $c = -\frac{1}{2}$, $a = -1$

and $$n = k \cdot l^{-1} E^{1/2} \rho^{-1/2}$$

$$= k \cdot \frac{1}{l} \sqrt{\frac{E}{\rho}}\,.$$

It should be noted that l here is not merely the length of the prongs ; it involves thickness as well. A. B. Wood in his book " A Textbook on Sound " states that the frequency of a fork is

given by $n=(1{\cdot}1937)^2 \cdot \frac{\pi}{8} \cdot \frac{t}{\surd 12 \, . \, l^2} \cdot \sqrt{\frac{E}{\rho}}$ or $n \propto \frac{t}{l^2} \cdot \sqrt{\frac{E}{\rho}}$, and at first sight the expression derived from dimensions does not seem to agree with this. It must be remembered, however, that the dimensions of t/l^2 is $[L]^{-1}$.

Effect of temperature on the frequency of a fork.

From the above, $n=k \, . \, l^{-1} E^{1/2} \rho^{-\frac{1}{2}}$

$$\text{or } \log n = \log k - \log l + \tfrac{1}{2} \log E - \tfrac{1}{2} \log \rho .$$

Taking differentials, $\dfrac{\delta N}{N} = -\dfrac{\delta l}{l} + \tfrac{1}{2}\dfrac{\delta E}{E} - \tfrac{1}{2}\dfrac{\delta \rho}{\rho}$.

If these changes are due to an increase in temperature $\delta\theta$, then, so far as l and ρ are concerned, we have $\delta l/l = \alpha \, . \, \delta\theta$ and $\delta\rho/\rho = -3\alpha \, . \, \delta\theta$, where α is the coefficient of linear expansion of the material. Concerning elasticity, it is known that the temperature coefficient of elasticity decreases with temperature (for metals at any rate), and if we may assume that a linear relationship such as $E_\theta = E_o(1-\beta\theta)$ is obeyed, where β is the temperature coefficient of elasticity, then $\delta E/E = -\beta \, . \, \delta\theta$.

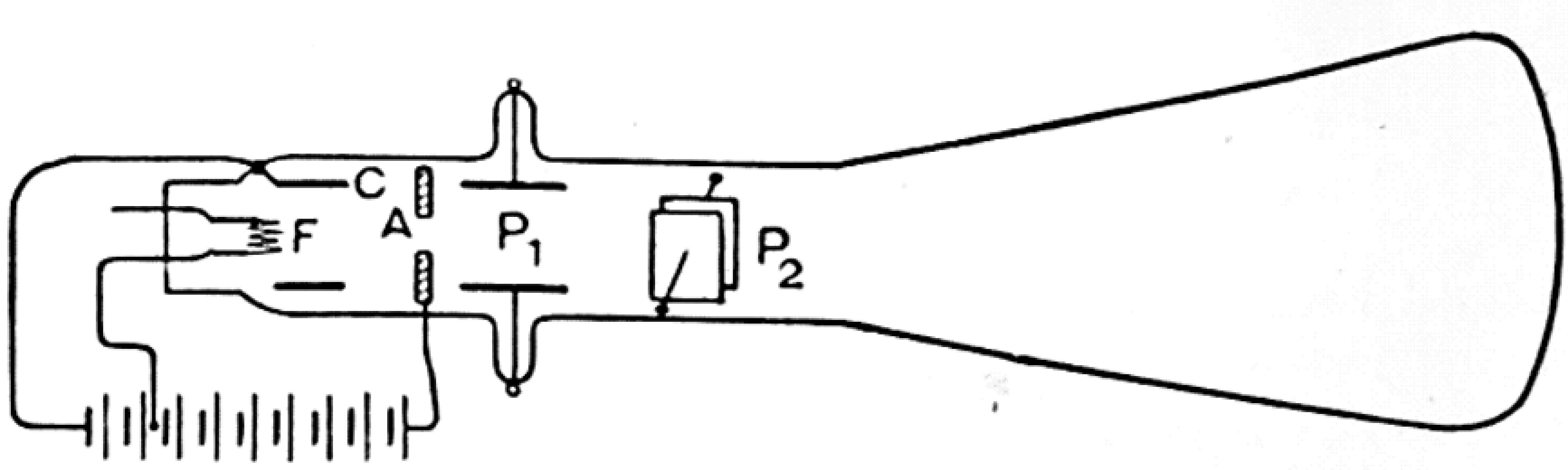
$$\text{Hence } \frac{\delta N}{N} = -\alpha \, . \, \delta\theta - \frac{\beta}{2} \, . \, \delta\theta + \frac{3\alpha}{2} \, . \, \delta\theta = -\left(\frac{\beta-\alpha}{2}\right)\delta\theta = -k \, . \, \delta\theta$$

(β is somewhat greater than α).

It will be seen from this relationship that frequency decreases with rise in temperature, and further, that the relationship between frequency and temperature is a linear one.

(4) **Cathode ray oscillograph.** Several references have been made in the text to the use of the cathode ray oscillograph in experimental acoustics, and there is no doubt that this instrument affords a rapid and an accurate means of demonstrating certain phenomena, especially those dealing with wave-motion.

Like many other instruments used in physics, the oscillograph is provided with a " pointer " (the electron beam), and sometimes



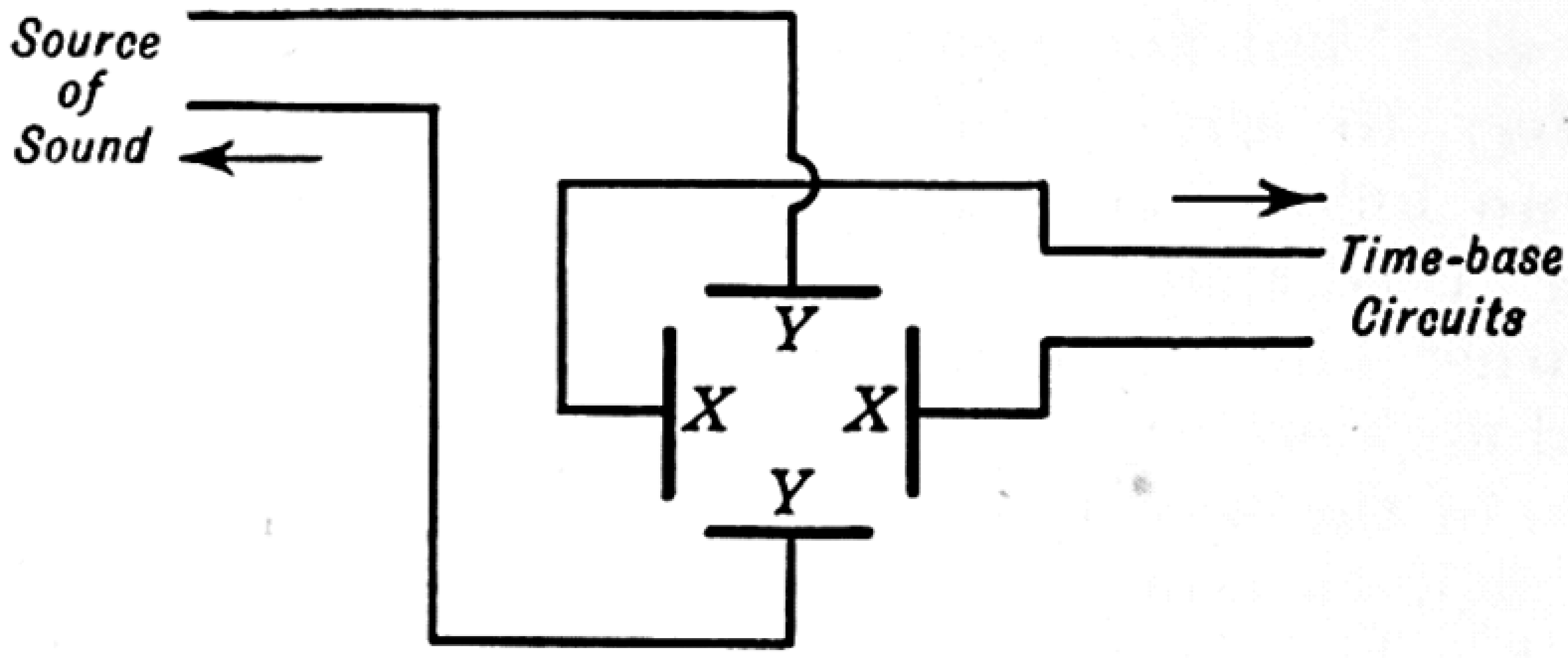
with a scale; but the pointer has no inertia and so responds immediately to any impulse impressed upon it. Hence the pointer can follow very rapid changes and portray them on a screen for visual analysis.

The earlier kind of oscillograph was of the *soft* type, which contained an inert gas such as argon; but most modern instruments are of the *hard* type in which the tube is exhausted to a state of high vacuum. Electrons are produced by the heated filament F, which is the cathode, and these are accelerated by a p.d. between the filament and the perforated anode A, the metal shield C concentrating the beam and directing more electrons through the hole in A. The electrons pass through A with considerable energy, and on striking the fluorescent screen S, they make it luminous at the point of impact. For better focusing of the electrons, two or more anodes can be used at successively higher p.d.'s.

The pairs of plates, P_1 and P_2, are both arranged parallel to the direction of the electron stream, but the plane of one pair is at right angles to that of the other. By applying fields to the plates, the stream can be deflected vertically or horizontally, and the character of the variations is shown on the screen as a fluorescent line; the fields applied to the plates may be either electrostatic, or magnetic (produced by coils outside the tube).

In using the oscillograph, the p.d. to be investigated is connected to the plates P_2, while the plates P_1 are connected to a *time-base* circuit in which voltage alternations of known frequency are maintained. If there is no time-base circuit and an alternating p.d. is applied only to P_2, the movement of the spot (on the screen) will be simple harmonic and will be indicated by a straight line across the screen. But when p.d.'s are applied to both pairs of plates simultaneously, the beam will move under two simple harmonic motions at right angles and the appropriate Lissajous figure will be traced out. If the motion of the spot due to the time-base is made at a uniform speed, the fluorescent track will be a time-displacement graph of the motion, and by synchronisation of the motion due to both sets of plates a stationary pattern can be obtained on the screen. The ideal time-base is what is known as a *linear time-base*, but for details of this and other features of the oscillograph the student should consult books on electricity.

Uses in experimental work. The general method of using the oscillograph in experimental work is indicated in the diagram (p. 312), in which XX and YY represent the plates P_1 and P_2 in the diagram on p. 310.



Whenever it is desired to examine visually any form of wave-motion by means of a stationary pattern which can, if necessary, be photographed and examined at leisure, the oscillograph is indispensable ; there are, of course, many other phenomena which can be investigated by using this instrument. Here we shall describe briefly only two examples, rather different in type, of its use in sound.

Comparison of frequencies. The frequency of any sound can be found if a calibrated oscillator of some type, either of fixed frequency or more preferably one in which the frequency is variable, is available. The output of the standard oscillator is applied to one set of plates, while the amplified output from a microphone located in the sound field of the source to be investigated is applied to the other set. The spot due to the electron beam will execute Lissajous figures, and an interpretation of these figures gives a means of obtaining the unknown frequency. The simplest plan is to use a variable standard and to alter this until the pattern on the screen is a steady circle, ellipse or straight line. The two frequencies are then of the same value, and it is only necessary to read the dial on the standard instrument to obtain the desired frequency.

Velocity of sound in air. Two microphones M_1, and M_2, situated in the sound field of a loudspeaker which is energised by a valve oscillator giving a pure tone of known frequency, are connected to the X and Y plates respectively. The two sources will produce the appropriate Lissajous figure which will, in general, be an ellipse ; but if the two microphones are separated by a distance equal to an exact number of half-waves, so that the phase difference is either zero or π, this figure will be a straight line. Hence M_1 and M_2 are first adjusted in position to produce a straight line on the screen, and then one is moved away until a similar trace is obtained ; the distance moved is clearly one wave-length. The process can be repeated to find a mean value of λ, and the velocity can be found from $V = n\lambda$.

QUESTIONS

These questions have been selected from recent examination papers for various levels of the General Certificate of Education and similar examinations. The examining bodies have kindly given their consent to the use in this way of their questions, the sources of which are indicated by initial letters following each question :

University of Bristol. (B)
Local Examinations Syndicate, University of Cambridge. (C)
Civil Service Commission. (CSC). By permission of the Controller of H.M. Stationery Office.
University of London. (L)
Joint Matriculation Board (Northern Universities). (N)
Oxford Local Examinations. (O)
Oxford and Cambridge Schools Examination. (O and C)
Central Welsh Board. (W)

CHAPTER I

Ordinary level.

1. How is sound transmitted from its source to the ear and what is its effect on the ear? Describe the construction of *either* a telephone or a microphone, and explain its method of operation. (L)

2. How is sound (*a*) produced, (*b*) transmitted, from place to place? Describe briefly two experiments to illustrate (*a*) and one to illustrate (*b*). (N)

3. Why is it supposed that sound consists of a wave motion propagated through air? Illustrate your answer by experimental examples. (O)

4. What are the main characteristics of wave motion? Illustrate your answer by a diagram. (L)

5. Explain what is meant by "wave motion". Distinguish between longitudinal and transverse wave motion, and give an example of each. (L)

6. Describe *one* experiment to explain each of the following: (*a*) The source of sound is a vibrating body. (*b*) Do sounds pass through a vacuum? (*c*) Sound is transmitted by a wave motion.

What evidence is there for believing that sound does or does not pass through solids and liquids? (O)

7. Describe experiments to show that (*a*) sound is caused by vibration, and that (*b*) sound is not transmitted in the absence of a material medium.

A spectator watches a game of cricket on a still summer day from a distance of half a mile. At what interval after seeing a player hit a ball will the spectator hear the sound? State how you obtain the answer. (The velocity of sound in air should be taken as 1100 ft. per sec.) (L)

Advanced level.

8. Give a general account of the mode of action of the human ear. What is meant by the threshold of audibility, and how does this vary with frequency for the average person?

What scale is used for comparing the loudness of sounds, and why is such a scale chosen? (CSC)

9. Describe the nature of the disturbance set up in air by a vibrating tuning fork and show how the disturbance can be represented by a sine curve. Indicate on the curve the points of (*a*) maximum particle velocity, (*b*) maximum pressure.

What characteristics of the vibration determine the pitch, intensity and quality respectively of the note? (N)

Scholarship level.

10. Show that the total energy of a particle performing simple harmonic motion is independent of time and that it is proportional to the square of the amplitude and to the square of the frequency of the motion.

The tension in the wire of a sonometer is equal to the weight of 100 gm. If the wire is 100 cm. long, and vibrates in its fundamental mode with a maximum amplitude of 0·1 mm., calculate the energy of the vibration. (The frequency n of vibration of the wire is related to the tension T, the mass per unit length m, and the length l by the formula

$$n=\frac{1}{2l}\sqrt{\frac{T}{m}}.) \qquad \text{(C)}$$

11. Describe how you would produce (*a*) a note of approximately 1,000 cycles per second, (*b*) an ultrasonic vibration of approximately 50,000 cycles per second. Explain how you would measure the wave-length of these disturbances. (C)

CHAPTER II

Ordinary level.

1. Describe one method for the experimental determination of the velocity of sound in free air.

If an observer records a time-interval of t sec. between seeing a flash and hearing the thunder, calculate the distance in miles of the flash from the observer. (The velocity of sound in air may be taken as 1100 ft. sec.$^{-1}$) (L)

2. Describe a laboratory experiment to determine the speed of sound in air.

Describe and explain the effect of a change of temperature on the frequency of the note given by (*a*) an organ pipe, (*b*) a violin string. (L)

3. (*a*) Describe a method of finding the velocity of sound in air.

(*b*) A balloon is 2,200 feet above ground, and the velocity of sound in air is 1,100 feet per second. How long does it take for sound to travel to the ground, be reflected, and return to the balloon?

(*c*) Sound travels from a ship to the bottom of the sea, is reflected, and returns to the ship in 0·08 second. The velocity of sound in sea water is 5,000 feet per second. How deep is the water?

(*d*) Suggest one reason why echo-sounding to find depth is useful at sea, but is of little use to the pilot of an aeroplane. (C)

4. Describe a method of determining the velocity of sound in the laboratory.

A man by the side of a lake sees the steam from the whistle of a steamer on the lake. Three seconds later he hears the whistle, and four seconds later still he hears the echo of the whistle from a cliff behind the steamer and at the other side of the lake. What is the width of the lake if the velocity of sound in air is 1100 ft. per sec.? (N)

5. " The velocity of sound in air is approximately 1100 ft.-sec.$^{-1}$."
Describe and explain two ways of verifying this statement experimentally, one being an open-air method and the other suitable for use in a laboratory. (L)

6. State the evidence from which it has been concluded that light and sound travel at very different rates.

In a 440 yds. race along a straight track two timekeepers stand at the " finish ". One starts his watch when he sees the smoke from the starter's pistol and the other when he hears its report. The times recorded for the winner of the race are 54·76 sec. and 53·62 sec. respectively. Use this data to find a value, in ft. sec.$^{-1}$, for the velocity of sound in free air. (L)

7. Describe a method of measuring the velocity of sound in air, showing clearly how the result is calculated.

In a determination of the depth of the sea, the echo from the sea-bed of a sound produced on a ship is received back 0·8 sec. after the actual sound. What is the depth of the sea at that point? Velocity of sound in sea water = 1500 metres per sec. (L)

Advanced level.

8. How does the velocity of sound in a medium depend upon the elasticity and density? Illustrate your answer by reference to the case of air and a long metal rod. The velocity of sound in air being 1100 ft./sec. at 0° C., and the coefficient of expansion 1/273 per degree, find the change in velocity per ° C. rise of temperature. (L)

9. How would you find by experiment the velocity of sound in air? Calculate the velocity of sound in air in cm./sec. at 100° C. if the density of air at S.T.P. is 0·001293 gm./c.c., the density of mercury at 0° C. 13·60 gm./c.c., the specific heat of air at constant pressure 0·2417 and the specific heat at constant volume 0·1715. (L)

10. How would you compare (*a*) the velocities of sound in wood and air and (*b*) the velocities of transverse and of longitudinal disturbances in a stretched wire?

A stretched wire 1 m. long gives a note of frequency 2000 vibrations per second when stroked lengthwise with a resined leather. How is the string vibrating and what is the speed of propagation along it of the vibrations? (L)

11. Explain how sound waves are propagated in solids, liquids and gases. Describe briefly three methods of measuring the velocity of sound, one for each type of medium. (N)

12. Derive an expression for the velocity of plane sound waves through a gas. Discuss the effect of (*a*) pressure, (*b*) temperature, on the velocity. Neglecting end corrections, calculate the change in frequency of a 10 ft. open-end organ pipe when the air temperature changes from 5° C. to 25° C. (O)

13. Describe an experiment to find the velocity of sound in air at room temperature.

A ship at sea sends out simultaneously a wireless signal above the water and a sound signal through the water, the temperature of the water being 4° C. These signals are received by two stations, *A* and *B*, 25 miles apart, the intervals between the arrivals of the two signals being $16\frac{1}{2}$ sec. at *A* and 22 sec. at *B*. Find the bearing from *A* of the ship relative to *AB*. The velocity of sound in water at t° C. $= 4756 + 11t$ ft. per sec. (N)

14. Explain *progressive waves, stationary waves.*

How would you determine the velocity of sound in a wooden or a metal rod. (C)

15. Distinguish between progressive and stationary waves.

Describe an experiment, based on the production of stationary waves, which shows that sound travels more slowly in carbon dioxide than in air at the same temperature. Show how to calculate the ratio of the velocities of sound in these two gases from the experimental measurements. (N)

16. Give an account of the propagation of sound in the atmosphere, discussing especially the effects of wind and vertical temperature gradients.

Describe how the velocity of sound has been measured in the open air and mention the chief experimental difficulties in such a determination.

An organ pipe emits a fundamental note of frequency 128 seconds^{-1}

at 10° C.; what will be the frequency of this note if the temperature rises to 21° C.? (O)

Scholarship level.

17. An explosive percussion signal on a rail is set off by a locomotive passing over it. A listener 1 km. away with one ear to the rail hears two reports. What is the time interval between them?

(Young's modulus for steel $= 2 \times 10^{12}$ dynes cm.$^{-2}$; density of steel $= 7{\cdot}8$ gm. cm.$^{-3}$; density of air $= 0{\cdot}0013$ gm. cm.$^{-3}$; ratio of specific heats of air $= 1{\cdot}4$; 1 atmosphere $= 10^6$ dynes cm.$^{-2}$) (C)

18. How does the velocity of sound in a gas depend on temperature and pressure?

The observer in an aeroplane flying horizontally at 240 m.p.h. releases a bomb and hears the sound of the explosion 20 sec. afterwards. Find the height of the aircraft, neglecting air resistance. (Velocity of sound in air $= 1100$ ft./sec.) (O)

19. Give an account of any important and characteristic wave phenomena which occur in sound. Why are sound waves in air regarded as longitudinal and not transverse?

An observer looking due north sees the flash of a gun four seconds before he records the arrival of the sound. If the temperature of the air is 20° C. and the wind is blowing from east to west with a velocity of 30 m.p.h., calculate the distance between the observer and the gun. The velocity of sound in air at 0° C. is 1,100 ft. per second. Why does the velocity of sound in air depend upon the temperature but not upon the pressure? (N)

20. A ship travelling due north at 1 m./sec. in a thick fog fires a detonator in the sea alongside and receives an echo from a buoy on the port side 1·2 sec. later. Fifteen minutes later a repetition of the experiment yields the same result. What is the bearing and distance of the buoy? Describe the type of apparatus you would use to make these measurements.

(Velocity of sound in sea water $= 1500$ m./sec.) (O)

21. An observer standing close beside an anti-aircraft gun notices that the shell explodes 5 sec. after it has been fired. The sound of the explosion reaches him 9 sec. later. If the angle of elevation of the gun is 45°, calculate to within the nearest hundred feet the height at which the shell explodes. (Velocity of sound in air $= 1110$ ft./sec.; $g = 32$ ft./sec./sec.) (C)

22. On what does the velocity of sound in a gas depend? Explain fully why an organ pipe blown in hydrogen might be expected to have a frequency two octaves higher than when blown in oxygen.

Calculate the velocity of sound in air which is saturated with water vapour at 18° C. and at a pressure of 76 cm. of mercury. Saturation pressure of water vapour at 18° C. $= 15{\cdot}5$ mm. of mercury.

Velocity of sound in dry air at 0° C. 332 m./sec. The relative densities of hydrogen, water vapour, air, and oxygen under the same pressure and temperature conditions are 1 : 9 : 14·4 : 16. The ratio of the two specific heats of air may be taken as unaffected by the moisture content. (N)

23. Give a brief account of the evidence in support of the view that sound is propagated as a wave-motion through the air. What are the physical factors that determine the velocity of propagation of such waves?

Indicate the chief sources of error in measuring the velocity of sound in the open air and describe a good method of finding this velocity. (N)

24. Describe a method of measuring the velocity of sound in a gas like hydrogen, of which only limited quantities are available. Explain carefully what length and what time ($V = L/T$) are measured in your method. Discuss on what properties of a gas the velocity of sound depends, and draw a rough curve showing how you would expect the velocity of sound in a mixture of oxygen and nitrogen to depend upon the composition of the mixture. (O)

25. Assuming the velocity of sound in a gas $= \sqrt{\dfrac{\text{adiabatic elasticity}}{\text{density}}}$

find an expression showing how it depends on temperature and molecular weight of the gas.

The planet Jupiter has an atmosphere composed principally of methane (CH_4) at a temperature of $-130°$ C. Estimate the velocity of sound on this planet, assuming the ratio of the principal specific heats of this gas to be 1·3.

($R = 8{\cdot}3$ joules per degree per gm. mol.) (O)

26. Why is sound believed to be a periodic disturbance in the medium through which it is transmitted?

What characteristics of this disturbance are related to the description of sound in everyday language? (O)

27. Apply the method of dimensions to find an equation for the velocity of sound in a medium of known density and elasticity. What form does the relation assume in the case of a gas? (O)

28. Discuss the factors which may influence the velocity of sound in the atmosphere.

Explain qualitatively why a noise is heard louder when a wind blows towards the observer. (C)

29. How would you show experimentally how the velocity of sound in air depends upon (*a*) the pressure, (*b*) the temperature and (*c*) the frequency of the note?

An organ pipe and a piano string both emit a note of frequency 200

at 0° C. How many beats will there be between them at 30° C.? (Assume the frequency of the string to be independent of temperature.) (O)

CHAPTER III

Ordinary level.

1. Describe an experiment which demonstrates that sound waves can be reflected.

A ship, A, is at anchor a distance of 1000 yds. from a vertical cliff on the shore. Another ship, B, is anchored between A and the cliff. When B gives a short blast on its siren the sound is heard twice at A, the time-interval between the arrival of the sounds being 3 secs. Find the distance apart of the ships.

(The velocity of sound in air may be taken as 1100 ft. sec.$^{-1}$) (L)

2. What is an echo? Describe how an echo may be used to determine the velocity of sound in air. How would you then calculate the velocity of sound at 0° C.? (L)

3. For each of the following statements explain one observation which illustrates its truth: (*a*) the transmission of sound through air is not instantaneous; (*b*) the velocity of sound is independent of the pitch of note; (*c*) sound is reflected from a plane, or nearly plane, surface.

What difference would you expect between the reflections from a wooden wall and from a padded wall? (N)

4. Explain the conditions required for an echo to be heard. A ship sounds its siren when it is 3850 ft. from a vertical cliff. Taking the velocity of sound in air as 1100 ft. per sec., calculate how long it is before the echo is heard. At the moment of sounding its siren, a small depth charge is exploded in the sea near the ship and its echo from the undersea part of the cliff is heard in hydrophones on the ship after $1\frac{4}{7}$ sec. What is the velocity of sound in water? Upon what factors does the velocity of sound in air depend? (L)

5. In order to determine her proximity to an iceberg towards which she is heading with uniform velocity, a steamer sounds her siren *once every minute*. The echo of the first blast is heard after 12 sec. and that of the second after 9 sec. Calculate the original distance of the steamer from the iceberg and also her velocity. (L)

6. How are echoes produced? Make a careful diagram illustrating how the waves from a source of sound are reflected at a plane surface.

The observer at a certain distance from a cliff notes that the interval between a sound he makes and its echo is 3 sec. He then walks 550 ft. nearer to the cliff and finds that the corresponding interval is 2 sec. Calculate (*a*) the velocity of sound, (*b*) the observer's original distance from the cliff. (N)

Advanced level.

7. Discuss the factors on which the velocity of sound in a gas depends.

A man standing at one end of a closed corridor 190 ft. long blew a short blast on a whistle. He found that the time from the blast to the sixth echo was 2 sec. If the temperature was 17° C., what was the velocity of sound at 0° C.? (C)

8. Explain the formation of echoes in terms of the behaviour of waves.

Describe how the velocity of sound in either (*a*) air or (*b*) water, may be determined by the use of echoes.

An observer *A* fires a pistol in the angle between two high walls at right angles to each other, and the sounds produced are heard by another observer *B*. *A* is 110 yards from one wall and 220 yards from the other; *B* is 440 yards from each wall. Draw a diagram to scale (110 yards = 1 inch) showing the paths of "rays" of sound from *A* to *B*, and calculate the time-interval between the sounds heard by *B*. Velocity of sound in air = 1,100 feet per second. (N)

Scholarship level.

9. What explanation can you give of the following:

(*a*) The sound of a rifle shot is sometimes reflected from a railing as a musical note.

(*b*) Sound can be heard abnormally large distances in foggy weather.

(*c*) The stringed and wind instruments of an orchestra do not remain in tune when the temperature of a concert hall changes.

Describe any simple experiments the results of which support your explanations. (O)

CHAPTER IV

Ordinary level.

1. Describe and explain the phenomenon of "beats". You are given two tuning forks of nearly the same frequency. Assuming that the frequency of one of these forks is known, how would you determine that of the other? (C)

Advanced level.

2. A brass wire and a steel wire, of the same length, diameter, and tension, give 5 beats per second when mounted on a sonometer and made to give their fundamental tones simultaneously. Calculate the frequencies of vibration of the two tones, given that the densities of brass and steel are 8·4 and 7·8 gm. per c.c. respectively. (O and C)

3. Two wires vibrate transversely in unison. The tension in one wire is increased by 1 per cent., and now, when they vibrate simul-

taneously, three beats are heard in 2 sec. What was the original frequency of vibration of the two wires? (N)

4. Describe the formation and properties of stationary wave-motion.

Describe how a resonance tube and sources of sound of known frequency may be used to determine the velocity of sound in air. Show how the velocity at 0° C. may be calculated from the value at room temperature. (N)

5. Explain the formation of beats, and deduce an expression for the beat frequency in terms of the frequencies of the two sources.

Two tuning forks have nearly the same frequency. Describe carefully how you would determine the frequency of one of them if the frequency of the other were known. (B)

6. What is meant by the terms *node*, *antinode*, in respect of sound waves? What are beats, and how are they produced?

Two open organ pipes, 80 and 81 cm. long, are found to give 26 beats in 10 seconds when each is sounding its fundamental note. Find the velocity of sound in air and the frequencies of the two notes. (End corrections may be neglected.) (C)

7. Distinguish between *progressive* and *stationary* waves.

Describe an experiment to illustrate the formation of stationary waves (with more than one loop) on a string or wire. If a copper wire 0·193 mm. in diameter is used, what must be the tension to form loops 30 cm. long when the frequency of vibration of the wire is 64 per sec.? Give the result in gm. wt. (The density of copper is 8·93 gm. per c.c.) (N)

Scholarship level.

8. Explain the formation of a system of stationary waves. The frequency of the harmonics emitted by an organ pipe are not exact multiples of the fundamental frequency. What explanation can you give of this and how would you test your explanation experimentally? (O)

9. What is meant by "stationary waves"? Illustrate your explanation by reference to the vibration of air in open and closed organ pipes.

Indicate methods by which the frequency of a musical note could be determined. (O)

10. Distinguish between progressive and stationary waves. A sounding organ pipe closed at one end is adjusted to resonance. Describe carefully the motion of the air in the pipe and the changes of pressure with time at various points along the pipe.

The second overtone of a pipe closed at one end has the same frequency as the third overtone of a pipe open at both ends. Neglecting end effects, compare the lengths of the pipes. (C)

11. Explain the production of stationary waves. A train of sound waves is propagated along a wide pipe and is being reflected from an open end. If the amplitude of the waves is 0·002 cm., the frequency 1000, and the wavelength 33 cm., find the amplitude of the vibration at a point 20 cm. from the open end inside the pipe. (C)

12. Distinguish between progressive and stationary waves.

Describe the motion of the air in a closed sounding organ pipe adjusted to resonance, and the changes of pressure with time at various points along the pipe. Describe some experimental evidence in support of your statements. (C)

13. A vibrating tuning fork is moving steadily with a velocity of 150 cm./sec. normally towards a wall from which the sound waves are reflected. If the frequency of the fork is 512 sec.$^{-1}$, what will be the frequency of the beats heard by a stationary observer who has just been passed by the fork?

(Velocity of sound = 330 m./sec.) (C)

14. Show that when two notes of frequencies f_1, f_2 are sounded simultaneously beats are produced of frequency $f_1 \sim f_2$.

An air-raid siren, situated 550 ft. from the vertical face of a sharply-rising cliff, emits a note which rises in frequency uniformly from 0 to 250 c./sec. in 5 sec. and then drops uniformly through the same range in the same time. Observers notice a beating effect which they attribute to reflection of the sound from the face of the cliff. Discuss the phenomenon, and show how you would expect the beats to vary in clearness and in frequency for observers in different situations. (Velocity of sound = 1100 ft./sec.) (C)

CHAPTER V

Ordinary level.

1. Describe a sonometer and explain how to use it to demonstrate the relation between the frequency of the note emitted by a stretched string sounding its fundamental note and the tension.

A stretched steel wire emits a note of frequency 256 when vibrated under a certain tension. When the tension is increased by 5 kg. the note emitted by the wire is of frequency 384. What was the original tension of the wire? (L)

2. Explain how a sound is caused when a violin string is bowed. On what factors does the pitch of the sound depend? Describe briefly how the vibrations are transmitted to the ear. (L)

3. Draw and describe briefly an apparatus suitable for studying the laws of vibration of strings.

A string is tuned to give a certain note. Explain how you would obtain the octave of this note (*a*) by changing the vibrating length only, (*b*) by changing the tension only, (*c*) without changing length

or tension. Give reasons in each case for the method you would use. (N)

4. What factors determine the frequency of vibration of a stretched string?

Two wires of the same material have lengths in the ratio 2 : 3. If their diameters are the same, what must be the ratio of their tensions for the shorter wire to give a note an octave higher than the longer? (C)

5. Describe a sonometer and explain how you would use it to verify the relation between the length of the vibrating wire when at constant tension and the frequency of the note emitted by it.

A wire, AB, 100 cm. long, is held taut by a constant force, and a bridge, C, is placed so that $BC = 60$ cm. When vibrated, BC emits a note of frequency 252 cycles per second. The bridge is now moved 10 cm. nearer to A. What will be the frequencies of the notes emitted by the two parts of the wire when vibrated? (L)

6. Distinguish between (*a*) transverse and longitudinal waves, (*b*) progressive and stationary waves. Give an example of each type.

When a certain sonometer wire is stretched by a load of mass 8 lb. it is found that a length of 60 cm. vibrates transversely in unison with a given source of sound. On substituting a load of mass 4·9 kg. for the 8 lb. mass a length of 70 cm. of the wire is in unison with the same source. Estimate the number of grams equivalent to 1 lb. (L)

7. On what does the frequency of the note emitted by a stretched string depend?

Compare the frequencies of the notes emitted by two sonometer wires, made of the same wire, stretched by forces of 5·76 kilogram weight and 4 kilogram weight respectively. What would be the effect on the frequencies of these wires if they were wrapped with a layer of very fine wire? (N)

8. Describe a sonometer and explain fully how it is used to compare the frequencies of vibration of two tuning forks.

A sonometer wire between a fixed bridge, A, and a movable bridge, B, 60 cm. from A, has a frequency of 360 cycles per sec. Through what distance and in which direction must B be moved to tune the wire to 300 cycles per sec.? (L)

9. The frequency f of the fundamental mode of vibration of a transverse vibration of a string stretched by a force F is given by $f = \frac{1}{2l}\sqrt{\frac{F}{\sigma}}$, where l is the length of the string and σ its mass per unit length. Describe and explain how you would use this relation to determine a value for the frequency of a tuning fork. Give an account of a method of verifying your result. (L)

10. Describe how you would arrange for a sonometer wire to have the same fundamental frequency as a given tuning fork. With such

an arrangement, a sounding-fork of double the frequency placed on the sonometer board causes the wire to vibrate. Explain this, and describe the motion of the wire. (N)

11. Explain what is meant by an *overtone*. How would you show that the overtones of a string with fixed length and tension are *harmonic* overtones.

A thin wire 60 cm. long has a mass of 0·90 gm. and is under a tension of 10 kg. wt. Calculate the frequency of its second overtone when the wire is vibrating transversely. (L)

Advanced level.

12. On what factors does the frequency of vibration of a stretched string depend? Why is the bass string of a violin wound round helically with silver wire?

When the wire of a sonometer is 73 cm. long it is in tune with a certain tuning fork. On shortening the wire by 0·5 cm. it makes 3 beats a second with the fork. What is the frequency of the fork? (W)

13. Explain how you would investigate the dependence of the frequency of the note emitted by a plucked string on its length and tension, if you were provided with a sonometer and a set of standard forks.

A sonometer wire is tuned to a fork of frequency 100 vibrations per second. When another sonometer wire of the same material, diameter and length is sounded with the first, three beats per second are heard. If the second wire gives the higher note, find the ratio of the tensions in the two wires. (N)

14. Describe an experiment which will enable you to measure the ratio of the velocity of a compressional wave in a gas to the velocity in a solid.

An iron bar of density 7·7 gm./c.c. and of length 100 cm., when clamped in the middle and stroked, emits a note which is in resonance with 16·2 cm. length of a sonometer wire. A fork of frequency 540 resonates with 72·6 cm. of the same wire under the same tension. Find the frequency of the note and Young's modulus for iron. (W)

15. Assuming that the frequency of transverse vibration of a stretched string is inversely proportional to its length, describe how you would investigate the relation between the frequency and the tension.

How would you plot your observations and what result would you expect?

A stretched string and an air column closed at one end both resound to a tuning fork of frequency 256 vib./sec., the vibration being the fundamental in both cases. State briefly the differences between the states of vibration of the string and the air column.

What is the next higher frequency to which (*a*) the string, (*b*) the air column will resound without their dimensions or the tension being altered? (L)

16. A sonometer wire is stretched between two bridges on a large wooden mount. When it is plucked sharply in the middle and released a musical note is heard. Describe carefully and as fully as you can, what is happening during the sounding of this note (*a*) to the wire, (*b*) to the mount, (*c*) to the air between the mount and the observer's ear.

The length of the wire is 80 cm. and its tension is adjusted so that it is in unison with a fork of frequency 256 $sec.^{-1}$ One bridge is accidentally displaced so that the separation becomes 80·4 cm. Calculate the frequency of the beats now obtained when the fork and the wire are sounded together and find the percentage alteration in the tension which would restore the pitch of the note to the original value. (O)

17. Write down expressions for the velocity of propagation along a stretched wire of (*a*) transverse waves, (*b*) longitudinal waves. Deduce *one* of the expressions.

Find the ratio of the fundamental frequencies of transverse and longitudinal vibrations for a steel wire 1 mm. diameter, mounted on a sonometer and stretched by a force of 10 kilograms weight.

(Young's modulus for steel $= 20 \times 10^{11}$ dynes per sq. cm.)

Describe and explain a method of finding the velocity of longitudinal waves in a steel rod. (N)

18. Describe and explain the mode of action of an electrically driven tuning fork or vibrator. How may such a fork or vibrator be used to verify the relation between the tension and wave-length of transverse waves on a stretched string or wire?

A fine wire 500 cm. long is fixed at both ends and is under tension, the fundamental frequency of transverse vibration being 50 cycles per second. At what distance from its centre must the bridge be placed in order that four beats per second may be heard when both sections of the wire are made to vibrate transversely? (N)

19. Two wires, *A* and *B*, made of the same metal, of equal lengths and with diameters in the ratio 7 : 4, when set into transverse vibration give notes whose frequencies are in the ratio 4 : 5. These notes are brought into unison when the tension in *B* is diminished by 1·5 kg. weight. Find the tension in *A* and the original tension in *B*. (N)

20. A weight of 5 kg. suspended from the lower end of a uniform wire of length 1 metre and diameter 0·36 mm. produces an extension of 2·5 mm. Find the frequency of the note emitted when the stretched wire is stroked by a resined cloth. The density of the material of the wire is 7·8 gm. per c.c. (N)

Scholarship level.

21. A cord 5 metres long has a total mass of 245 gm. It is stretched with a constant tension of 1 kg. weight. If it is fixed at one end and shaken by hand at the other, what frequency of shaking will make it break up into three vibrating segments? (C)

22. A stretched steel wire, 0·5 mm. in diameter, is supported at two points 30 cm. apart. What tension must be applied to make its lowest characteristic frequency of transverse vibration 1000 cycles per second?

What approximations do you make?

(Density of steel = 7·8 gm./c.c.) (C)

23. Two strings of the same material and of the same cross-section are suspended on a sonometer. One is loaded with 12 kg. and the other with 3 kg. The first string is tuned to the first harmonic of the second string. If the second string is 100 cm. in length, what is the length of the first string? (C)

24. Prove that the velocity of transverse waves in a stretched string is $(\text{Tension}/\text{mass per unit length})^{1/2}$.

A wire is stretched between two fixed points so that its natural length is increased by 1 per cent. at room temperature. The coefficient of linear expansion of the wire is $2{\cdot}30 \times 10^{-3}$ per cent. per degree C. and the temperature coefficient of its Young's modulus is 0·012 per cent.

Calculate the percentage change in frequency of transverse vibrations when the temperature of the wire rises by 10° C. (O)

25. Devise a method of determining experimentally the frequency of vibration of a stretched string. (O)

26. State, but do not prove, an expression for the velocity of waves along a stretched wire, and use it to deduce the fundamental frequency of vibration of a wire fixed at both ends.

A tightly stretched wire can be used to measure the strain in a large girder in the following way. Its ends are fixed to two points A and B in the girder so that, as the girder is strained the distance between A and B is changed slightly. The natural frequency of the wire is adjusted, before straining the girder, to be the same as that of another similar wire mounted on unmoving supports, and when the strain is set up the beats between the frequencies of the two wires are observed. Show how, by observation of the beat frequency, and a knowledge of any other necessary quantities, you would determine the strain set up between the two points A and B. (C)

CHAPTER VI

Ordinary level.

1. What is the essential difference between a musical note and a noise? What determines the (*a*) loudness, (*b*) quality, (*c*) pitch, of

a musical note? Describe *one* experiment by means of which the last of these may be found in the laboratory. (L)

2. What is meant by (*a*) the frequency, (*b*) the wave-length, of a sound?

A note of frequency 272 per sec. is sounded on a day when the velocity of sound in air is 1088 ft. per sec. What is the wave-length?

Describe how you would measure the frequency of the note emitted by a tuning fork. (O and C)

3. Describe some form of siren. Why does its operation lead to the production of a musical sound? Explain how (*a*) the pitch, (*b*) the loudness, of the note can be changed.

If the frequency of the note is 300 vibrations per sec. and the velocity of sound in air is 1100 ft. per sec., calculate the wave-length of the sound waves produced. (N)

4. What do you understand by (*a*) the frequency, (*b*) the amplitude, of a vibration? How does a noise differ from a musical note? Describe a method of finding the frequency of a note given by a tuning fork. (N)

5. What are the physical characteristics which determine pitch, interval, and loudness, as applied to musical notes?

Describe a method of measuring the velocity of sound through the open air, pointing out the sources of error and explaining how one of these errors may be allowed for. (C)

6. Two monochord wires are tuned approximately, but not exactly, to the same note. Describe and explain what you will hear if they are sounded together.

When one wire is sounded alone, the note produced if it is plucked near one end has a different quality from the one produced if it is plucked in the middle. Why is this? (C)

7. What factors determine (*a*) the pitch, (*b*) the loudness, of the note given out by a sounding object? Describe experiments to illustrate each of these factors.

What effect on the pitch of the note given out by a stretched wire would be produced by (*a*) decreasing its length, (*b*) decreasing its tension, (*c*) raising its temperature? (L)

8. One musical note differs from another in pitch, loudness and quality. Explain the meaning of these characteristics of a musical note, giving experimental evidence to illustrate your explanation. (L)

9. A piano and a violin string are tuned and middle C is played on both instruments. In what respect may the notes differ and in what respect may they agree? Explain carefully the meanings of the terms you use. (L)

Advanced level.

10. Explain why the motion of a source of sound affects its pitch as heard by a stationary observer. How can the phenomenon be demonstrated in the classroom?

What is the velocity of the source along the line joining the source to the observer if, as a result of the motion, the frequency of the note heard is (*a*) increased in the ratio 16 : 15, (*b*) decreased in the ratio 15 : 16? Assume the velocity of sound in air is 1,120 ft. per sec. and give the results in feet per second.

Derive any formula employed. (N)

11. Derive expressions showing how the apparent frequency of a note heard by an observer is affected by (*a*) motion of the source, (*b*) motion of the observer, in each instance the motion being along the line of propagation of sound. A motor-car is fitted with twin horns differing in frequency by 256 vibrations per second. Calculate the difference of frequencies of the notes heard by an observer when the car, sounding its horns, is approaching him at 40 m.p.h. Velocity of sound is 1,120 feet per second. (N)

12. (*a*) As two trains are approaching each other, one travelling at 50 m.p.h. and the other at 30 m.p.h., the whistle of the former is sounded. The frequency of the whistle is 500 per second. Explain why the driver of the other train hears a note of different frequency and calculate its value. Would this be altered if the speeds of the trains were interchanged?

Velocity of sound in air = 1,120 feet per second.

(*b*) How has the analogous effect in light been used in astronomy? (N)

13. Give a general account of the Doppler effect, with examples in sound. Consider the bearing which the velocity of the wind may have on the effect.

A plane is travelling horizontally in a straight line in still air with a velocity of 400 ft. per sec. Find the percentage difference between the frequency of the sound heard by an observer on the ground at the instant when he sees the plane vertically overhead and the true frequency.

Assume the velocity of sound in air to be 1,120 feet per second. (N)

14. Explain the effect of the motion of the source and of the observer on the apparent pitch of a note.

An express train blowing its whistle travels through a station, and an observer standing on the platform notices that as the engine passes him the drop in pitch of the whistle corresponds to an interval of 6/5. What is the speed of the train? (Speed of sound in air = 1,100 ft. per sec.) (O and C)

15. Explain why the note emitted by a closed organ pipe differs in quality from that of a note emitted by (*a*) an open pipe, (*b*) a violin.

The fundamental note emitted by a stretched string vibrating transversely between two bridges 50 cm. apart is in unision with that of an organ pipe when the temperature is 12° C. Find approximately the change in length between the bridges necessary to restore unison when the temperature of the air in the pipe rises to 20° C., the tension in the wire remaining unaltered. (N)

Scholarship level.

16. Give an account of the formation of beats and deduce an expression for the frequency of the beats in terms of the frequencies of the sources.

Two trains approach one another along the same line, each moving with a velocity of 10 ft. per sec. A whistle of frequency 250 per sec. is sounded on each train. Find the frequency of the beats between the two sources as heard by an observer on either train. (C)

17. Explain what is meant by the Doppler effect and obtain an accurate expression for it when an observer is moving towards or away from a fixed source.

An object is dropped from an aeroplane travelling horizontally in a straight line with a velocity of 250 kilometres per hour at a height of 5 kilometres. Immediately the object strikes the ground a hooter near the place where it strikes is sounded and emits a note of frequency 300 vibrations per sec. Neglecting effects due to air resistance, calculate the time that elapses between the moment the object leaves the aeroplane and that at which the pilot first hears the hooter, and also the initial frequency of the note heard.

($g=980$ cm. sec.$^{-2}$, Velocity of sound in air = 330 m./sec.) (C)

18. A source emitting sound of natural frequency n moves in a straight line l with constant velocity v. A stationary observer is at a point not on the line. Prove that, to him, the apparent frequency at any instant is $nc/(c-v\cos\theta)$, where c is the velocity of sound and θ the angle between l and the line joining observer and source.

Calculate the rate of change of frequency at this instant and show that if the source passes *close* to the observer he will notice a *sudden* drop of pitch. (O)

19. A man is standing beside a railway line observing the whistle of a passing train. The whistle, which has a natural frequency of 1000 cycles/sec., suffers an apparent change of frequency of 100 cycles/sec. as it passes the observer. What is the speed of the train?

(Velocity of sound = 1100 ft./sec.) (C)

20. A source of sound gives out waves of frequency n. Discuss the effect of motion (*a*) of the source, (*b*) of the observer, (*c*) of both the source and the observer, on the frequency of the sound as heard by the observer. The motion in each case may be assumed to be along the line joining the source and observer.

A train is moving with uniform velocity v on a straight track

between two bridges A and B over the track, the motion being towards A. An observer on the train hears the echo of the train's whistle reflected from each of the bridges. If the velocity of sound is V, find the ratio of the wave-lengths of the waves reflected from A and B and the ratio of the frequencies of the echoes heard by the observer. (C)

21. Give an account of the Doppler effect in sound and light.

A man directs a beam of sound of frequency 12,000 cycles/sec. at an approaching motor car. He listens to the beats between the reflected sound and that emitted. What is the frequency of these beats if the car is travelling at 30 m.p.h. (44 ft./sec.)? (C)

22. Obtain the formula for the Doppler effect when the source is moving with respect to a stationary observer. Give examples of the effect in sound and light.

A whistle giving out 500 vibrations per sec. moves away from a stationary observer in a direction towards, and perpendicular to, a flat wall with a velocity of 5 ft./sec. How many beats per second will be heard by the observer? (Take the velocity of sound as 1,120 ft./sec. and assume there is no wind. (C)

23. How does the frequency of the sound received by a stationary observer depend on the motion of the source of the sound?

What is the effect if both source and receiver are in motion?

Two cars are approaching one another along two straight roads at right angles. Both cars are travelling at 30 miles/hr. and car A emits a note of frequency 256 vibrations/sec.

Determine the frequency of the sound heard by an observer in car B when the line joining the two cars makes an angle of 45° to the two roads.

(Assume that the velocity of sound in still air = 1,142 ft./sec.) (B)

24. What do you understand by the Doppler effect?

A whistle of 1,000 cycles/sec. pitch is attached to one end of a light tube 2 ft. long so that it can be sounded while the tube is rotating freely in a vertical plane about a horizontal axis through the other end. If the velocity of the whistle when the tube is horizontal is 16 ft./sec., find the upper and lower limits of the pitch of the sound heard by an observer on the ground viewing the motion end-on.

(Velocity of sound = 1,120 ft./sec.) (O)

25. What do you understand by the Doppler effect? A seaplane flying at 300 m.p.h. close to the sea passes an observer a quarter of a mile off. Find the direction from which the observer will hear the sound when the seaplane is nearest to him and show how the pitch of the sound will vary as it passes him.

(Velocity of sound is 1,100 ft./sec.) (O)

26. Discuss *pitch* and *tone* in music from the standpoint of physics, illustrating your discussion by an analysis of the sound produced by at least three musical instruments. (O)

27. Notes of the same frequency produced by different musical instruments are said to be of different quality. Give a physical description of this difference and say how it could be investigated experimentally. (O)

CHAPTER VIII

Ordinary level.

1. Explain what is meant by resonance in sound.

A vibrating tuning fork is held just above the top of a tall narrow jar full of water. The water is gradually run out of the jar and when the length of the air column above the water is 33 cm., the column is found to resound loudly. Describe carefully the state of disturbance of the air at different points in the column under these conditions.

Given that the velocity of sound in air is 33,800 cm. per sec., calculate the frequency of the tuning fork. Ignore any correction for the diameter of the jar. (C)

2. A long vertical brass tube, open at the top, contains water the level of which can be adjusted. Explain carefully how the air in the tube can be set into resonant vibration by means of a tuning fork.

When a fork of frequency 512 is sounded, the difference in level of the water between two successive positions of resonance is found to be 33 cm. What is the velocity of sound in air? (N)

3. Describe how you would use an air column of variable length to compare the frequencies of the tuning forks.

If an air column 13·2 cm. long, closed at one end, resounds to a tuning fork of frequency 256, what is (*a*) the wave-length of the note emitted by the fork, and (*b*) the velocity of sound in air at the temperature of the experiment? (N)

4. Being provided with a resonance tube, a tall gas-jar full of water, and a tuning fork of frequency 512 cycles per sec., how would you determine the frequency of an unmarked tuning fork?

A resonance box is to be made for use with a tuning fork of frequency 439 cycles per sec. when the velocity of sound in air is 341 metres per sec. What must be approximately the shortest length of the box if it is to be closed at one end? (L)

5. Explain the phenomenon of the resonance tube. A vertical tube, 1 metre long, is filled with water which is allowed to run out gradually from the bottom. For how many positions of the water surface will it be possible to obtain resonance with a tuning fork of frequency 512?

(The velocity of sound may be taken as 330 metres per sec.) (C)

6. What is meant by the wave-length of a note sounded in air? Explain how it is affected by (*a*) an increase in the frequency of the note, (*b*) a rise in temperature of the air.

Describe how you would determine the frequency of a tuning fork from observations of the resonance of an air column, the velocity of sound in air being known. (N)

7. What is meant by *resonance* in the theory of sound?

Using sketches where appropriate describe:

(*a*) a mechanical experiment which illustrates your explanation of resonance;

(*b*) an experiment with a stringed instrument (for example, sonometer or piano) in which resonance is demonstrated; and

(*c*) an experiment in which a column of air shows the phenomenon of resonance.

Name *one* musical instrument which makes use of the effect demonstrated in (*c*). (N)

8. A tuning fork is sometimes mounted on a resonance box. Explain the action of the box.

The boxes of two similar forks mounted in this way are placed in line with the open ends together. One of the forks is set into vibration and then stopped. Explain why the sound still persists.

What happens if the end of one fork is loaded with wax and both forks are then sounded together? (L)

9. What do you understand by resonance? Describe experiments you would make in the laboratory to show under what conditions resonance occurs. (O, part)

10. Explain what is meant by resonance, giving two examples.

Calculate approximately the length of the resonance box, closed at one end, suitable for a tuning fork of frequency 384. The velocity of sound in air may be taken as 1,120 ft. per sec. (L)

Advanced level.

11. What is meant by the term *resonance* in sound? Give examples of its application in musical instruments. Describe in particular the operation of an organ-pipe, and explain why an open pipe has a fundamental frequency which is approximately twice that of a closed pipe of the same length.

Discuss the use of a sounding-board in a piano. Does it make use of resonance effects? (O and C)

12. Distinguish between *free, forced,* and *resonant* vibrations, giving *one* example of each.

A student, performing an experiment with a tuning fork and a resonance tube closed at one end, obtained resonance when the length of the air column was 49·5 cm. and again when it was 82·9 cm. Comment on these results and calculate the frequency of the tuning fork if the temperature of the air was 15° C. and the velocity of sound in air at 0° C. is 33,150 cm. per sec. (N)

13. Give an account of the modes of vibration of open and closed columns of air. A pop-gun consists of a cylindrical barrel 3 sq. cm.

in cross-section closed at one end by a cork and having a well-fitting piston at the other. If the piston is pushed slowly in, the cork is finally ejected, giving a " pop " the frequency of which is found to be 512. Assuming that the initial distance between the cork and the piston was 25 cm. and that there is no leakage of air, calculate the force required to eject the cork. (The atmospheric pressure may be taken as 1 kgm. per sq. cm. and the velocity of sound in air as 340 metres per second.) (C)

14. Explain what is meant by " resonance ", illustrating your answer with *one* mechanical and *one* acoustical example.

How may the frequency of a tuning fork be found by a resonance tube method when the velocity of sound in air at 0° C. is known? Discuss briefly how the observations and the final result would be affected by a rise in temperature. (N)

15. Describe and explain how, being provided with a tuning fork of known frequency, you would determine the velocity of sound in air at 0° C.

An expression for the velocity of sound in a gas was first obtained by Newton. Later on his expression was modified by Laplace. How was Newton's expression defective and how did Laplace correct it? (N)

16. How would you determine the velocity of sound in air by means of a vertical tube which is partly filled with water?

It is found that the shortest length of such a tube, with its upper end partly obstructed, which will resound to frequencies of 200, 300, 400 and 600 vibrations per second are 37·50, 23·67 and 9·83 cm. respectively. Show that these results may be represented by a linear graph. Determine from the graph, or otherwise, the velocity of sound in air and the end correction for the tube. (L)

17. Write an essay on resonance, illustrating your answer by examples from various branches of physics. (N)

18. Explain the meaning of the term resonance, giving in illustration two methods of obtaining resonance between the stretched string of a sonometer and a tuning fork of fixed frequency. A sonometer wire of length 76 cm. is maintained under a tension of 4 kg. weight and an alternating current is passed through the wire. A horse-shoe magnet is placed with its poles above and below the wire at its mid-point, and the resultant forces set the wire in resonant vibration. If the density of the material of the wire is 8·8 gm. per c.c. and the diameter of the wire is 1 mm., what is the frequency of the alternating current? (L)

19. Explain the phenomenon of resonance, and illustrate your answer by reference to the resonance tube experiment. In such an experiment with a resonance tube the first two successive positions of resonance occurred when the lengths of their columns were 15·4

and 48·6 cm. respectively. If the velocity of sound in the air at the time of the experiment was 34,000 cm./sec., calculate the frequency of the source employed and the value of the end correction for the resonance tube. If the air column is further increased in length, what will be the length when the next resonance occurs? (W)

20. Describe the way in which the air layers in different parts of an open organ pipe are vibrating when the pipe is sounding its fundamental note. What other modes of vibration are possible for this pipe?

An open organ pipe in which the air is at a temperature of 15° C. and a sonometer wire of frequency 512 vib./sec., when sounded together, give 5 beats/sec., the organ pipe emitting its fundamental note. If a slight reduction in the tension of the sonometer wire produces unison between the two notes, what change in the temperature of the air in the organ pipe would have produced unison with the original frequency of the sonometer wire? (W)

21. Explain in general how stationary undulation is produced and state the features of this kind of motion.

A resonance tube has a jagged end and it is used to find the velocity of sound in air. A tuning fork of frequency 250 causes it to resound when it is filled with water to a mark 28 cm. below a reference mark near the open jagged end. A fork of frequency 500 causes resonance when the water reaches a mark $11\frac{1}{2}$ cm. below the reference mark. Both these resounding lengths are the shortest possible with these forks. Find the velocity of sound from these results and also the position of the antinode at the upper end in relation to the mark. (L)

22. Distinguish carefully between progressive and stationary waves and explain the terms node and antinode.

How would you demonstrate the existence of nodes and antinodes in an open organ pipe sounding its first overtone?

The shortest length of a resonance tube closed at one end which resounds to a fork of frequency 256 is 32·0 cm. The corresponding length for a fork of frequency 384 is 20·8 cm. Calculate the end correction for the tube and the velocity of sound in air. (N)

Scholarship level.

23. Describe what experiments you would perform to find the frequency of an organ pipe.

An open organ pipe of length l stands close to a second organ pipe of the same diameter which is longer by a small amount x. When both pipes are sounded, show that the number of beats per second is approximately $\frac{Vx}{2l^2}$, where V is the velocity of sound in air. (O)

24. Describe a manometric flame, and the mode of applying it to investigate the condition of the air in a sounding pipe.

Show that the timbres of two pipes, the one open and the other closed, which sound the fundamental, are necessarily different. (O)

25. The note which is emitted when the cork is rapidly withdrawn from an empty bottle is due to the air in the neck acting as a piston to the air in the bottle itself, which contracts and expands adiabatically as a whole.

Find the note at N.T.P. emitted from a Winchester quart bottle ($2\frac{1}{2}$ litres) the length and diameter of whose neck are each 2 cm.

Density of air at N.T.P. $= 1{\cdot}293$ gm. per litre.

One atmosphere $= 10^6$ dynes per sq. cm. $\gamma = 1{\cdot}41$. (O)

26. What are the resonance frequencies of the air in a narrow pipe one metre long open at both ends at 20° C. and 740 mm. of mercury?

(Ratio of specific heats of air $= 1{\cdot}4$.
Density of air at N.T.P. $= 0{\cdot}0013$ gm./c.c
Density of mercury $= 13{\cdot}6$ gm./c.c.
Value of g $= 981$ cm./sec.^{2}) (C)

CHAPTER X

Ordinary level.

1. A cog-wheel having 25 teeth is rotated 3 times per sec., and a thin strip of metal is fixed so that it is struck by the cogs. Assuming that the velocity of sound is 33,150 cm. per sec., what is the wave-length of the note emitted? (O)

2. Describe a method of measuring directly the frequency of vibration of a tuning fork. (A value for the velocity of sound in air must not be assumed.)

A fork of unknown frequency gives 3 beats per second when sounded with another of frequency 256 cycles sec.$^{-1}$ The fork is then loaded with a piece of wax and it again gives 3 beats per second when both forks are sounded together. Account for this result. (L)

3. What is the meaning of the terms *frequency* and *wave-length* as applied to sound? Describe an experiment you would make to compare the frequencies of two tuning forks.

If the wave-length of sound is 4 ft. and the velocity of sound in air is 1100 ft. per sec., what is the frequency? (B)

4. Describe the dropping-plate method for determining the absolute pitch of a tuning fork. In a particular experiment the plate dropped from rest through a distance of 5 cm., and while it was dropping through the next 15 cm. 25 vibrations were made by the fork. Calculate the frequency of the fork. (C)

5. Give an account of some method of determining the frequency of a tuning fork, and show how the result would be obtained from the observations made.

Two tuning forks have frequencies 256 cycles sec.$^{-1}$ and 252 cycles

sec.$^{-1}$ Describe and explain what is heard when the two forks are sounded together. (L)

6. Define *frequency*. What are the factors which control the frequency of the note emitted by a sonometer wire when this is plucked or bowed?

Describe and explain a method of comparing the frequencies of two tuning forks. (L)

7. What do you understand by *frequency* of a tuning fork? Describe an experiment to determine its value.

If the velocity of sound in air is 1,100 ft. per sec., what is the wavelength of the sound emitted by a fork of frequency 256? How would the value be affected by a rise of temperature? (L)

Advanced level.

8. Describe and explain the determination of frequency by (i) Kundt's tube method, (ii) the dropping plate method, and give an indication of the frequencies for which each method is suitable.

In what respects do musical tones differ from one another, and what is the physical explanation of such differences? (N)

9. How does the pitch of a note emitted by a stretched string depend upon (*a*) the stretching force, (*b*) the length of the string, (*c*) the mass of the string per unit length?

Describe experiments which could be made with a sonometer to test the truth of your statements.

How would you cause a stretched wire to emit its different harmonic overtones? (O and C)

10. One end of a long thin wire is attached to the prong of a tuning fork. The other end passes over a pulley and is attached to a weight Draw a diagram of the apparatus and discuss briefly the ways in which the wire may vibrate with the same frequency as the fork when the load is varied.

If the weight is 250 gm., eight segments are formed in 420 cm. length of the wire. If the mass of this length is 1·8 gm., what is the frequency of the fork? (L)

11. Assuming the expression for the velocity of transverse waves on a stretched string, viz. $V=\sqrt{\frac{T}{m}}$, where T is the tension in the string and m the mass per unit length, deduce the relation for the fundamental frequency of transverse vibrations of a string fixed at both ends.

Describe how you would investigate experimentally the relation between frequency and tension.

Two identical wires A and B are attached to a sonometer and are stretched by nearly equal weights, the vibrating length being in each case 100 cm. A has a frequency of 104 and B is loaded with 10 kilos.

When both wires vibrate, four beats are heard per second. How would you determine, experimentally, which wire has the higher frequency? If A has the higher frequency, what change must be made in (i) the tension, (ii) the length of B in order that the wires may vibrate in unison? (N)

12. Describe the phenomenon of beats and show that the beat frequency is the difference of the frequencies of the tones producing the beats. Describe one quantitative use of the phenomenon.

A certain fork is found to give two beats per second when sounded in conjunction with a stretched string vibrating transversely under a tension of either 10·2 or 9·9 kgm. wt. Calculate the frequency of the fork. (N)

13. Describe in detail how to determine the frequency of a tuning fork by the falling plate method.

Describe how the frequency of an alternating current, such as that from the A.C. mains, may be determined with a sonometer. (N)

14. Describe a laboratory apparatus for investigating the laws of vibration of stretched strings, and give an account of its use. Explain the fact that the same string under precisely the same physical conditions may emit sounds of different quality, according to the manner in which it is brought into vibration.

Two notes on a piano are four octaves apart. If the lower note is given by a wire 135 cm. long and the higher by a wire 40 cm. long, and if the tensions are the same, calculate the relative mass of each wire per unit length. (W)

15. It is found that when the length l and the tension T of a string are varied so as to keep the frequency of transverse vibration of the string constant, Tl^p is constant, p being a numerical index. Describe how you would verify the result.

Two wires A and B, made of the same metal, of equal lengths and with diameters in the ratio 7 : 4, when set into transverse vibration give notes whose frequencies are in the ratio 4 : 5. These notes are brought into unison when the tension in B is diminished by 1·5 kg. wt. Find the tension in A and the original tension in B. (N)

Scholarship level.

16. Describe in detail the stroboscopic method of determining the frequency of vibration of a tuning fork. Outline *two* other methods which are available for measuring the frequency and discuss the respective merits of all the methods you have described.

An outer ring of 12 dots and an inner ring of 8 dots, all uniformly spaced, are painted on a disc which can be rotated about its axis and which is viewed stroboscopically by means of a tuning fork of frequency 100 vibrations per second, the disc being seen once in each vibration of the fork. Calculate the minimum rate at which the disc

must be rotated so that the outer ring shall appear at rest with the normal spacing between the dots.

The disc is then speeded up until *both* rings appear to be at rest. Determine the rate of revolution when this first occurs and describe and account for any peculiarity in the appearance of the outer ring.

Mention some other application of the stroboscope principle. (N)

MISCELLANEOUS QUESTIONS

Ordinary level.

1. Describe an experiment to show that sound does not travel through a vacuum. A normal human ear responds to sound waves with wave-lengths ranging from 10 metres to 0·02 metre. Assuming the velocity of sound in air to be 340 metres sec.$^{-1}$, calculate (*a*) the frequencies corresponding with these limiting wave-lengths, (*b*) a value for the number of octaves in the range of audibility. (L)

2. Describe how the velocity of sound in air can be measured.
What is the wave-length in air of a sound with a frequency 400 when the velocity of sound is 1,100 feet per second?
Two notes of frequency 256 and 258 are sounding simultaneously Describe and explain what you hear. (C)

3. Describe how the velocity of sound in air may be determined without the use of a resonance tube.
The distance in miles of a lightning flash from an observer may be found approximately by counting the number of human pulse beats between the lightning flash and the thunder-clap and dividing the number by 6. Justify this rule, assuming that the human pulse beats 75 times per minute and that the velocity of sound is 1080 ft. per sec. (C)

4. Explain briefly how echoes arise. A man standing before a high cliff fires a gun and hears the echo from the cliff 1·6 sec. later. If the velocity of sound in air is 1100 ft./sec., what is his distance from the cliff? (O and C)

5. Explain how the frequency of a vibrating string depends on (*a*) its length, (*b*) its tension.
Describe fully an experiment or experiments which would enable you to prove your statements correct. Give a drawing of the apparatus you would use. (O)

6. A musical sound can be made by blowing across the top of a tube. If you had eight equal test tubes, how would you use them to form an octave scale? (L, part)

7. The disc of a siren has 30 holes and makes 1200 revs. per min. Find a value for the shortest length of a closed pipe which, at a temperature of 0° C., will respond to the note of the siren. How

will this length change when the temperature rises to 27° C.? (The velocity of sound in air at 0° C. may be taken as 331 metres $sec.^{-1}$) (L)

8. Assuming that the velocities of sound in air and carbon dioxide at room temperature are 340 m. $sec.^{-1}$ and 266 m. $sec.^{-1}$ respectively, find the approximate frequency of the fundamental tone produced by (*a*) a tube of length 64·2 cm. open at both ends and containing air, (*b*) a tube of length 25·0 cm. closed at the lower end and containing carbon dioxide. Describe what would be heard if the gases in both tubes were in vibration simultaneously. (L)

9. What is meant by an echo and what conditions are necessary for its production? Give a practical application of the use of echoes.
What is *one* possible cause of the poor acoustical properties of many public halls and what steps could be taken to rectify it? (L)

10. How may a single stretched wire be used to compare the frequencies of two tuning forks? A thin iron wire is stretched with a known load and a length l is found to emit a note in unison with the note of a tuning fork. What length of iron wire of half the diameter will be in unison with the same fork when the thinner wire is stretched with a load equal to half that used on the first wire? (L)

11. Describe and explain an experiment to find the frequency of a tuning fork, using a long glass tube of variable length open at both ends. What difference would be made in the calculation of the result if the tube were closed at one end? (L)

12. Describe and explain what is heard when a bottle is filled with water from a tap. (L)

Advanced level.

13. State briefly how you would show by experiment that the characteristics of the transmission of sound are such that (*a*) a finite time is necessary for transmission, (*b*) a material medium is necessary for propagation, (*c*) the disturbance may be reflected and refracted.
The wave-length of the note emitted by a tuning fork, frequency 512 vibrations per second, in air at 17° C. is 66·5 cm. If the density of air at S.T.P. is 1·293 gm. per litre, calculate the ratio of the two principal specific heats of air. Assume the density of mercury is 13·6 gm. per c.c. (N)

14. What factors determine the velocity of sound (*a*) in a gas, (*b*) in a stretched string? Given that the ratio of the specific heats of oxygen at constant pressure and constant volume is 1·40 and the gas constant is $2{\cdot}62 \times 10^6$ ergs per gram per degree C., find the velocity of sound in oxygen at 27° C. (C)

15. (*a*) Explain Huyghen's construction for the propagation of waves in a medium and use it to show that a bullet travelling through

air with a uniform velocity u, greater than that of sound v, gives rise to a conical wave having the bullet at its apex and a semi-vertical angle $\sin^{-1} v/u$.

(*b*) Give a brief account and explanation of the audibility of sounds over great distances. (N)

16. Give an account of an experiment by which acoustic beats of adjustable frequency can be produced. Show how the beats are related to the frequencies of the oscillations from which they are formed. Give a diagram to show how the beats are produced and how their wave-form is related to that of the waves which produce them. (O)

17. Explain the terms "frequency", "wave-length", "amplitude", and "phase", as applied to wave-motion.

A long uniform rod of circular section is clamped at its centre and is caused to vibrate longitudinally by stroking with a resined cloth. Describe in detail how you would determine the frequency of the note emitted by the rod, assuming that the velocity of sound in air at 0° C. is known.

Explain the effect on the frequency of reducing the length of the rod to half its original value. (N)

18. Discuss the terms "intensity" and "loudness" as applied to sound.

A concert hall has a domed ceiling which is a portion of a sphere whose centre of curvature is at floor level. Assuming the source of sound to be at this level, explain why, from an acoustical standpoint, the design is not good and would be much improved by doubling the radius of curvature of the dome without altering its maximum height above the floor.

Explain why the acoustical properties of a concert hall may vary with the size of the audience. (N)

19. Theory shows that the fundamental frequency n of transverse vibration of a stretched string of length l, radius r, tension T and density ρ is given by $n = \frac{A}{rl}\sqrt{\frac{T}{\rho}}$, where A is a constant. Describe how you would use a sonometer and tuning forks of known frequency to verify this relationship. (O and C, part)

20. Describe and explain the way in which a Kundt tube may be used to determine the ratio of the specific heats of a gas. A Kundt tube is excited by a brass rod 150 cm. long and the distance between successive nodes in the tube is 13·6 cm.; what is the ratio of the velocity of sound in brass to that in air? (L)

21. Write a short essay on methods of recording and reproducing sound. (N)

Scholarship level.

22. What is meant by reverberation and reverberation time?
What factors determine the reverberation time of a concert hall and why does the magnitude of this time affect the suitability of the hall for speech and music?
Indicate briefly other aspects of the hall which affect its acoustical quality. (N)

23. Give an account of the factors which affect the audibility of distant sounds. (C)

24. Give a short account of the conditions affecting the propagation of sounds in the atmosphere. (C)

25. Give an account of the behaviour of sound waves of audible frequency in air, illustrating your answer by everyday phenomena. (C)

26. Write a short essay on gramophones. (O)

27. Write a short essay on " The Acoustics of Buildings ". (O)

28. What is meant by saying that a room has " good " or " bad " acoustical properties? Illustrate your answer by reference to a room with which you are familiar. (O)

29. Explain why :
(*a*) A wind instrument rises in pitch when warmed.
(*b*) The whistle of an approaching engine appears to have a higher pitch than the same whistle receding.
(*c*) There is a difference in tone between different types of musical instruments.
Suggest an experiment to test the correctness of your explanation in each case. (O)

ANSWERS

CHAP.				
I.	7. 2·4 sec.	10. 0·242 erg.		
II.	1. $\frac{5}{24}t$ miles.	3. (*b*) 4 sec., (*c*) 200 ft.	4. $1\frac{1}{24}$ miles.	
	6. 1158 ft. per sec.	7. 600 metres	8. 2 ft. per sec.	
	9. 38,830 cm./sec.	10. 4×10^5 cm./sec.	12. 2 vib./sec.	
	13. 53° 8′.	16. 130·4 c.p.s.	17. 2·85 sec.	18. 4,200 ft.
	19. 4,554 ft.	20. 900 m. ; 24° from second position.		
	21. 6,900 ft.	22. 344 m./sec.		
	25. $3{\cdot}1 \times 10^4$ cm./sec.	29. 10·7.		
III.	1. 450 yd.	4. 7 sec. ; 4,900 ft./sec.		
	5. 2,200 yd. ; 18·75 m.p.h.	6. (*a*) 1,100 ft./sec. ; (*b*) 550 yd.		
	7. 1,100 ft./sec.	8. 0·533, 0·397, 0·331 sec.		
IV.	2. Brass 131·6 sec.$^{-1}$, steel 136·6 sec.$^{-1}$	3. 300 sec.$^{-1}$		
	6. 33,700 cm. sec.$^{-1}$; 208 c.p.s. ; 210·6 c.p.s.	7. 39·2 gm. wt.		
	10. Open pipe twice length of closed one.	11. 0·00317 cm.		
	13. 4·4 sec.$^{-1}$			
V.	1. 4 kgm.	4. 16 : 9.	5. 216, 504 c.p.s.	6. 450 gm.
	7. 1·2 : 1.	8. 12 cm. away from *A*.	11. 639·3 c.p.s.	
	12. 435 c.p.s.	13. 1 : 1·061.		
	14. 2,420 c.p.s., $1{\cdot}8 \times 10^{12}$ dynes cm.$^{-2}$			
	15. String 512 c.p.s. (damped in middle) ; 768 c.p.s. (plucked in middle). Air column : 768 c.p.s.			
	16. 1·3, 1%.	17. 1 : 40.	18. 4·8 cm.	
	19. $4\frac{1}{6}$ kgm. wt., $8\frac{1}{6}$ kgm. wt.	20. 4,961 sec.$^{-1}$.		
	21. 4·25 sec.$^{-1}$.	22. 56·2 kgm. wt.	23. Also 100 cm.	
	24. 3·35%.			
VI.	2. 4 ft. 3. $3\frac{2}{3}$ ft.	10. (*a*) $74\frac{2}{3}$ ft. sec.$^{-1}$, (*b*) 70 ft. sec.$^{-1}$	11. 269.	
	12. 556·2 c.p.s., 554·6 c.p.s.	13. 14·6%.	14. 100 ft. sec.$^{-1}$	
	15. 7 mm. decrease.	16. 4·6 sec.$^{-1}$		
	17. 50·63 sec., 262·5 c.p.s.	19. 55 ft. sec.$^{-1}$		
	20. $\frac{\lambda_A}{\lambda_B}=\frac{V-v}{V+v}, \frac{n_A}{n_B}=\frac{(V+v)^2}{(V-v)^2}$.	21. 960 c.p.s.		
	22. $4\frac{1}{2}$ beats sec.$^{-1}$	23. 270 c.p.s.		
	24. Either 1,018 and 990 c.p.s. ; or 1,010 and 983 c.p.s.			
	25. $\sin^{-1} \frac{2}{5}$.			

CHAP.

VIII. 1. 256 c.p.s. 2. 337·9 metres/sec.
3. (*a*) 52·8 cm., (*b*) 135·2 metres/sec. 4. 19·4 cm.
5. 3 10. 9·7 in. 12. 510 c.p.s. 13. 1·52 kgm.
16. 332 metres/sec. 18. 49·6 c.p.s.
19. 527 c.p.s., 1·2 cm., 81·8 cm. 20. 5·7° C.
21. 330 metres/sec.; 5 cm. above mark.
22. 1·6 cm., 344 metres/sec. 25. 131·7 c.p.s.
26. 171 c.p.s. × 1, 2, 3 . . .

X. 1. 442 cm. 3. 275 c.p.s. 4. 247·6 c.p.s. 7. 4·3 ft.
10. 72 c.p.s. 11. (i) 0·82 kgm. increase, (ii) 3·8 cm. decrease.
12. 264·2 c.p.s. 14. 2·2 : 1.
15. $A = 8$ kgm. wt., $B = 4$ kgm. wt.
16. $8\frac{1}{3}$ rev. sec.$^{-1}$, $12\frac{1}{2}$ rev. sec.$^{-1}$, 24 dots seen on outer ring.

MISCELLANEOUS QUESTIONS

1. (*a*) 34 and 17,000 c.p.s., (*b*) 9 (approx.). 2. 2·78 ft.
4. 880 ft. 7. 13·79 cm.; new length 14·45 cm.
8. (*a*) 265 c.p.s. (approx.), (*b*) 266 c.p.s. (approx.), 1 beat sec.$^{-1}$.
10. $\sqrt{2l}$. 13. 1·39. 14. 332 metres/sec. 20. 11·0 : 1.

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